CONTENTS

1. Background equations

1.1 Model

The forces acting on a control volume can be seen in Figure 1.1,

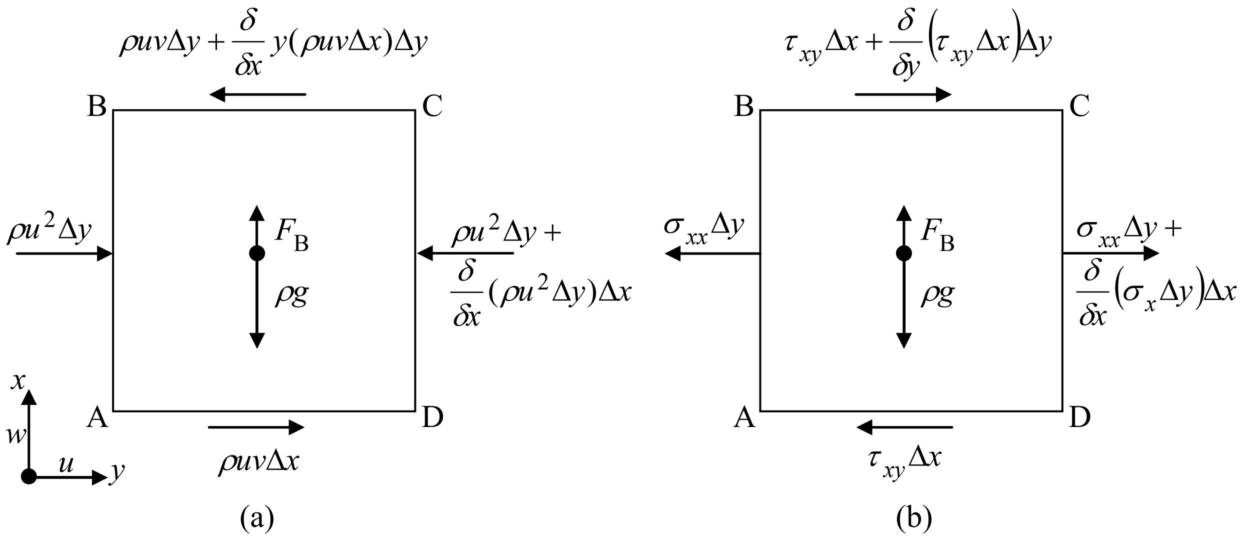

Figure 1.1: Control volume of a two dimensional buoyancy dominated flow. In (a), diagram of linear momentum force. Side AB: inflow momentum, side BC: momentum cross term in the opposite direction, side CD: reaction force opposing the momentum force and side DA: cross term in the direction of momentum force. In (b), shear force diagram corresponding to momentum forces.

If we analyse the forces acting on a control volume, the rate of change of momentum is equal to the forces acting on a control volume. Forces acting on the surface of the control volume, $\mathbf{F}_\mathrm{S}$, are normal stresses (pressure force) and tangential forces (shear forces due to momentum and turbulence), and body forces are acting within the control volume per unit volume, such as gravitational forces (due to gravity and buoyancy). The equation of motion in matrix format is given below,

$$\frac{\partial}{\partial t}\int_{\mathrm{CV}} \rho \mathbf{u} \mathrm{d}v = \mathbf{F}_\mathrm{S} + \left(\int_{\mathrm{CV}} \rho \mathbf{g} \mathrm{d}v - \mathbf{F}_\mathrm{B}\right) - \int_{\mathrm{CS}} \mathbf{u} \rho \mathbf{u} \mathrm{d}\mathbf{A} \qquad (1.1)$$

in vector notation $\mathbf{u} = u, v, w$. For flows occurring in buildings are steady and the gravitational force is modified by buoyancy acceleration $\mathbf{g}'$. Rearranging equation (1.1) we obtain,

$$\int_{\mathrm{CS}} \mathbf{u} \rho \mathbf{u} \mathrm{d}\mathbf{A} = \mathbf{F}_\mathrm{S} + \int_{\mathrm{CV}} \rho(\mathbf{g} - \mathbf{g}')\mathrm{d}v\,. \qquad (1.2)$$

For the downward oriented jet the control volume diagram becomes as shown in Figure 1.2,

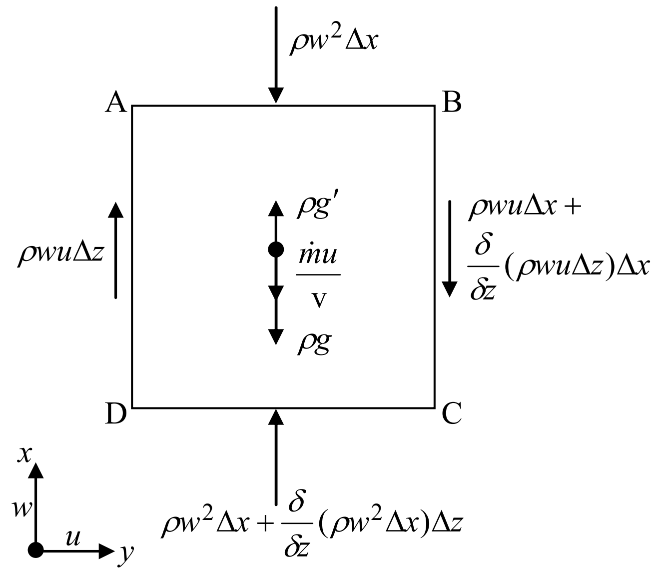

Figure 1.2: Control volume of a two dimensional buoyancy dominated flow of downward oriented jet in a meridional plane.

The momentum flux is conserved at every plane along the flow field of a symmetrical jet,

$$\int_0^\infty \rho w^2 dx = \mathrm{C}\,. \tag{1.3}$$

For simple calculations of hot air, this can be given in the form of surface force,

$$M_J = \rho U^2 \mathrm{A} \tag{1.4}$$

For calculations involving temperature differences a reference temperature should be defined. For analytical calculations in thermodynamics, the reference is usually taken at the absolute temperature of the freezing point of water. In the experiments carried out in this work, the temperature of the chamber tends to equalise to the external lab temperature in order to achieve thermal equilibrium, $(q_m - q_{lab}) = c \cdot (T_m - T_{lab})$. The lab temperature is maintained relatively constant. Hence, this is used instead of the absolute zero for the calculation of buoyancy force. The relationship between density and temperature for room temperatures is found by using the Taylor series expansion,

$$\rho = \rho_o + \frac{\partial \rho}{\partial T}(T - T_o) + \frac{\partial^2 \rho}{\partial T^2}(T - T_o)^2 + \cdots \tag{1.5}$$

neglecting all terms above the first order, we obtain,

$$\rho = \rho_o + \frac{\partial \rho}{\partial T}(T - T_o) \tag{1.6}$$

and rearranging, leads to the Boussinesq approximation,

$$\frac{\rho_o - \rho}{\rho_o} = \frac{T - T_o}{T_o} = \frac{\Delta T}{T_o}. \tag{1.7}$$

The expansion coefficient at the lab temperature, T_o=273+T_{lab} [K], is defined below,

$$\beta = \frac{1}{273 + T_{lab}} \tag{1.8}$$

and for the room temperature of 20°C, $\beta = 3.41 \times 10^{-3}\,\mathrm{K}^{-1}$.

The temperature difference between the air inside the chamber with the external air in the lab introduce the buoyancy parameter given by,

$$g' = g\frac{(T - T_o)}{T_o}. \tag{1.9}$$

The buoyancy force acting on a control volume is defined as the ratio of the temperature difference between the medium and the reference temperature over the reference temperature multiplied by the gravitational acceleration, g[m/s²] and the density of the fluid, ρ[kg/m³]. There are two buoyancy sources used in the experiments of this work. The buoyancy force per unit volume for each terminal is calculated as follows,

for the hot air supply outlet, $$(F_{HS})_B = \rho g \frac{T_{HS} - T_o}{T_o} \tag{1.10}$$

and the cold air supply outlet, $$(F_{CS})_B = \rho g \frac{T_{CS} - T_o}{T_o} \tag{1.11}$$

by subtracting equation (1.11) from equation (1.10), we obtain the buoyancy force for the room per unit volume, $(F_R)_B$,

$$(F_R)_B = \rho g \frac{T_{HS} - T_{CS}}{T_o} \tag{1.12}$$

The effect of the shear forces due to turbulence are considered in the small scale where the Richardson number (Ri) can be applied. A standard form of the Archimedes number most commonly found in literature (or overall Richardson number, Ri_o) relates momentum and buoyancy to the hydraulic diameter of the room. A similar ratio may be more appropriate which is another form of the Archimedes number that can be derived directly from the momentum-to-buoyancy force as a function of the inlet area per unit volume,

$$R_{BJ} = \frac{(F_R)_B}{(M_{HS})_J} = \frac{g\beta\Delta T}{u^2 A}, \tag{1.13}$$

due to the momentum conservation of the jet this quantity is the same for each horizontal plane for a constant buoyancy force along the z-axis of the jet.

1.2 Time-scale of heat conduction

The time scale can be investigated by using equations of heat conduction between two objects, similar to the equations of charge-discharge cycle of a capacitor in electronics. The zeroth law of thermodynamics introduces the concept of thermodynamic equilibrium between two objects at different temperatures. The amount of heat transfer, ΔQ, is proportional to the temperature difference between the objects,

$$\Delta Q(t) = C\Delta T(t) \tag{1.14}$$

The capacitance for heat flow, $C=h$A [W/K], and ΔT [°C] measured from the reference point and ΔQ [W] is the corresponding heat flow for the given temperatures.

The change of temperature between the objects can be described by the following equations,

$$\Delta T(t) = \Delta T_{\max}\left(1 - e^{-t/\tau}\right) \tag{1.15}$$

$$\Delta T(t) = \Delta T_{\max} - \Delta T_{\max}\left(e^{-t/\tau}\right) \tag{1.16}$$

From the theory of flow of electrons through a capacitor, over one period, the temperature differences, ΔT, increase approximately by $1\text{-}e^{-1}=0.632$. In steady-state conditions, one cycle is complete over $4RC=4\tau$. This can be shown below in Figure 1.3,

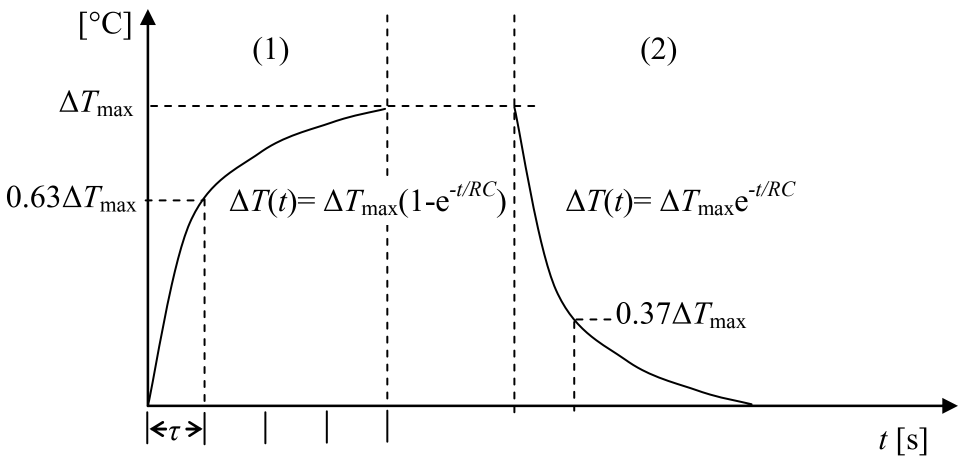

Figure 1.3: Charge and discharge cycle of chamber.

For practical calculations, the time-scale can be calculated from the equation below,

$$t = \frac{D_h \times A_R}{Q_v} = 4\tau \tag{1.17}$$

where D_h is the hydraulic diameter of the room in [m], A_R is the surface area of the room in [m] and Q_v is the supply flow rate in [m³/s].

1.3 Thermal efficiency

Thermal efficiency, η [%], is defined as the ability of the chamber medium to achieve temperatures higher than the external reference temperature, T_o, and closer to the mean input temperature T_{in} of the air supply jets. This is given below,

$$\eta = \frac{T_m - T_o}{T_{in} - T_o}. \tag{1.18}$$