Citation for published version:
A. Bannour and Yichuang Sun, “Duality of antennas and subcarriers in massive MIMO-OFDM downlink system”, *Electronics Letters*, Vol. 51(14): 1115-1117, July 2015.

DOI:
10.1049/el.2015.0355

Document Version:
This is the Accepted Manuscript version.

Duality of Antennas and Subcarriers in Massive MIMO-OFDM Downlink System

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Massive multiple-input multiple-output (MIMO) can outperform conventional MIMO in terms of spectrum efficiency and link reliability significantly . For massive MIMO, there are still theoretical and practical issues that have to be addressed. In this work, we derive and analyze the capacity of massive MIMO-OFDM downlink system and demonstrate the duality of antennas and subcarriers in such system analytically and by simulation. A detailed comparison between massive MIMO, massive MIMO-OFDM and MIMO-OFDM with large subcarriers is presented.

Introduction: To satisfy the demands for high throughput communications massive numbers of antennas (massive MIMO) [1, 2] can be adopted for 5G systems, since massive MIMO helps to reduce transmit power, noise and fast fading. The reliability and spectral efficiency are much more improved compared to conventional MIMO techniques. Large-scale MIMO can also provide a real capacity enhancement. However there are a number of problems that need to be solved such as the estimation of large number of channels between n_t transmit antennas and n_r receive antennas, the use of tremendous pilot overhead which results in pilot contamination and the increase of the computational power. While the theoretical aspects of massive MIMO systems have gained significant attention in the research community [3, 4], much less is known about practical transmission schemes. As pointed out in [5], practical realizations of large-scale MIMO systems will require the use of low cost and low-power radio-frequency (RF) components. Practical wireless channels typically exhibit frequency selective fading, which requires high cost RF component. The combination of massive MIMO with Orthogonal frequency division multiplexing OFDM can overcome the frequency selective problems and reduce the burden on RF components.

This paper discusses massive MIMO-OFDM based on capacity and achievable rate. The contribution of the research are :

- Analytic capacity of massive MIMO-OFDM system is derived.
- We prove theoretically and by simulation the duality between the number of antennas in MIMO and the number of subcarriers in OFDM.
- We show the benefit of MIMO-OFDM with high subcarrires compared to massive MIMO and massive MIMO-OFDM.

System Model: The MIMO-OFDM system in baseband model, equipped with n_t transmit antennas and n_r users antenna is considered $(n_r = n_t)$. To simplify the discussion we give the overall mathematical model as presented in [6]. MIMO-OFDM symbols are supposed to be transmitted over frequency and time selective Rayleigh channel. The channel taps are assumed constant when transmission occur. The channel impulse response (CIR) distribution follow the Rayleigh law between the q^{th} transmitting antenna and p^{th} receiving antenna and $\boldsymbol{h}_k^{p,q} \sim \mathrm{Raylei}(\sigma_h\sqrt{\frac{\pi}{2}}, \frac{4-\pi}{2}\sigma_h^2)$. Note that $h_k^{p,q}(l)$ is the l^{th} path from the q^{th} transmitting antenna to the p^{th} receiving antenna at time k and L is the largest order among all impulse responses. The mathematic model of the k^{th} MIMO-OFDM symbol at the p^{th} receiving antenna is

$$\boldsymbol{y}_k^p = \sum_{q=1}^{n_T}\sum_{l=0}^{L-1} h_k^{p,q}(l)\boldsymbol{u}_k^q(k-l) + \boldsymbol{w}_k^p \tag{1}$$

where $\boldsymbol{u}_k^q$ is the symbol vector transmitted by the q^{th} antenna and $\boldsymbol{w}_k^p$ is zero mean white Gaussian complex noise of variance $\frac{N_0}{2}$.

Capacity of Massive MIMO-OFDM System: The mathematic expression of MIMO-OFDM system using the maximization of mutual information is presented in detail in a previous work by *Bannour et al* in [6–8]. To achieve the maximum capacity for a large number of antennas, one can derive the theoretical limit of the capacity $\boldsymbol{C}$ as when n_t goes to infinity as :

$$\begin{aligned}\lim_{n_t\to+\infty}\boldsymbol{C} &= \lim_{n_t\to+\infty}\log_2\det\left(\boldsymbol{I}_{n_cn_t} + \frac{1}{\sigma_w^2}\boldsymbol{H}^H\boldsymbol{H}\right) \qquad (2)\\ &= \lim_{n_t\to+\infty}\log_2\det\left(\boldsymbol{I}_{n_cn_t} + \frac{n_cn_t}{\sigma_w^2}\frac{\boldsymbol{H}^H\boldsymbol{H}}{n_cn_t}\right)\end{aligned}$$

Based on the big number law and large scale random matrix theory one can develop :

$$\begin{aligned}\lim_{n_t\to+\infty}\frac{\boldsymbol{H}^H\boldsymbol{H}}{n_cn_t} &= \lim_{\underbrace{n_tn_c}_{N}\to+\infty}\frac{\boldsymbol{H}^H\boldsymbol{H}}{n_cn_t} \qquad (3)\\ &= \lim_{N\to+\infty}\frac{\boldsymbol{H}^H\boldsymbol{H}}{N}\\ &= \frac{\left[\mathrm{tr}(\|\boldsymbol{H}\|^2)\right]}{N}\boldsymbol{I}_N\\ &= \sigma_h\sqrt{\frac{\pi}{2}}\boldsymbol{I}_N\end{aligned}$$

So when combining (3) and (2) one can get:

$$\begin{aligned}\lim_{n_t\to+\infty}\boldsymbol{C} &= \log_2\det\left(\boldsymbol{I}_{n_cn_t}(1+\frac{n_cn_t}{\sigma_w^2}\sigma_h\sqrt{\frac{\pi}{2}})\right) \qquad (4)\\ &= n_cn_t\log_2\left(\frac{\sigma_h}{\sigma_w^2}\sqrt{\frac{\pi}{2}}n_cn_t\right)\end{aligned}$$

If we set $\sigma_h = 1$ then:

$$\lim_{n_t\to+\infty}\boldsymbol{C} = n_cn_t\log_2\left(\frac{\sqrt{\frac{\pi}{2}}}{\sigma_w^2}n_cn_t\right) \tag{5}$$

The capacity performed over n_cn_t narrowband channels can be written as

$$\lim_{n_t\to+\infty}\boldsymbol{C}_{AV} = \log_2\left(\frac{\sqrt{\frac{\pi}{2}}}{\sigma_w^2}n_cn_t\right) \tag{6}$$

From equation (6), one can conclude that a small antenna number n_t can achieve similar performance as a large antenna number when an OFDM modulator is included due to the factor of n_cn_t. Rather than focusing on the antenna number, one can target the number of subcarrier n_c, in another word we speak about (n_c, n_t) duality. Without loss of generality, we start with the couple $(n_t, n_c)_{n_t\to\infty}$ at this stage and we can inverse the situation to $(n_c, n_t)_{n_c\geq\beta}$ where β is a big subcarrier number. In general, a very large number of antenna will result in much more RF modules, including Low Noise Amplifier (LNA), frequency down-converter, and analog-to-digital converter (ADC). To reduce the cost associated with RF modules, the OFDM technique can be used to achieve the needed capacity with a smaller number of antennas.

Simulations: The Cumulative Complementary Density Function (CCDF) is used as tool to give the probability that the capacity C is larger than the capacity abscissa C_x. Fig. 1 depicts the CCDF of the capacity developed in (6), for massive MIMO, MIMO-OFDM and massive MIMO-OFDM configurations.

In Fig. 1, the CCDF windows is spread for up to 10dB. The capacity is larger than 18 bits/s/Hz, 24 bits/s/Hz and 32 bits/s/Hz respectively for massive MIMO, MIMO-OFDM and massive MIMO-OFDM for 90% of probability. The capacity increase when the couple (n_c, n_t) increase. We note gap of 10 bits/s/Hz at 60% of probability, between massive MIMO and massive MIMO-OFDM and a negligible gap between massive MIMO-OFDM and conventional MIMO-OFDM with high number of subcarriers $n_c = 1024$. It could be noticed that for high number of subcarrier n_c the high number of antennas is no longer needed, because we can achieve the theoretical limit of the downlink capacity with reduced antenna number due to the n_cn_t effect.

Fig. 2 shows clearly the impact of subcarrier number on the capacity. We notice 10 bits/s/Hz gap at 600 antennas between massive MIMO and MIMO-OFDM with high n_c values, and a

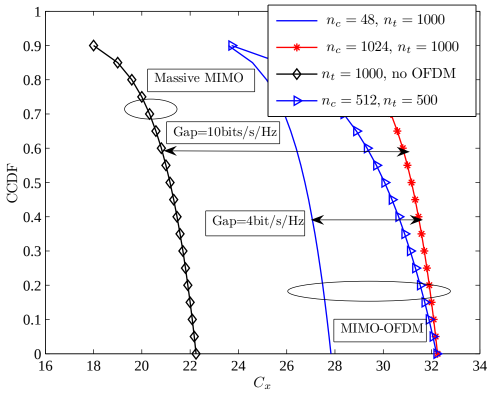

Fig. 1 *Complementary Cumulative Density Function of massive MIMO, MIMO-OFDM and massive MIMO-OFDM, SNR=10dB*

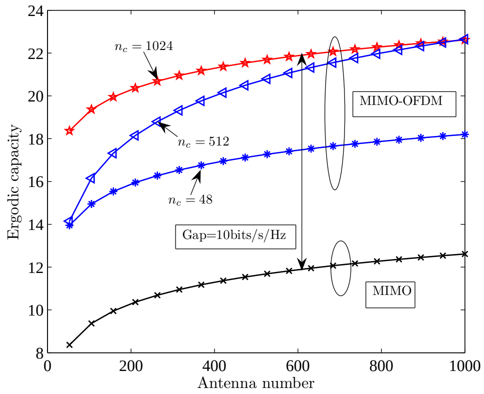

Fig. 2 *Asymptotic behavior of capacity for massive MIMO, MIMO-OFDM and massive MIMO-OFDM, SNR=10dB with respect to antenna number*

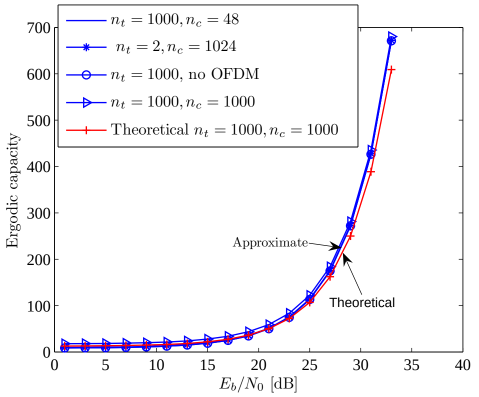

Fig. 3 *Capacity for massive MIMO, MIMO-OFDM and massive MIMO-OFDM with respect to SNR*

negligible gap between massive MIMO-OFDM and MIMO-OFDM with high n_c.

In Fig 3, for both the approximate capacity expression, as function of SNR per receive antenna, derived in (6) and the theoretical result using (2), is shown that the match between the two gets closer as the SNR increases.

Fig. 4 illustrates the impact of subcarrier number on the capacity for both small and very large numbers of antennas. We notice that the capacity increases asymptotically as the number of subcarrier increases. It can be observed also, that the gap between the two curves get wide as the number of subcarriers increases and increasing the number of subcarriers is more effective for large number of antennas than a small number of antennas because the duality of subcarriers and antennas only exists when when the number of antennas is very large.

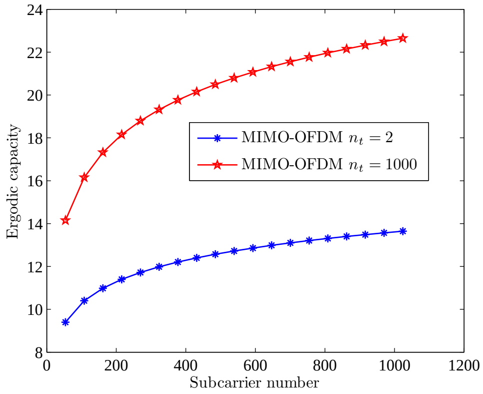

Fig. 4 *Capacity for MIMO-OFDM and massive MIMO-OFDM with respect to subcarrier number, SNR=10dB*

Conclusion: We studied a MIMO-OFDM configuration in massive MIMO context. It has been shown analytically that in rich scattered channels, MIMO-OFDM can achieve the same capacity efficiencies with high subcarrier number n_c, as those using the recently proposed massive MIMO scheme. Simulations showed the effectiveness of the proposed method for the case where n_c is high and n_t still in conventional value. The proposed scheme can use a fewer transmit antennas combined with a high number of OFDM subcarriers to reduce the high cost of RF components in practical wireless systems.

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