THE VIABILITY OF WEIBULL ANALYSIS OF SMALL SAMPLES IN PROCESS MANUFACTURING

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بِسۡمِ ٱللَّهِ ٱلرَّحۡمَٰنِ ٱلرَّحِيمِ

﴿ وَقَضَىٰ رَبُّكَ أَلَّا تَعۡبُدُوٓاْ إِلَّآ إِيَّاهُ وَبِٱلۡوَٰلِدَيۡنِ إِحۡسَٰنًاۚ إِمَّا يَبۡلُغَنَّ عِندَكَ ٱلۡكِبَرَ أَحَدُهُمَآ أَوۡ كِلَاهُمَا فَلَا تَقُل لَّهُمَآ أُفّٖ وَلَا تَنۡهَرۡهُمَا وَقُل لَّهُمَا قَوۡلٗا كَرِيمٗا ٢٣ وَٱخۡفِضۡ لَهُمَا جَنَاحَ ٱلذُّلِّ مِنَ ٱلرَّحۡمَةِ وَقُل رَّبِّ ٱرۡحَمۡهُمَا كَمَا رَبَّيَانِي صَغِيرٗا ٢٤ ﴾

In the name of Allah, Most Gracious, Most Merciful:

{Thy Lord hath decreed that ye worship none but Him, and that ye be kind to parents. Whether one or both of them attain old age in thy life, say not to them a word of contempt, nor repel them, but address them in terms of honour ✷ And, out of kindness, lower to them the wing of humility, and say: "My Lord! bestow on them thy Mercy even as they cherished me in childhood. }.

The Holy Quran : Al-Isra' : 23-24

This PhD is the bear fruit of my parents and my brother love, encouragement, precious support and sacrifice. Therefore, I dedicate this work to them as a mark of my gratitude, respect and thanks.

ABSTRACT

This research deals with some Statistical Quality Control (SQC) methods, which are used in quality testing. It investigates the problem encountered with statistical process control (SPC) tools when small sample sizes are used. Small sample size testing is a new area of concern especially when using expensive (or large) products, which are produced in small batches (low volume production).

Critical literature review and analysis of current technologies and methods in SPC with small samples testing failed to show a conformance with conventional SPC techniques, as the confidence limits for averages and standard deviation are too wide. Therefore, using such sizes will provide unsecured results with a lack in accuracy.

The current research demonstrates such problems in manufacturing by using examples, in order to show the lack and the difficulties faced with conventional SPC tools (control charts). Weibull distribution has always shown a clear and acceptable prediction of failure and life behaviour with small sample size batches. Using such distribution enables the accuracy needed with small sample size to be obtained. With small sample control charts generate inaccurate confidence limits, which are low. On the contrary, Weibull theory suggests that using small samples enable achievement of accurate confidence limits. This research highlights these two aspects and explains their features in more depth. An outline of the overall problem and solution point out success of Weibull analysis when Weibull distribution is modified to overcome the problems encountered when small sample sizes are used.

This work shows the viability of Weibull distribution to be used as a quality tool and construct new control charts, which will provide accurate result and detect non-conformance and variability with the use of small sample sizes. Therefore, the new proposed Weibull deduction control charts shows a successful replacement of the conventional control chart, and these new charts will compensate the errors in quality testing when using small size samples.

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CHAPTER 1

INTRODUCTION

1.0 CHAPTER ONE REVIEW

The drive for quality improvement has lead to the use of Statistical Process Control (SPC) techniques to monitor and maintain low reject levels. For high scale production large samples can be used to measure with a high level of confidence.

When low volumes is required, or processes with high piece cost, it can be expensive to collect large samples for analysis. Statistical analysis will always offer higher confidence levels as samples sizes increase.

This research is based on investigating the stability of Weibull analysis to analyse small samples. In particular, how the use of small samples (<10) can act as a predictive tool in low volume manufacturing. The principal aim of this research is to establish how quality and reliability techniques may be combined to offer feasible analysis for the statistical control in manufacturing processes.

The above definitions show that quality is linked directly to customers or the end users. Therefore, it is the quality provider's mission to satisfy customer need and ensure that the customer is enchanted by the product or service provided.

Unmitigated quality may be achieved when the product or service exceeds customer expectation. This technique may be considered as every establishment current goal due to the globalisation of the market and the tough competition, which exist in the world industries and services. In the ordinary situation in an industry, the gap between the customer satisfaction –or expectation- and the product - or services – reflects the quality measurement scale. The gap is inversely proportional to the quality scale, in other words, when the gap scale is large it means that the quality standards are low, and if the gap is small the quality characteristics are high. This Situation is demonstrated in Figure 1:1.

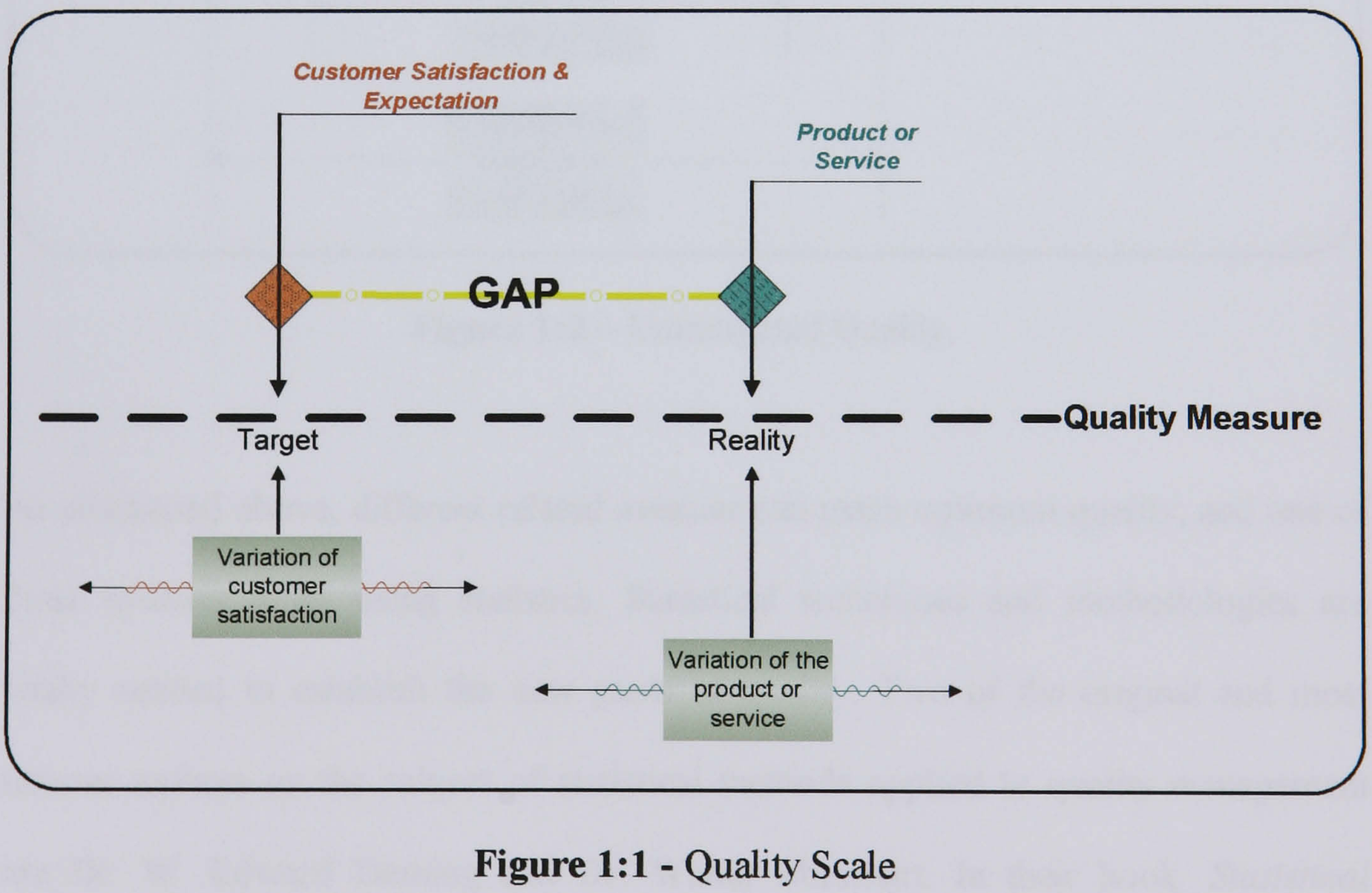

Figure 1:1 - Quality Scale

Ideally, the product or service variations should exceed the customer expectation in order to obtain unmitigated quality, which is shown in figure 1:2. Due to such facts, the reality of a product or service should equal to the target that the customer is expecting in the item in which the customers' money will be invested. For that reason, it can be argued that a company should set its goal to exceed the customer needs and expectations, which will grant the company a world-class quality characteristic.

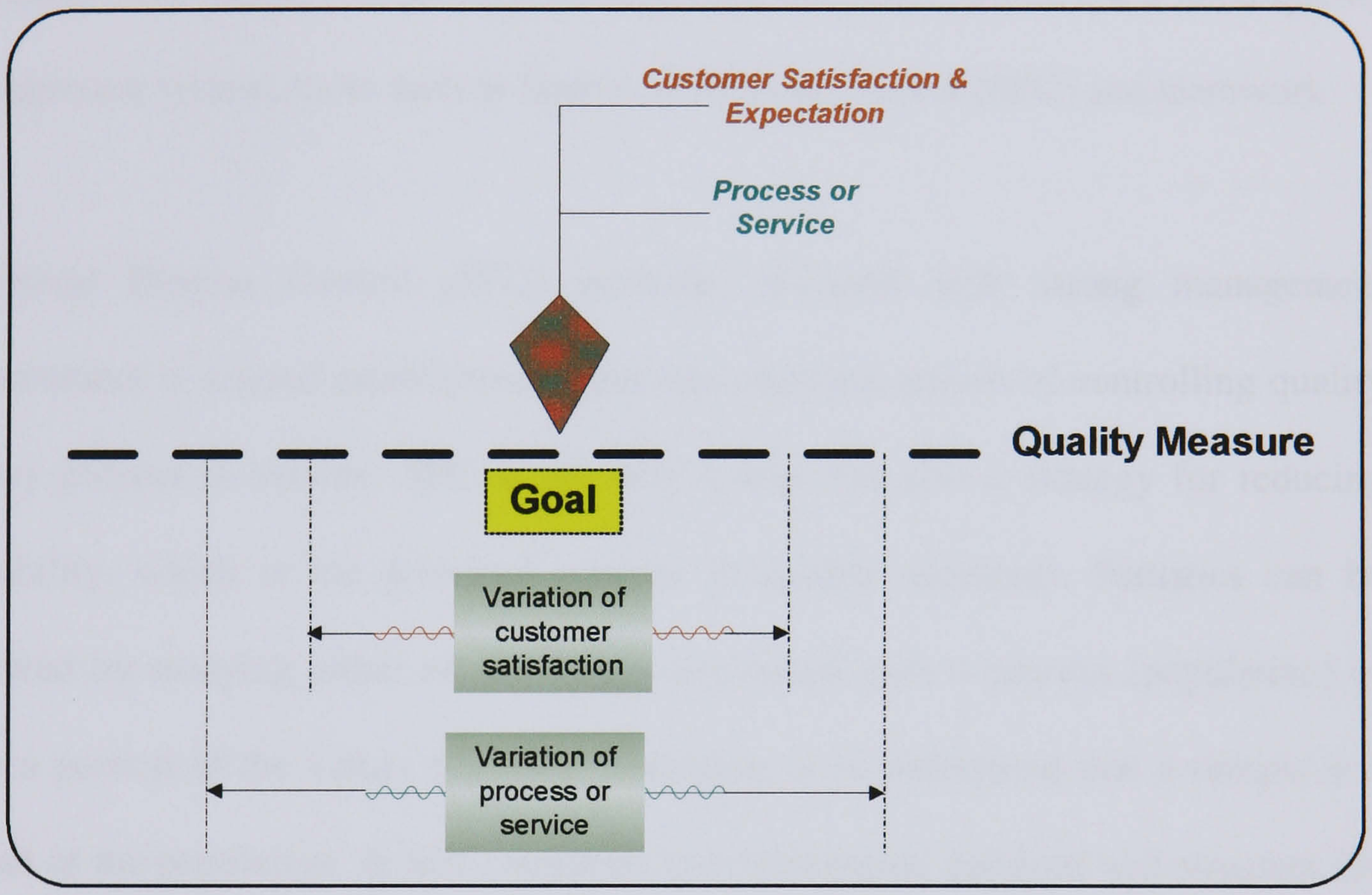

Figure 1:2 – Unmitigated Quality.

As mentioned above, different related avenues can reach optimum quality; and one of these avenues is by using statistics. Statistical techniques and methodologies are vitally needed to establish the new gaols of quality. Two of the original and most famous authors on the subject of statistical methods applied to quality management are Dr. W. Edward Deming and Dr. Walter Shewhart. In their book, *Statistical Method for the viewpoint of Quality Control* [5], they wrote: *"The long-range*

contribution of statistics depends not so much upon getting a lot of highly trained statisticians into industry as it does on creating a statistically minded generation of physicists, chemists, engineers and others who will in any way have a hand in developing production processes of tomorrow". This phrase was written in 1939, and it can obviously be true today. Total Quality Management (TQM) is concerned with identifying customer requirements and tries to meet them based on a defined quality approach. This requires three basic management essentials, which are a good quality management system, tools such as Statistical Process Control (SPC) and teamwork.

Statistical Process Control (SPC) methods, affirmed with strong management commitment in a good establishment provides objective means of controlling quality in any process or service. SPC is not only a tool, but also a strategy for reducing variability, which is the principal concern in quality problems. Statistics can be gathered by studying either all the values associated with a process (population) or only a portion of the values (sample). Therefore, it is understood that a sample is a subset of the population. In SPC, numbers and information gathered will structure the bases of a managerial decision and action. Data recording is a basic element of a comprehensive quality framework. SPC tools vary, but there are some common tools, which may be applied to explain fully and involve maximum use of data. These simple tools offer the organisation an uncomplicated method of collecting data, presenting and analyse.

1.1.2 - Reliability

Reliability is an essential aspect of both product and process design. Sophisticated equipment used today in such areas as transportation, communication, and medicine requires high reliability. High reliability can also provide a competitive advantage for many consumer goods. As the overall quality of products continues to improve, consumers expect higher reliability with each purchase; they are simply not satisfied with products that fail unexpectedly. However, the increased complexity of modern products makes high reliability more difficult to achieve. Likewise in manufacturing, the increased use of automation, complexity of machines, low profit margins, and time-based competitiveness make reliability in production processes a critical issue for survival of the business.

Reliability can be generally defined as "*the probability that an item will fail over given time*". However, the probability distribution of failures is usually a more convenient figure to use in reliability computations. Weibull distribution can be an effective distribution to be used in order to calculate reliability and predict failures.

One of the principal advantages of Weibull is the unique method by which handles distributions. This approach allows for predictions to be made with small sample sizes. Unfortunately, the theory is based on a finite lower value with a defined upper limit. Therefore, it is not a direct comparison to SPC that bases analysis on first and second order moments, e.g. μ and σ.

1.2 SMALL SAMPLE SIZE CONSEQUENCES ON PRODUCTION INSPECTION

Inspection is an imperative technique to check the quality standards that have been set to attain the elite quality required to fulfil customer satisfaction. Inspection can be a useful way to examine the behaviour of a process and detect the variation that may occur within the process. Inspection can be made on the whole production lot, and at that time is called 100% inspection, as every item will be thoroughly checked. Such a technique is a time consuming procedure.

Sample inspection is considered a more effective and efficient way to inspect variation or non-conformance of a process. Thus, many factors affect such techniques, it is considered to be a modern method to detect default and assure quality. Sampling inspection should submit to different guidelines, which are[6]:

I. Sample should be rational – Sample should reflect the population behaviour, a chosen sample ought to be homogeneous, as the non-conformance should be clear and appear between samples, while it need not be noticed within each sample. By this principle, spotting deviation within the process across a certain time period can be precisely predicted by mathematical formulas.

II. Sample size should be diminutive – as the size of a sample may be proportional to some financial aspects. Many managerial opinions support the idea of having small sample size always. The sample size is important when financial criteria are involved. Small sample are preferable specially when low volume size, and highly cost product are being inspected. If the

inspection involve destructive testing the a company can not risk testing large size sample due to the financial impact, which will cause the retail price to increase and the competitive virtue will decrease.

III. Sampling frequency (rate of recurrence) - Using large sample size frequently with short time period lags will be desirable for inspectors to maintain high quality standards and detect every variation, which may occur in the process. But due to economical reasons this behaviour cannot be useful and cost effective. For that reason, a balance should be imposed between the frequency of sampling and the cost of quality needed. Practically, this issue is determined by the experience of the inspector and the quality designer.

Typically, small size samples are desirable, as sample size has an economical impact. The breakeven point is the standard of quality required to achieve customer expectations. Practically, it was found that a sample size of 30 could achieve a sensitive detection for non-conformance[7]. The sample size of 30 could be still considered as a large figure in such manufacturing venues such as defence and space industry, as the cost of the product is extremely high and high quality and reliability is essentially needed when risk should be minimised.

As a result of the above-mentioned factors, this research has taken reducing the sample size accompanied with obtaining a sensitive inspection technique as a main goal to be accomplished.

1.3 GENERAL DESCRIPTION OF THE CONTRADICTING OUTCOMES BETWEEN SPC AND WEIBULL ANALYSIS

Recent global competitiveness has made companies look for a new strategy to increase their profit, gain market reputation, and strengthen their industry. Quality control (SPC) and reliability can ensure these goals for any company if they are used in a correct manner; they are regarded as effective tools when large sample size (n>50) is being tested. The problems is that with small samples, which means when a high value low volume is being manufactured- such as military, satellite, and medical parts, and normally these parts have an expensive financial value. Within this type of manufacturing, safety and life cycle computation is the most vital element to ensure the success of such products. Using the conventional SPC control charts does not ensure the detection of variability and non-conformity due to sample size restrictions.

Weibull distribution has always shown a clear and acceptable prediction of failure and life behaviour with small sample size batches. Using such distribution enables the accuracy needed with small sample size to be obtained[8]. While, on small samples SPC Charts generate inaccurate confidence limits, which are low. Additionally, Weibull theory suggests that using small samples enable achievement of accurate confidence limits.

Small samples testing failed to show a conformance with conventional SPC techniques, as the confidence limits for averages and standard deviation are considered to be too wide. Hence, using such sizes will provide unsecured results with a lack in accuracy.

Therefore, in this research a new idea will be investigated and examined to use a reliability model such as Weibull to be used as a Statistical Process Control Model for the expensive, low volume production. However, to achieve this, the difference between their analyses must be addressed.

1.4 CURRENT PHD RESEARCH AIMS AND OBJECTIVES

This research concentrates on quality and reliability methods, which are used in quality testing. It will investigate the potential problems encountered with Statistical Process Control (SPC) tools when small sample sizes are used. Small sample size testing is a new area of concern especially when using expensive products that are produced in small batches (low volume production).

These stated concerns are demonstrated in problems with respect to manufacturing, in order to show the lack and the difficulties faced with conventional SPC tools (control charts). The examples used are dimensional parameters of products, and failure rates.

Subsequently, the research hypothesis is:

It is suggested that remodelling small Weibull samples to accommodate populations will produce data suitable for measuring non-conformance.

To examine the above hypothesis the consecutive aim and objectives are established.

Aim:

I. **To identify how small samples affect statistical analysis to monitor processes with the use of Weibull data.**

II. **To propose a method of using Weibull analysis for statistical control of low volume processes.**

Objectives:

I. **To establish the principal limitations of small samples for process control.**

II. **To determine the relationship of Weibull for controlling the process.**

III. **To develop a Weibull model for the process.**

IV. **To generate a charting process control with small samples.**

1.5 THESIS LAYOUT

Chapter 1 : Introduction.

Chapter 2 : Critical literature review of SPC, Control charts, Weibull parameters with small samples.

Chapter 3 : Methodology to analyse the problem and set new ways to achieve a solution

Chapter 4 : Primary Investigation in Shewhart control charts and its limitations, and Weibull analysis when small sample sizes are adopted.

Chapter 5 : Modelling new control charts based on Weibull distribution, which will overcome the problem encountered with small sample size use.

Chapter 6 : Discussion for the main finding and results.

Chapter 7 : Conclusion, Recommendations and Future Work.

1.6 CHAPTER ONE CONCLUSION

This chapter has introduced the impact of using small sample size in inspection. It demonstrated the important incongruity between SPC and Weibull analysis outcomes. Also, it has addressed a clear understanding of the problems associated with using small sample size and obtaining a suitable sensitivity to detect variations.

Chapter one has given a general overview of the importance of quality and reliability principles and methods in a manufacturing environment. It has also explained the research aims and objectives. A brief layout of the thesis was explained in a logical manner to test the hypothesis set to tackle such problem.

CHAPTER 2

LITERATURE REVIEW

2.0 CHAPTER TWO REVIEW

In order to have a clear view and understanding to the problem encountered with small samples size, a comprehensive review to present and past research will be introduce and critically present each idea associated with small sample size problem in statistical quality analysis.

2.1 HISTORICAL DEVELOPMENT OF QUALITY

Quality is an ancient idea developed along with human society maturity across the years. It can be clearly seen that the old civilisations used Excellence as a parameter of their progress in providing good life standards for its nations. Going backward 3000 years B.C., it can be evidently noticed that the ancient Egyptian civilisation used measurement instrumentations to maintain and inspect the dimension of their carving in walls, pyramids, and temples. Therefore, such procedure made the ancient Egyptians succeed in their work and left their monuments as a remarkable print in the history and undoubtedly proof of their Excellence, superb development for human kind luxury and promise of human capabilities.

During the Middle Ages in Europe, craftsmen were totally skilled and able to manufacture the whole product, to satisfy definite purpose of the customer. However, in the middle of the 18th century, Honoré Le Blanc, a French gunsmith, was the first to person to develop a system for manufacturing muskets to a standard pattern using interchangeable parts[9]. Due to this development, products became more complex and hard to be manufactured by one person. Therefore, the idea of interchangeable parts dictated a close look at standards and the overall inspection for the finished product.

Quality as a terminology flourished at the beginning of the twentieth century. Especially, when Dr. William A. Shewhart set, at Bell telephone Laboratories in 1925[10], new statistical charts to monitor and control product variability and standard non-conformity. Dr. Shewhart stated " The long range contribution of statistics depends not so much upon getting a lot of highly trained statisticians into industry as

it does on others who will in any way have a hand in developing production processes of tomorrow"[11]. It can be clearly seen; Dr Shewhart believed that statistical techniques and methodologies are vitally needed to establish the goals of quality. Also F. Dodge and H.G. Roming developed a new methods based on sampling inspection and both of the above published the first tables constraining such method to check quality and standards, based on acceptance sampling which may assure 100% inspection[12]. Afterwards, many people started to research in the field of quality and they developed many philosophies in quality and techniques to examine quality standards and approve them.

In order to understand quality as a concept and methodology, there should be a clear consideration of the quality definitions, philosophises, and techniques. Such recognition needs to be explored through a thorough analysis of the previous people who stated their opinion, depending on their knowledge and experience, and developed a unique understanding to the philosophy of quality. Furthermore, the set of standards play an influential role in easing the understanding of quality and the way it should be applied to each area of use.

Quality became one of the concerns that occupy manager activities, as quality is associated with money; managers tried to reduce quality cost to gain profit, on the contrary, quality should reach a certain standard to satisfy customer expectations and to ensure a good demand for the product or service provided.

2.2 A COMPARISON IN THE QUALITY PHILOSOPHY DEVELOPMENT

2.2.1 Shewhart Quality Philosophy

It is undoubted that Dr. W. Shewhart led the way to modern quality control[13]. He was the first to adopt statistical methods to develop a method to control quality. Shewhart established a statistical chart to constrain quality standards; hence these charts became the first tool to be used in Bell Laboratories and other companies afterward to manage quality and to monitor the behaviour of a process, in order to detect variability and distortions within a process. Shewhart control charts were the foundation of quality assurance.

Shewhart, using a literal definition of quality (Latin *qualitas*, from *qualis*, meaning "how constituted"), defined two common aspects of quality[14]:

1. Objective quality – which handles the quality of an item as an objective reality, without the influence of the human;
2. Subjective quality – which handles the quality of a thing relative to what the human thinks, feels, or senses as a result of the objective reality.

Shewhart believed that there is an objective state in quality control, which allows a possible prediction of quality within limits even though the sources of variability are not clear. Based on such beliefs, it is feasible to achieve the following aspects:

1. Decreasing the cost of inspection;
2. Cutback the cost of rejection;
3. Attainment of maximum benefits from quantity production;

4. Achievement of uniform quality even though the inspection test is destructive;
5. Reduction in tolerance limits, where quality measurement is indirect.

Shewhart was widely recognised afterwards as the pioneer of quality control. In May 1932, he was invited to England to attend a meeting at the British Standard Institute with representatives of manufacturing industries. Shewhart's developments in the field of statistical quality control, and its practical applications and benefits to industry were examined.

The meeting gave rise to a committee, which responsible for producing a report on the application of statistical methods in standardisation and specification of quality. In 1935, after lots of discussions, the committee produced the famous BS600[15], based on the comments of Shewhart and the work of Dr. Egon Pearson.

It can be seen that Dr. Shewhart had left a remarkable fingerprint in the quality control field, due to his astonishing use of statistics in quality, and his development of quality control charts. Despite some modern opinions, which consider Shewhart's methods to be orthodoxy, it is indeed the foundation of quality control science.

2.2.2 Deming Quality Philosophy

Statistician W. Edward Deming never defined or described quality in a precise manner. Deming stated " A product or service possesses quality if it helps somebody and enjoys a good and sustainable market"[16]. The Deming philosophy focuses on bringing about improvements in product and service quality by reducing uncertainty and variability in design and manufacturing process. In Deming's point of view, variation is the chief culprit of poor quality.

Deming established 14 points, which improve the quality and reduce variation. These 14 points are [17]:

1. Create and publish to all employees a statement of the aims and purposes of the company. The management must demonstrate constantly their commitment to this statement.
2. Learn the new philosophy to top management, and everyone.
3. Understand the purpose of inspection, for improvement of processes, and reduction of cost.
4. End the practice of awarding business on the basis of price tag alone.
5. Improve constantly and forever the system of production and service.
6. Institute training.
7. Teach and institute leadership.
8. Drive out fear. Create trust. Create a climate for innovation.
9. Optimise toward the aims and purposes of the company efforts of teams, groups, and staff areas.
10. Eliminate exhortations for the work force.

11. a) Eliminate numerical quotas for production. Instead, learn methods for improvement.

    b) Eliminate MBO (management by objective). Instead, learn capabilities of processes.

12. Remove barriers that rob people of pride of workmanship.
13. Encourage education and self-improvement for everyone.
14. Take action to accomplish the transformation.

Deming's philosophy and his 14 points caused some confusion and misunderstanding among business people, because Deming did not provide a clear rationale or foundation for them. As a comparison, Shewhart used the idea of technical quality to encourage adopting his philosophy; on the other hand, Deming concentrated on the development of the concept of quality as an economical philosophy, which was hard for the Americans to adopt, as they did not expect the competition of other countries, and they did not associate customer needs as a matter of profitability.

After the Second World War there was an economic recession in many American companies, so as a result of this recession Deming was no longer welcome in American industry. In 1951, he went to Japan upon an invitation by JUSE (Japanese Union of Scientists and Engineers), Deming held seminars and training courses for Japanese industry to assist the languished Japanese industry in statistical questions during its process of reconstruction. Deming focused on quality as a strategic economical goal, which could enable Japanese industry to compete in global markets[18].

The results of Deming philosophy appeared clearly when Japanese industries became a competitor in the world market, while the US industry began to lose share against Japanese industry during the 1960's, due to the precise significance to quality in the Japanese product[19].

Deming has proposed a "chain reaction", which links quality, productivity, market share, and jobs. This chain is illustrated in figure 2:1.

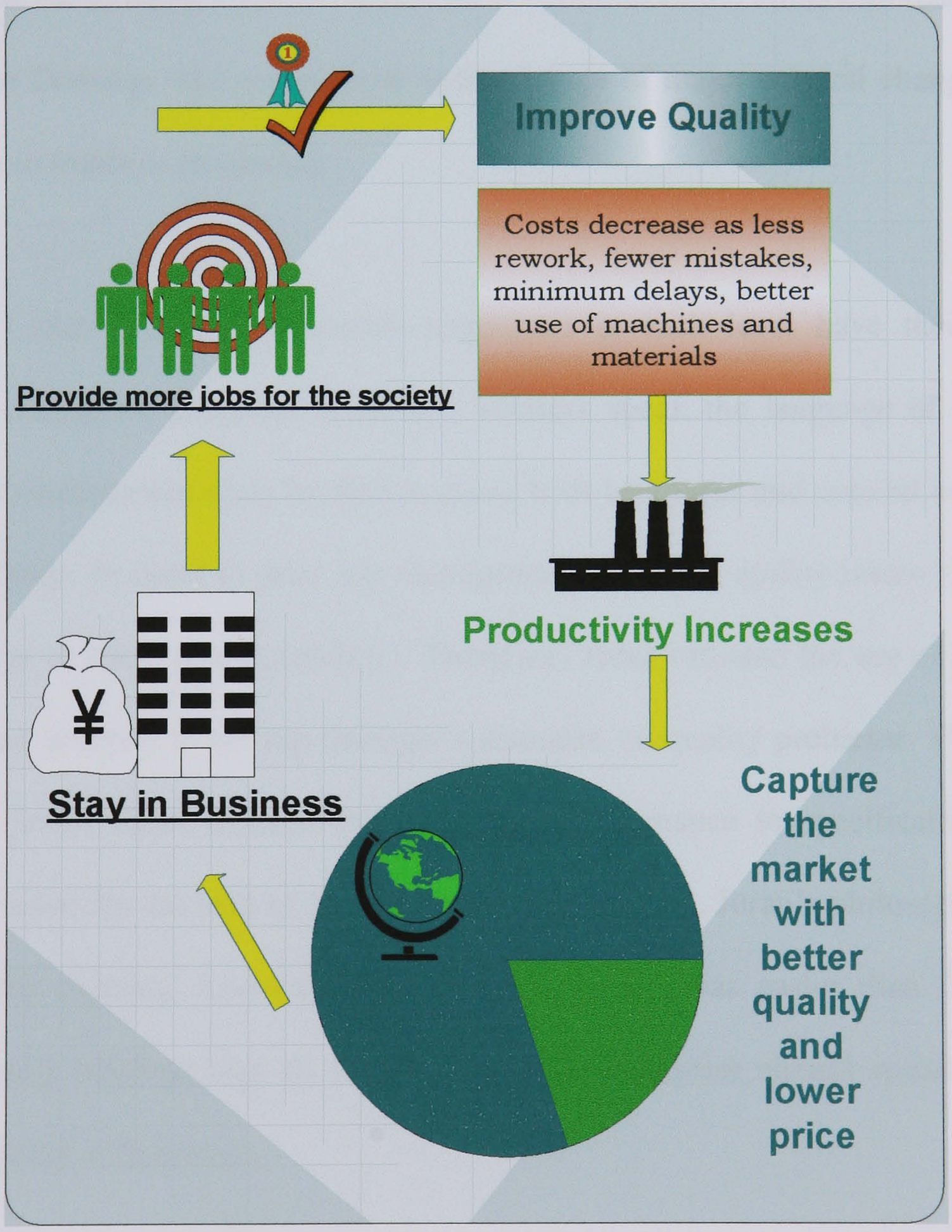

Figure 2:1 – Deming quality chain reaction.

2.2.3 The Juran Philosophy

In the 1920s, Joseph Juran initiated the development of statistical methods for quality in Western Electric Corporation. Juran spent his working life as a corporate industrial engineer involved with quality concepts, analysis and applications. In 1951, Juran published a book entitled "Quality Control Handbook". This handbook became later one of the most basic references in quality science. Juran improved quality by involving with the existing systems, which were common to the American managers, differing from Deming, who adopted the methodology of major cultural changing in the enterprise to improve its quality.

Juran noticed that in any organisation employees at each level have their own languages[20], manager's language is dollars, workers speak the language of things, while middle management must be able to speak both languages and convert between dollars and things. In order to draw top management attention, quality issues must be in the language of these people (dollars). Therefore, Juran initiated the use of quality accounting and analysis to get top manager's attention on quality problems. While at the worker's level, Juran focused on increasing conformance to specification and rejecting the defect by the help of statistical tools for analysis. Juran's philosophy was adopted by the existing American organisations, thus it was easier than Deming philosophy, who believed that all the people in the enterprise should speak in the common language of Statistics.

Quality from Juran's point of view is simply summarised as "fitness for use"[2]. Juran defines quality as "product performance that results in customer satisfaction", in other

words, freedom from product deficiencies, which avoid customer dissatisfaction. Juran tackled quality from four aspects, which are:

1. Quality of design- that concentrates on market investigation, and items or service concept.
2. Quality of conformance- that contains technology, staff involved and management.
3. Availability- that focuses on reliability, maintainability and logistical support.
4. Field service- that comprises promptness, competence and integrity.

Juran's prescriptions focus on three major processes, called the *Quality Trilogy*, which are[13]: (refer to Table 2:1 for details)

1. Quality planning- the process of preparing to meet quality gaols.
2. Quality control- the process of meeting quality gaols during operations.
3. Quality improvement - the process of breaking through to unprecedented levels of performance.

Quality planning	**Quality control**	**Quality improvement**
Determine who the customer are	Evaluate actual product performance	Establish the infrastructure
Determine the customer needs	Compare actual performance to product gaols	Identify the improvement projects
Develop product feature to satisfy customer needs	Act on the difference	Establish project teams
Develop processes able to produce the product feature		Provide the team with resources, training, and motivation.
Transfer the plans to the operating forces		Diagnose the cause, and stimulate remedies

Table 2:1 – Juran Trilogy of quality

Many of the ideas which Deming and Juran came up with, are similar so there is a closeness in their philosophy to a reasonable extent; as an example: Juran and Deming concentrated on top management commitment to improve the quality in their organisations. However, it is a fact that Juran and Deming were different in many issues in the case of Quality improvement. Juran established a well-specified mechanism to improve quality. His mechanism includes proving the urgency of quality improvement, specifying detailed projects for improvement, diagnoses of dissatisfactory causes affecting quality, and providing control to maintain the improvement of quality. Deming stated that management ought to drive out fear, on the contrary, Juran thought that Deming is wrong is this statement, and he mentioned that " Fear can bring out the best in people"[21].

2.2.4 The Crosby Philosophy

In 1979, Philip Crosby established Philip Crosby Associates, it was founded to develop quality and provide training programs and constancy. Crosby worked in American industry and was involved with quality adoption within various establishments. He was the corporate vice president for quality at International Telephone and Telegraph (ITT). Crosby wrote many books concerned with quality and management, such as "Quality is free" which was his first published book.

Crosby's core of quality philosophy was based on two major concepts, which are:

(a) Absolutes of quality management;

(b) Basic elements of improvements.

These two concepts consist of many points such as[22]:

1. Quality means conformance to requirements, not elegance;
2. There is no such thing as a quality problem, as quality originates in functional departments, not in the quality department;
3. There is no such thing as the economics of quality; doing the job right first time is always cheaper;
4. The only performance measurement is the cost of quality, which is the expense of non-conformance;
5. The only performance standard is "Zero Defects (ZD)".

Crosby's Zero defects (ZD) methodology was a performance standard not a motivational programme. The theme of ZD is do it right the first time, which means concentrating on preventing defects rather just finding and fixing them.

It can be seen that quality can be looked after from different angles, and each one can have a significantly different approach by implementing organisational changes to achieve quality. Hence, quality is understood to be everyone's responsibility in an organisation.

Juran's philosophy was to provide changes within the current system, as quality is fitness for use, and his quality trilogy provides this concept in the system, while Deming's philosophy was based on improving products and services by reducing uncertainty and variation. Conversely, Crosby's philosophy was based on behaviour changes rather than using statistical techniques to maintain a quality standard within an organisation, which also require a change in corporate culture and attitude.

A unique interesting issue in Crosby's philosophy is the detail he provided about how organisations stated the enhanced features of managing quality. Moreover, Crosby focused in his philosophy on the methodology of managerial thinking toward quality. He advised mangers to take as their duty shaping and adopting the best methods, which is appropriate to their organisations based on the organisation's individual situation, due to some implementation problems occurring when some organisation tried to adopt Deming philosophy[23].

2.2.5 The Modern Quality Philosophy

Due to the importance of quality in daily life in raising the standard of living and ensure a luxurious environment for mankind to enjoy their life, people were attracted to quality. Quality has developed dramatically in the last century. Experts who researched in the field of quality have provided many philosophies. Despite the difference in their philosophies, all of them agreed that quality should be a commitment to everyone in the enterprise. For this reason, many authors tried to use this idea to manage the application of quality and the methods to be implemented; in order to have state of the art quality.

Competition was the driving factor to enhance quality and adopt new strategies to establish a quality environment, which produces a quality product or service that satisfies customer needs, and achieve profit and reputation for the survival of the company in turbulent markets.

Many scrupulous people in the field of quality tried to develop a comprehensive strategy to apply the knowledge provided by previous researches. Despite the differences and the nature of each company's activity many developed new ideas based on the experience, which they gained across the years working in quality. For example, Armand V. Feigenbaum was researching in measuring conformance to technical specifications. He set the concept of Total Quality Management (TQM) [24]. Feigenbaum defined TQM as "Total Quality is an effective system for integrating the quality-development, quality-maintenance, and quality improvement effort of various groups in an organisation so as to enable marketing, engineering, production, and

service at the most economical levels which allow for full customer satisfaction"[4]. Therefore, Feigenbaum perceived quality as a comprehensive strategic business tool, which required everyone's involvement within the organisation. His strategy was based on three main pillars, which are: quality leadership, modern quality technology and organisational commitment. His strategy argued continuous management improvement based on realistic planning not only reducing the error or failure. Also, new methods of evaluating the conformity should be implemented in order to satisfy customers. Finally, training and motivation for the whole organisation's workers should be continuously and constantly provided, to ensure quality enhancement in each aspect of the company's activity, and to establish a comprehensive commitment in each person.

Dr. Kaoru Ishikawa was also a pioneer in Japanese quality strategies and methods. He encouraged the ideas of Total Quality Control (TQC), which involves refining the application of different statistical tools to quality problem[25]. Ishikawa understood that every individual in the organisation ought to participate with quality monitoring, improvement, and quality problem solving. Dr. Ishikawa took a large part in shaping the Japanese quality movement[26]. The Japanese Quality characteristics were emphasised by Ishikawa aspects, which gives Japanese quality a different scope to that of western countries. These aspects can be summarised as follows: Quality begins and ends with education and training, quality should be associated with customer requirements, the ideal state of quality occurs when inspection is no longer necessary, roots of cause should be removed not the symptoms, differentiation between means and objectives, quality should be established first by which long term profit will results, marketing the quality nationwide, and the majority of the company problems

(95%) can be solved by using simple statistical tools, therefore, statistical methods should be utilized[27].

Dr. Ishikawa used the term "Company Worldwide"[28] to relate to the principles of Feigenbaum TQC[29], which relate to the Japanese industrial environment. Ishikawa stated that every Japanese company wishing to transform to company Worldwide quality control status should adopt and train all its employees on Statistical Quality Control (SOC). Some researchers such as Barrie G. Dale draw a conclusion through their study of Japanese organisations that Ishikawa's definition of company Worldwide is only manipulating semantics, in other words, Japanese effort to enhance the quality can be described by Feigenbaum's western definition of Total Quality Control (TQC)[30].

It is clearly seen that quality methods are in continuous improvement due to the rapid change in today's industry. Many scientists and researchers are trying to develop new models to fill the gaps, which might occur in industry. Despite the common view of Deming, Juran and Crosby regarding quality, each of them chooses a different ideology to implement quality. Quality for all of them needs commitment, but commitment varies, for example, Deming stressed on management commitment as managers dictate all the quality specifications and monitor the behaviour of strategy within the organisation, on the contrary, Ishikawa states that quality is like a process and every individual in the organisation is working in counted as a part of this process, so every one is committed and responsible for quality. For quality to succeed in an organisation, the organisation should set its gaols based on the behaviour of it nature in the market, then it should adopt the appropriate strategy which fits such

nature and gaols. Moreover, managers need to fully digest the diversities and resemblances in the most important quality philosophies and build up a quality management approach tailored to their organisations.

As seen previously, quality definition is a complex endeavour; it varies depending on the strategy and ideology adopted. It is the author's opinion that quality, in today's ideology, is not only achieving customer needs and expectation, it is "getting on target with minimum variance". Specification is the aid to achieve quality; hence, productivity is doing something efficiently, while quality is doing the right thing efficiently. Achieving quality needs a high level of commitment for everyone involved in producing the item or service.

2.3 QUALITY STANDARDS AND AWARDS

Due to the enormous contribution to quality by different people across the world, many diverse philosophies and methodologies were present. As world markets became more competitive and the technology reached point where the world became like a small village, the urgency of regulating the inter-changeability crop up. Today momentum of technological advance makes conventional technical agreements between manufacturers hard to formulate and keep, hence barriers were established and the idea of having a common ground of understanding was obvious[31].

A standard is simply a decision concerning materials, performance, capability, arrangement, condition, action, methods, procedures, formalities, responsibility, concept, …etc[32]. Therefore, many institutes tried to form their own standards to improve the efficiency and provide a high level of production or service. The main aims of standardisation can be summarised in the following points: overall economy and reduction of cost that is associated with a good product or service, protection of customer interests and safety, provision of a means of expression and communication[33]. British Standard institution (BSI)- founded in 1901- was the pioneer in the standardisation field. Also other standards we established such as the European committee for standardisation (CEN), International Standards Organisation (ISO), Japanese Industrial Standards (JIS).

At 1978, BSI initiated setting guidelines to standardise a quality management system, this system was issued and published in 1979 under the code of BS5850[34]. BS5750 is considered the leading standard for quality management; it is not a product

specification, nor a guarantee of product quality. BS5750 splits into four main parts which are:

1. BS5750 Part 0 (equivalent to ISO9000 & EN29000) – is a guide to the selection of appropriate parts of the overall quality management system and its elements within the standard.
2. BS5750 Part 1 (equivalent to ISO9001 & EN29001) – is related to quality specifications for design, development, production installation and services.
3. BS5750 Part 2 (equivalent to ISO9002 & EN29002) – sets out requirements where a firm is manufacturing goods or offering a service to a published specification or to the customer's specification.
4. BS5750 Part 3 (equivalent to ISO9003 & EN29003) – specifies the quality system to be used in final inspection and test procedure.

In order to enhance the use of standards and market quality improvement to ensure a better customer oriented industry, many awards have been launched to boost the awareness of quality importance. There are two main prestigious quality awards, which are the Deming Prize Award for industrial Achievement, and the Malcolm Baldrige award.

In June 1951, less than a year after Deming's first lecture on quality control in Tokyo, Japan instituted the Deming Prize for industrial achievement[35]. This prize was based on Deming 14 points, which epitomize a challenge to leadership in quality assurance. Many Japanese companies achieved excellence by adopting Deming's strategy and gaining his prize for industrial achievements.

In the United States, it took 30 years, until 1981, before an equivalent American incentive, named for the late Secretary of Commerce, Malcolm Baldrige, was established to encourage higher American quality, then known as Malcolm Baldrige National Quality Award (MBNQA). The MBNQA criteria define key practices in categories of leadership, customer and market focus, strategic planning, human resource development, and process analysis.

Deming was not an advocate of the Baldrige award[36]. The competitive nature of the Baldrige award is fundamentally at odds with Deming's methodology. Nevertheless, most of the Deming's principles are implicitly associated with the Baldrige award criteria. A good example for such fact is symbolised by Zytec – an electronic corporation in the US, Zytec adopted the 14 points of Deming to improve their quality system, and as a result of such improvement they succeeded in obtaining the MBNQA[6].

It is clearly seen that quality awards have a huge influence on quality improvement, as they stimulate the companies to reach an acceptable level of quality and customer satisfaction.

2.4 QUALITY DIMENSIONS

Quality of a product may be measured in different ways and can be evaluated based on many point of views regarding many conceptual understandings of the criteria. Garvin set comprehensive key elements to evaluate quality[37]. These keys are called Dimensions of quality, they are:

1. *Performance* (will the product do the intended job?).
2. *Reliability* (how often does the product fail?).
3. *Durability* (how long does the product last?).
4. *Serviceability* (how easy is it to repair the product?).
5. *Aesthetics* (what does the product look like?).
6. *Features* (what does the product do?).
7. *Perceived* Quality (what is the reputation of the product?).
8. *Conformance to Standards* (is the product made exactly as the designer intended?).

2.5 THE ECONOMICS OF QUALITY

The quality for a product or a service has a cost value attached to it. The cost of quality will influence the profit margin; therefore, the cost of quality contributes to the overall profit of the company. In order to establish a clear view on quality cost, figure 2:2 and 2:3 will illustrates the idea behind the quality cost[38].

Figure 2:2 - Quality Economics

Figure 2:2 shows the cost of quality versus the return of quality; from this figure it can be seen there are various regions that a company can operate in. These regions will reflect the amount of profit earned by the company. As seen in figure 2:2, equilibrium should be established between the cost of quality and the return of quality in order to reach a profitable breakeven point. This point is point B in the above figure, which will guarantee high level of quality accompanied with high return.

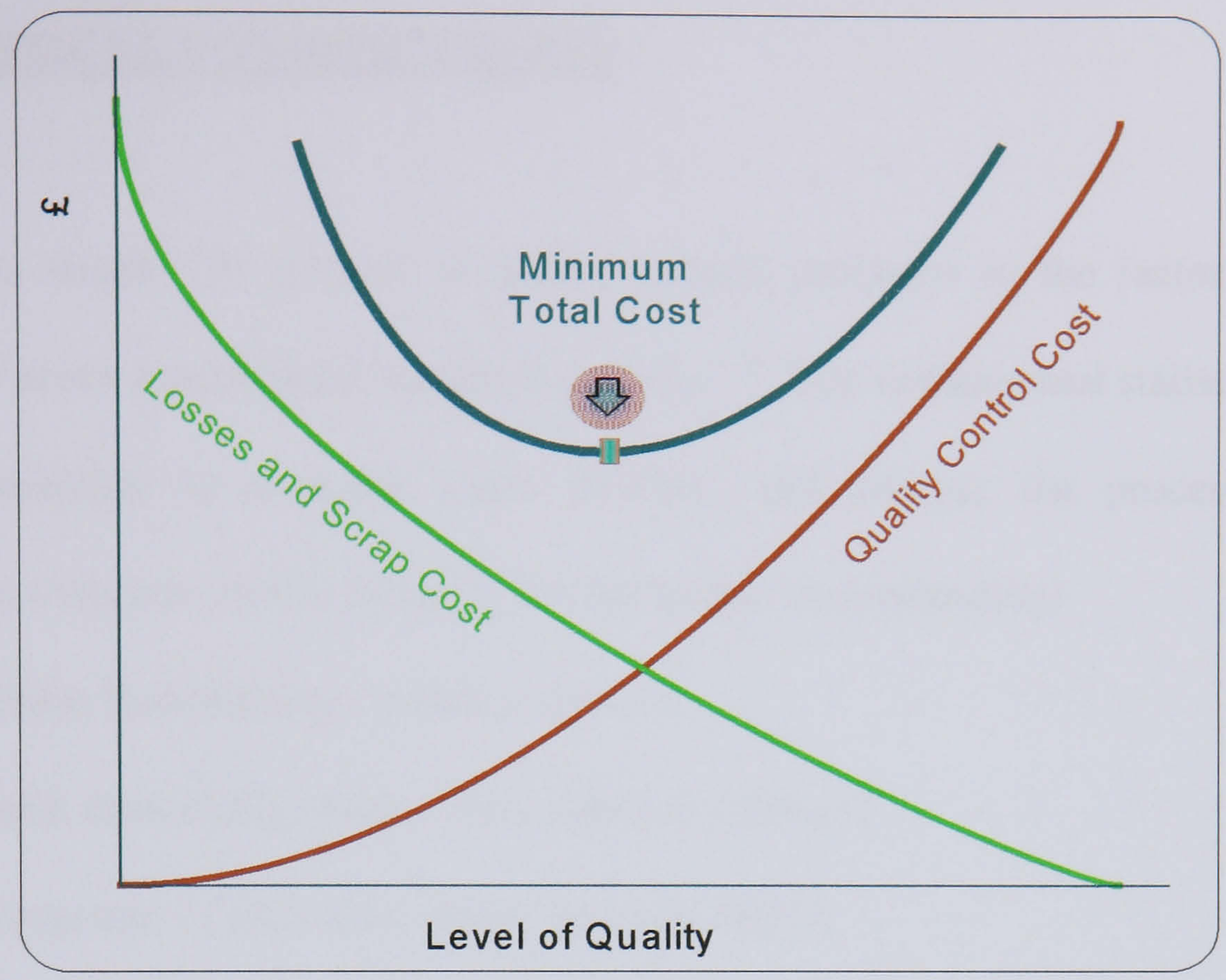

Figure 2:3 - Total Cost of Quality

Figure 2:3 illustrates the relationship between total cost, quality control cost and scrap cost. The total cost is a function of quality cost and cost of scrap, rework, and loss of goodwill. However, the magnitude of the total cost is a combination of both quality control cost plus the cost of losses. Within the quality level there is a point of minimum cost where the optimum operating conditions (specification) are present. Therefore, failures and not reaching specifications within the product or the manufacturing process may result in money waste and quality disappointment, which will be reflected on the overall cost and profit.

2.6 STATISTICAL CONTROL CHARTS

As Ishikawa stated, "95 percent of quality related problems in the factory can be solved with seven fundamental quantitative tools" [27]. The fundamental statistical tools aid the researcher to examine, scan, monitor, and analyse the process. These fundamental tools are: (Refer to figure 2:4 for further understanding)

1. Process flowcharting - *{what is done?}*
2. Check sheets/tally charts - *{how often is it done?}*.
3. Histograms - *{what does variation looks like?}*.
4. Pareto analysis - *{which are the big problem?}*.
5. Cause and effect analysis and brainstorming - *{what causes the problem?}*.
6. Scatter diagrams - *{what are the relationships between factors?}*.
7. Control charts - *{can the variation be represented in a time series? And which variation to control and how?}*.

As seen above, Shewhart Control Charts is one of the seven quantitative quality tools. Control charts enhance the analysis of a process by showing how that process is performing over time. Therefore, combining these charts with an appropriate statistical summary will provide a clear understanding for those who are studying certain process, and enable them to make decisions concerning future production. Control charts describe whether the process is in terms of current performance or not. Generally Control charts serve two basic functions, which are[39]:

1. Control charts are considered as decision-making tools. They provide an economic basis for making a decision as to investigate for potential problems, to adjust the process, or to leave the process as it is.

2. Control charts are problem-solving tools. They assist in the identification of problems in the process. They help to provide a basis on which to formulate improvement actions.

Modern quality goal is to produce a product or a service that exhibits little or no variation if afforded. Variation -where no two items or services are exactly the same- exists in all process. Variation varies depending on the criteria of investigating them and tackling these variations. Variation has mainly three types (a) within piece variation (b) piece to piece variation (c) time to time variation. Normal variation within certain processes is studied by sampling the process. Control charts monitor the variation within the process and using statistical measurements process variation is recorded on different control charts, which show changes in the process, allowing early detection of process changes, which reduce rework, scrap, process delays and money loss[13].

Two main hazardous criteria should be tackled and omitted from any production process, as they represent a risky situation on quality. These two criteria are (a) deviation from target specifications, and (b) excessive variability around target specification.

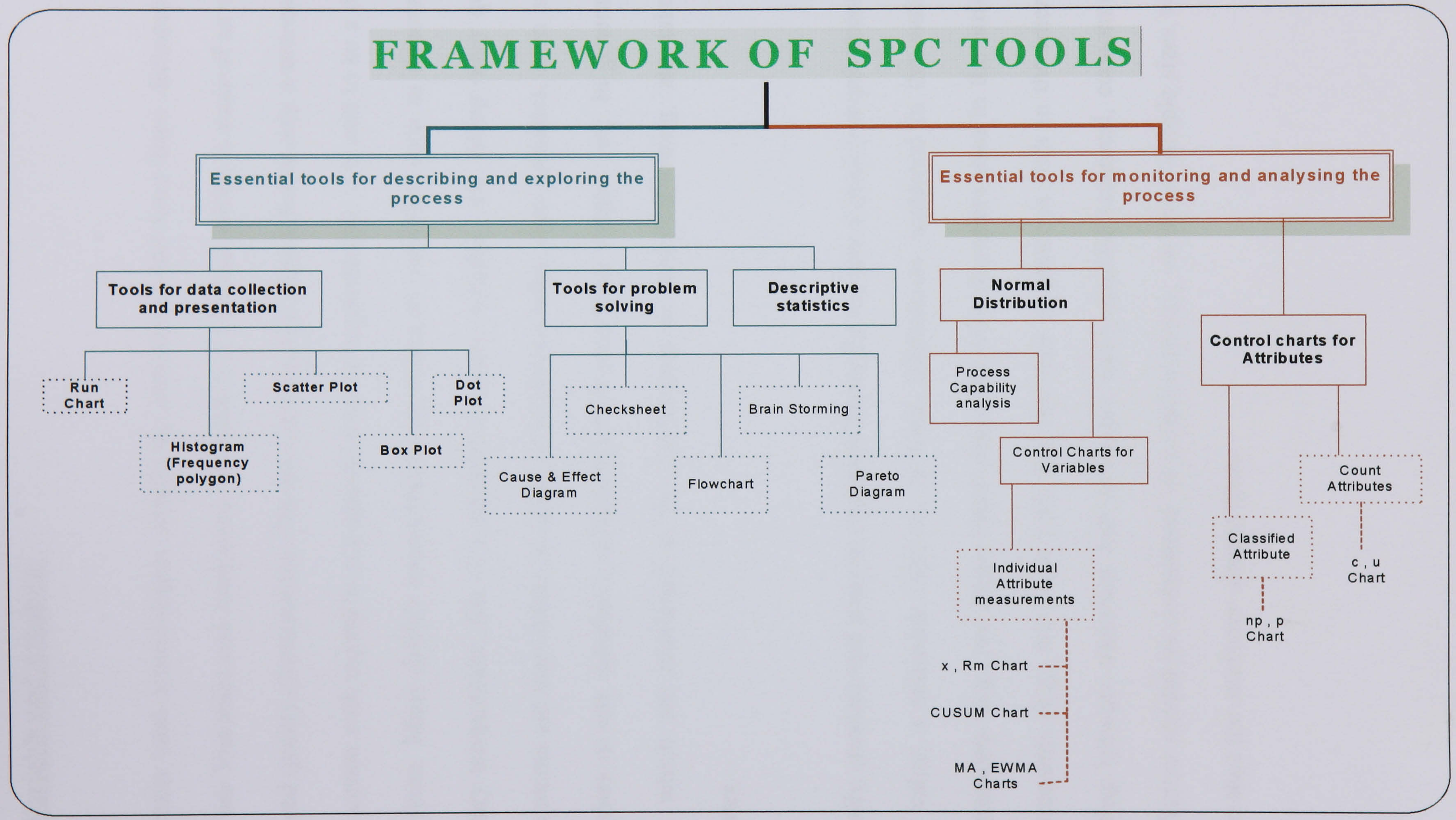

Figure 2:4 – Framework of Statistical Process Control Tools

2.6.1 Variable and Attribute

Two main vital terminologies should be understood and have been distinguished; which are Variables and Attributes. A Variable is a record, which is made of an actual measured quality characteristic. On the contrary, if a record shows only a summary or classification with regard to any specified set of requirements; it is said to be a record of attribute. Many quality requirements are stated as variables, such as dimensions, boundary temperature, life of a product in hours, weights…etc. Many other quality requirements are also stated in terms of attributes rather than variables, such as if a pen writes or not, whether a surface finish is smooth or rough …etc. In general, the items, which are examined to be conforming or non-conforming, are taken as attributes.

Normally, product and process engineers typically express a quality requirement as a target value, a tolerance interval, or both. Information contained in the variable's measurements (e.g., we know more about the wire diameter) than in the attributes measurements (e.g., we know only that the wire diameter is with in the interval)[7]. Choosing between variables and attributes, many specific technical, economic and time factors should be considered. In round numbers, variables sample sizes will be smaller than the attributes sample sizes.

2.6.2 Control Charts for variable Measurements

Control charts, like any other basic tools for quality improvement, are relatively simple to use. Control charts have three basic applications: (1) to establish a state of statistically controlled process, (2) to monitor a process when the process goes out of control, and (3) to determine process capability.

This current research is concentrating on small samples of variable data, and their behaviour using the Conventional Shewhart SPC charts. While the attribute data assume only two values, good or bad, pass or fail. Attributes usually cannot be measured, but they can be observed and counted and are useful in many practical situation. Usually, attributes data are easy to collect, often by visual inspection. Many accounting records, such as percent scrapped, are readily available. However, one drawback in using attributes data is that large samples are necessary to obtain valid statistical results. For these reasons, the main interest in the current investigations is to understand the background knowledge of variable control charts such as $\overline{X}$, R Charts.

There are four major models of control charts for variable measurement, which are[40]:

1. Shewhart Control Charts for Variables.
2. Cumulative-Sum (CUSUM) Control Charts.
3. The Exponentially Weighted Moving-Average (EWMA) Control Charts.
4. Moving average control charts.

In 1920's, Dr. Walter A. Shewhart set elaborated charts, which test, monitor and control variability within a process. Shewhart developed control charts to detect

various variability and distortion in the process. Shewhart Control charts for variables are:

1. $\overline{X}$, R chart (Average, range chart).
2. $\overline{X}$, s chart (sample average and standard deviation chart).

The first step in developing $\overline{X}$, R chart is to gather data. Usually, about 25 to 30 samples are collected. Samples between size 3 and 10 are generally used, with samples size of 5 being the most common. The number of samples is indicated by k, and n denoted the sample size. For each sample I, the mean is denoted $\overline{X_i}$ and the range by R_i are computed. The values are then plotted on their respective control charts. Next, the overall mean and overall average range calculations are made using equation 2.1 and 2.2, and these values specify the centre lines for the $\overline{X}$, R chart.

$$\overline{\overline{X}} = \frac{\sum_{i=1}^{k} \overline{X}_i}{k} \qquad \cdots \textit{Equation } 2.1$$

$$\overline{R} = \frac{\sum_{i=1}^{k} R_i}{k} \qquad \cdots \textit{Equation } 2.2$$

The average mean and average range are used to compute control limits for $\overline{X}$, R chart. Control limits are easily calculated using the Shewhart formulas, as shown in equation 2.3, 4, 5, and 6[41].

$$\textit{Upper Average Control Limit} = UCL_x = \overline{\overline{X}} + A_2 \overline{R} \qquad \cdots \textit{Equation } 2.3$$

$$\textit{Lower Average Control Limit} = LCL_x = \overline{\overline{X}} - A_2 \overline{R} \qquad \cdots \textit{Equation } 2.4$$

$$\textit{Upper Control Range Limit} = UCL_R = D_4 \overline{R} \qquad \cdots \textit{Equation } 2.5$$

$$\textit{Lower Control Range Limit} = LCL_R = D_3 \overline{R} \qquad \cdots \textit{Equation } 2.6$$

Where the constants D_3, D_4 and A_2 depend on the sample size and can be found in special tables. Figure 2:5 shows a standard shape for $\overline{X}$, R chart.

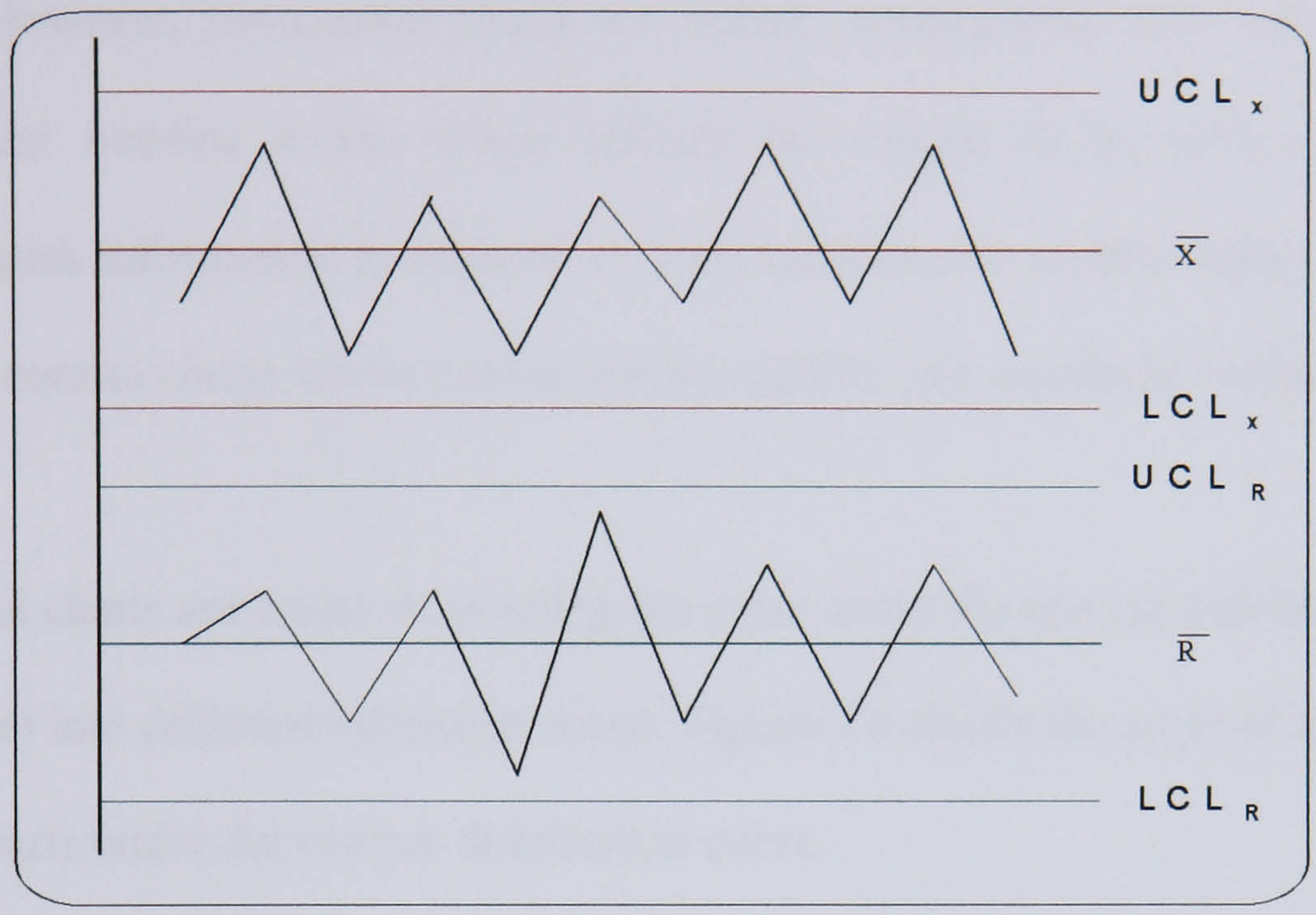

Figure 2:5 – Control chart

The control limits represent the range between which 99.73% of all points are expected to fall if the process is in statistical control. If any points fall outside the control limits or if any unusual patterns are observed, then some special cause has probably affected the process. The process should be studied to determine the cause. If special causes are present, then they are not representative of the true state of the statistical control and all the calculation for the centreline and control limits will be biased. The corresponding data points should be eliminated, and new values for the average of mean, average of range, and control limits should be computed.

2.7 PRE-CONTROL CHART

Pre-control charts are used as any other control charts to detect variation within a process. However, pre-control charts are mainly distinguished from other charts by having clear warning zones, which indicate the weight of the error, and provide primary quick information to respond to such variation for variable data only. Unlike, Shewhart control charts where control limits ought to use calculated control limits[42].

Pre-control charts are based on dividing the areas under the normal distribution curve (bell curve) into different indication zones. Figure 2:6 shows the areas of the pre-control charts under the normal distribution curve.

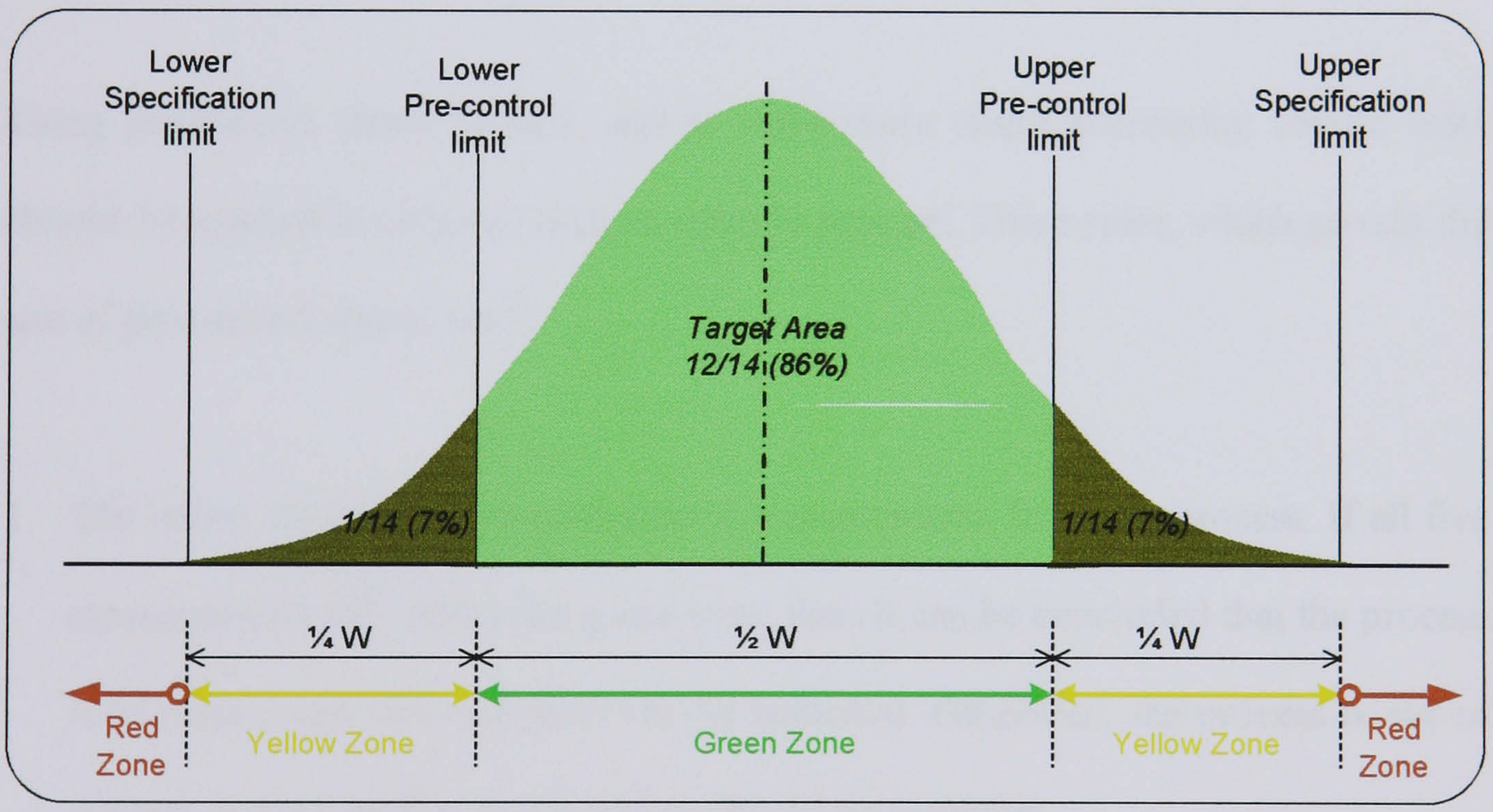

Figure 2:6 – Pre-control Areas

As seen above, the target area (Green zone) represents 86% of the population. The area between the Upper Pre-control limit (UPCL) and the Upper specification limit equal to 7%, while on the other side, the area between the lower pre-control limit and

the lower specification is also equal to 7%. Out of these three areas is the red zone, which represents the out of control status. Also, Figure 2:7 represent a pre-control chart.

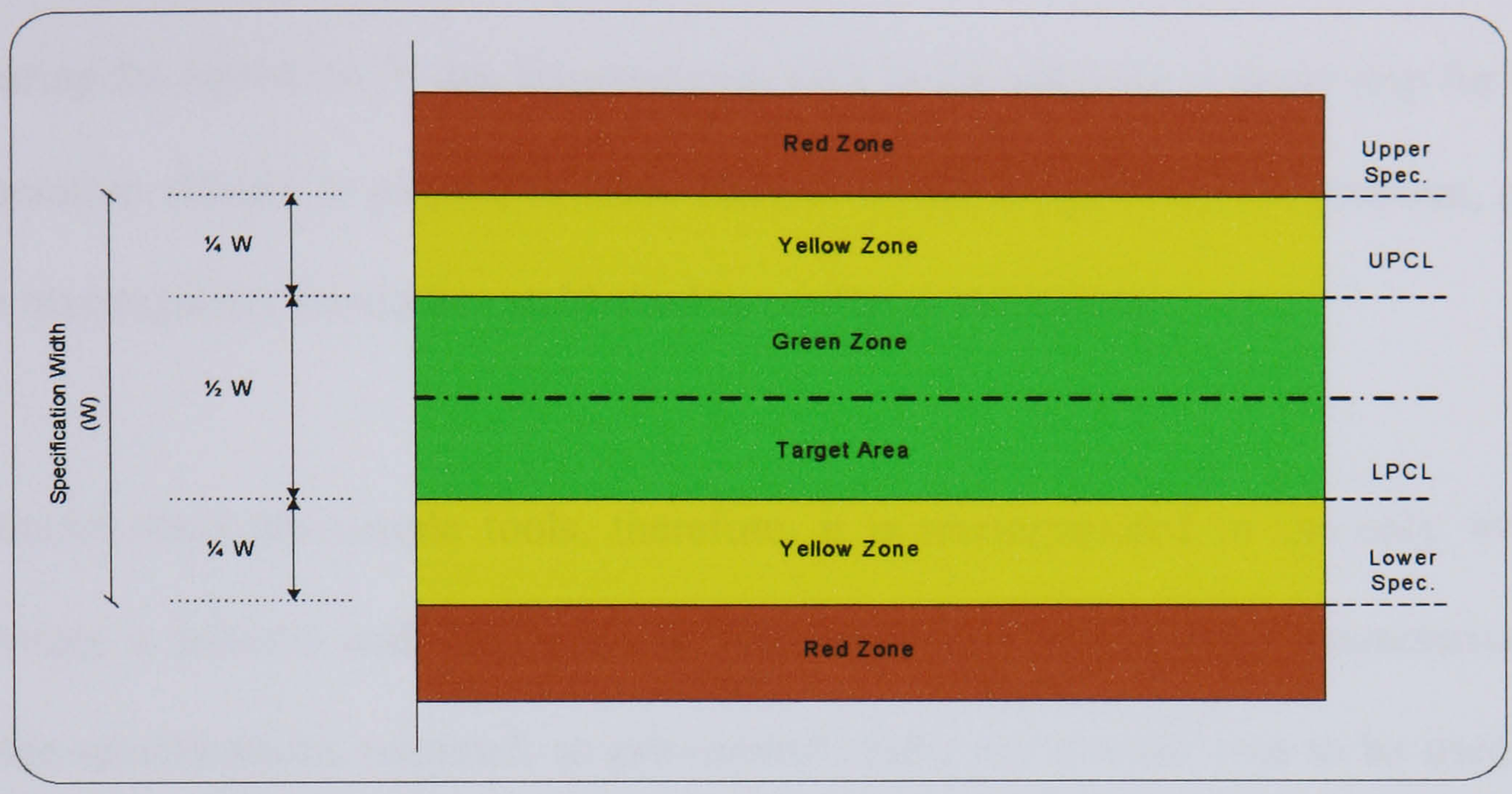

Figure 2:7 – Pre-Control Chart

Using pre-control charts is easy, and to make these charts successful; certain rules should be applied in order to analyse specific process. These rules, which govern the use of pre-control charts, are[43]:

1. The initial sample of five consecutive measurements from the process. If all five measurements fall within the green zone, then it can be concluded that the process is in control and full operation can be launched. Otherwise, the process is out of control, and a specified investigation should be launched.
2. During the operation, two consecutive measurements from the process are periodically taken, and if:

   - Both are in the green zone, or if one is in the green zone and the other in the yellow zone, then continue the operations.

- Both fall in the same yellow zone, and then adjust the operation setting.
- Both fall in different yellow zones, then stop the operation and investigate the causes of increased variation.

3. During the operation, if any measurement falls in the red zone, a direct stop for the operation should be placed, because there is an out of specification problem, and an investigation should be established to configure the causes.

Pre-control chart are simple tools, therefore, it is recommended to use only when monitoring a process and verifying the conformance of the process characteristics with the specifications required, as pre-control charts are a weak tool to be used to improve the process, and it is a major disadvantage for such type of quality control charts.

Some researchers does not encourage companies to use such control charts, as this type of charts is based on specification limits as the red zone area, however, control limits should be the out of control limitation for the process. Some managers, using this chart, often draw wrong conclusion when they take specifications as their limits. This is wrong, because control charts are based on variability of the process, while specification limits are determine by designers before the start of the process. It is obviously seen that there is now relation between the two limits. Also, specification limits are based on individual measurement, while the control charts are based on average measurements of samples. For such reasons, it is wrong to base decisions on specification limits.

2.8 ZONE CONTROL CHART

The zone control chart is another type of control chart, which relies on weighting each measurement in the operation based on its location from the mean line. If the point is near the centre line, it has low weight, and if it far away it has a high weight.

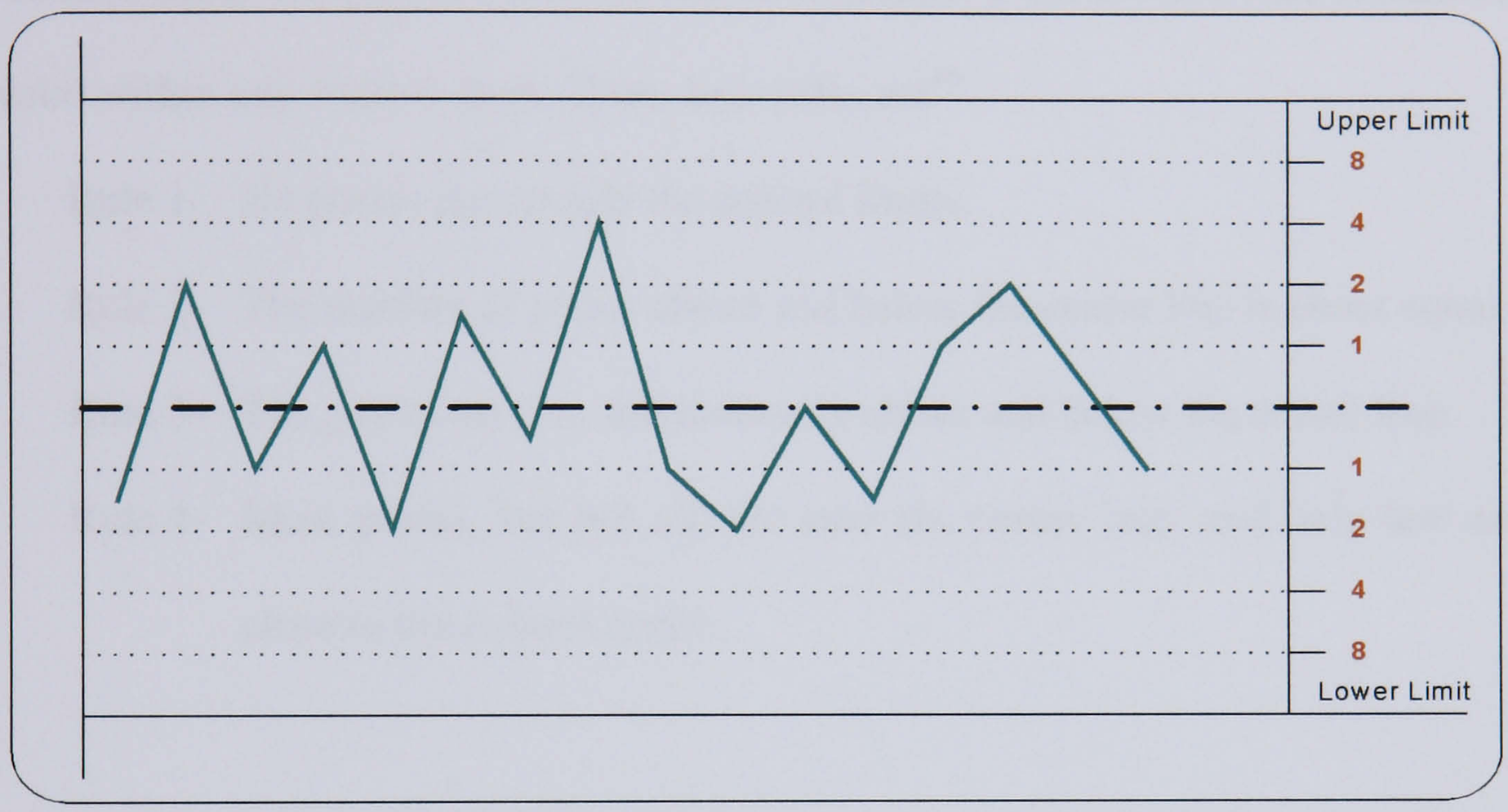

Figure 2:8 - Zone control chart

Figure 2:8 shows a zone control chart, where there is a weight scale on the right hand side of the chart. Each point in the control charts is given a score of 1, 2, 4 or 8, depending on which band it falls into. Therefore, it is concluded empirically that the process changes if the cumulative summation of the score exceeds 7, noting that the cumulative sum is reset to zero whenever the plot crosses the centreline[45].

2.9 CONTROL CHART DECISION RULES

Control charts can normally present process behaviour; control charts will give a general view on the process behaviour and whether it is in control (stable) or out of control (unstable). There are four general rules, which can give a quick decision about any control chart. Therefore, a process can be in control if all of these four conditions are valid within any control chart. These four rules are[46]:

Rule 1- No points are outside the control limits.

Rule 2- The number of points above and below the centre line is about equal.

Rule 3- The points seem to fall randomly above and below the centre line.

Rule 4- Most points, but not all, are near the centre line, and only few are close to the control limits.

The assumption behind these four rules is that the distribution of sample means is normal. The central limit theorem in statistics states that the distribution of sample means tends to be a normal distribution as the sample size increases regardless of the original distribution. For small sample sizes, the distribution of the original data ought to be reasonably normal for this assumption to be valid. Furthermore, using the Central Limits Theorem (CLT), which states that despite the nature of the data distribution, averages of samples are normally distributed[47].

Rule 1 originated from the fact that the lower & upper control limits are computed to be three standard deviations from the overall mean. Thus, the probability that any sample mean falls outside the control limits is very small. Rule 2 and 3, are based on the fact that the normal distribution is symmetric, therefore, the same number of

points fall above as below the centre line. Hence, since the mean of the normal distribution is the median, about half of the points fall on either side of the centre line. Rule 4 relays on the fact that 68% of a normal distribution falls within one standard deviation (1σ) of the mean (μ); thus, most, but not all, points should be close to the centre line. Knowing that these characteristics will hold provided that the mean and variance of the original data have not changed during the time the data (measurements) were collected, means, that the process is stable.

2.10 INTERPRETING PATTERNS IN CONTROL CHARTS

Control charts reflect the behaviour of a process through monitoring it by selecting samples and analysing them. Measurements on control charts follow certain pattern. These patterns represent different points, which helps analysts to detect variability (out of control status) and its cause. These patterns are:

A. ***One point outside control limits*** - in special cases a measurement can be out of the control limits. Usually R chart provide a similar situation of oddness for such measurement (see figure 2:9). The reasons why this happens can be an error in calculation with in the control charts, or it can happen by chance, otherwise it can happen due to sudden change in the process such as sudden power surge, tool failure, or incomplete process.

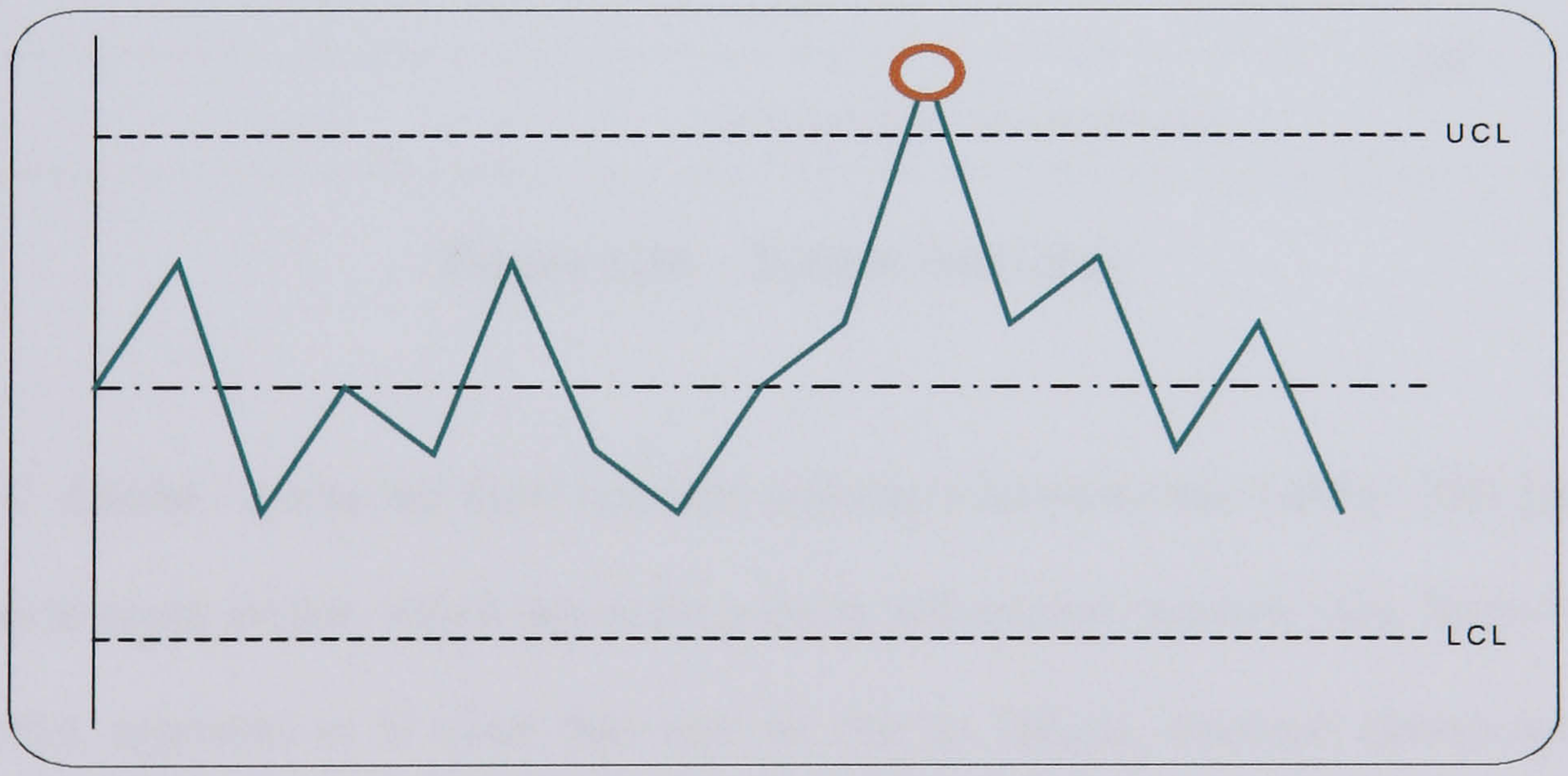

Figure 2:9 - Point out of limit

B. ***Sudden shift in the process average*** - it is when consecutive points (normally eight points)[48] fall on one side of the centre line. This is caused by a change in the machine set-up or new operator existence. If the shift is above the centre line in the R chart that means the process is less uniform. On the other hand, if the shift in the R chart is down of the centre line then, it means the uniformity of the process has improved. Another case, which indicates a sudden shift in the process average, happens when two of three consecutive points are above two standard errors of the centre line. Also, if four of five points below one standard error. (See figure 2:10).

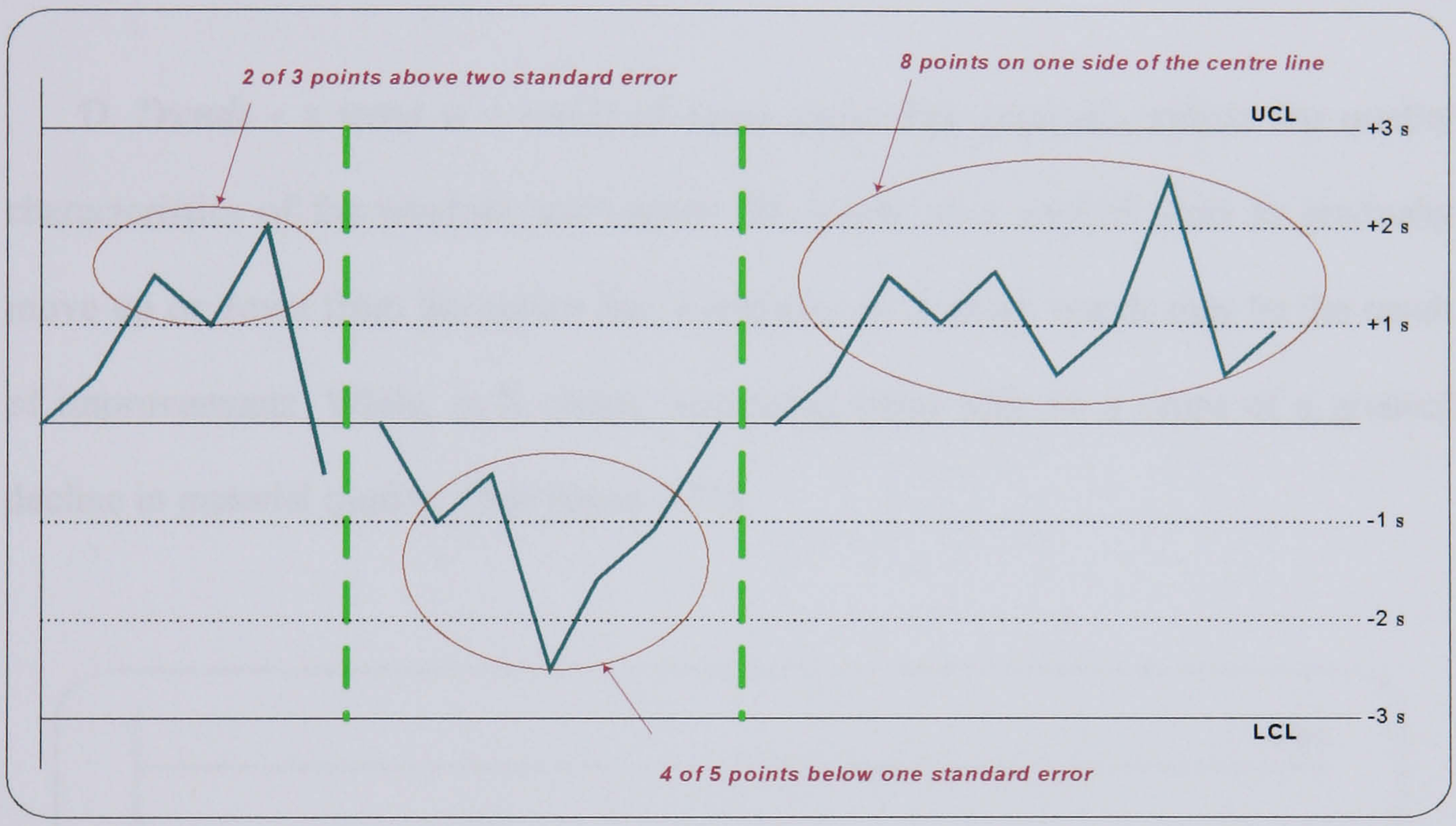

Figure 2:10 – Sudden shifts chart

C. ***Cycles*** - cycles are short repeated patterns with peaks and valleys. This pattern is due to some causes, which are in the process and appear regularly (see figure 2:11). If cycles appeared in X chart that may be due to fatigue, seasonal causes such as temperature or humidity, or changes between day and night. But if it appears in R charts, that may be due to maintenance schedules, or differences between shifts.

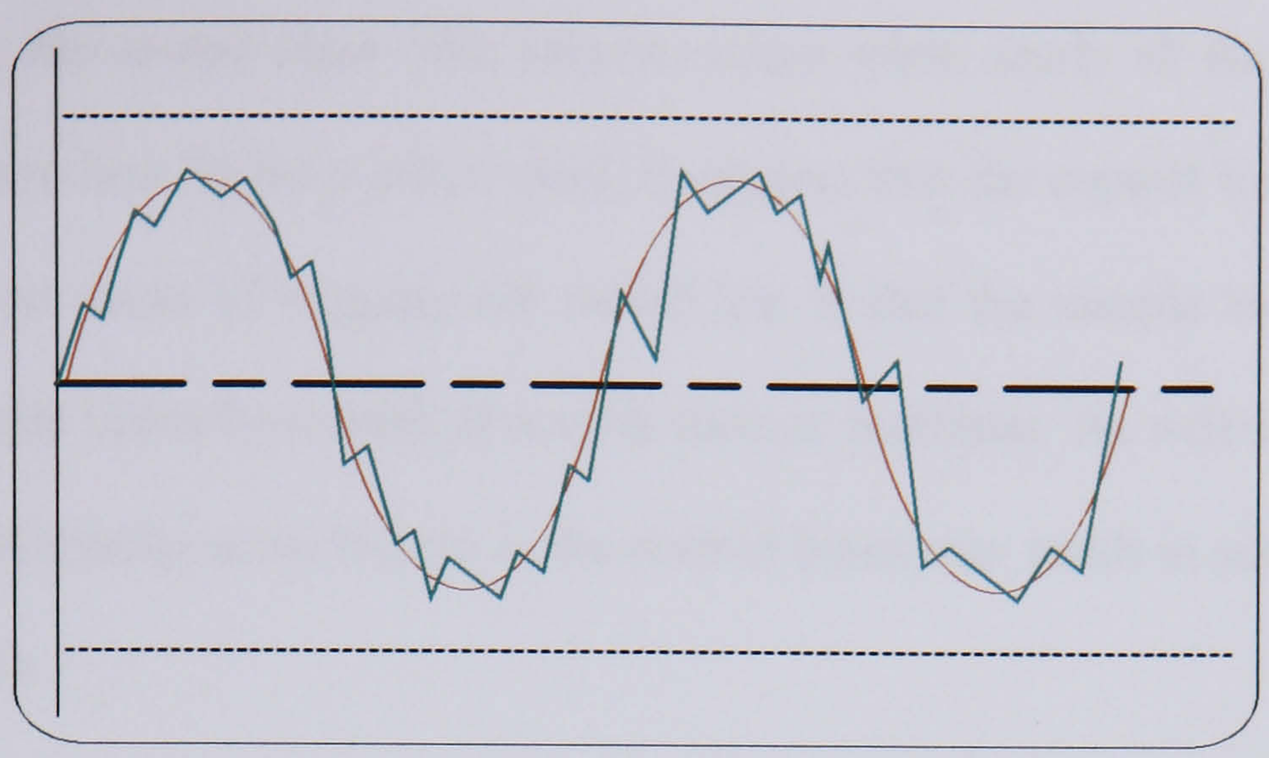

Figure 2:11 - Cycles

D. ***Trends*** - a trend is a result of some cause that gradually affects the quality characteristics of the product and causes the points on a control chart to gradually move up or down from the centre line. Generally, in X chart, trends may be the result of improvements. While, in R charts, increasing trend may be a cause of a gradual decline in material quality. (See figure 2:12).

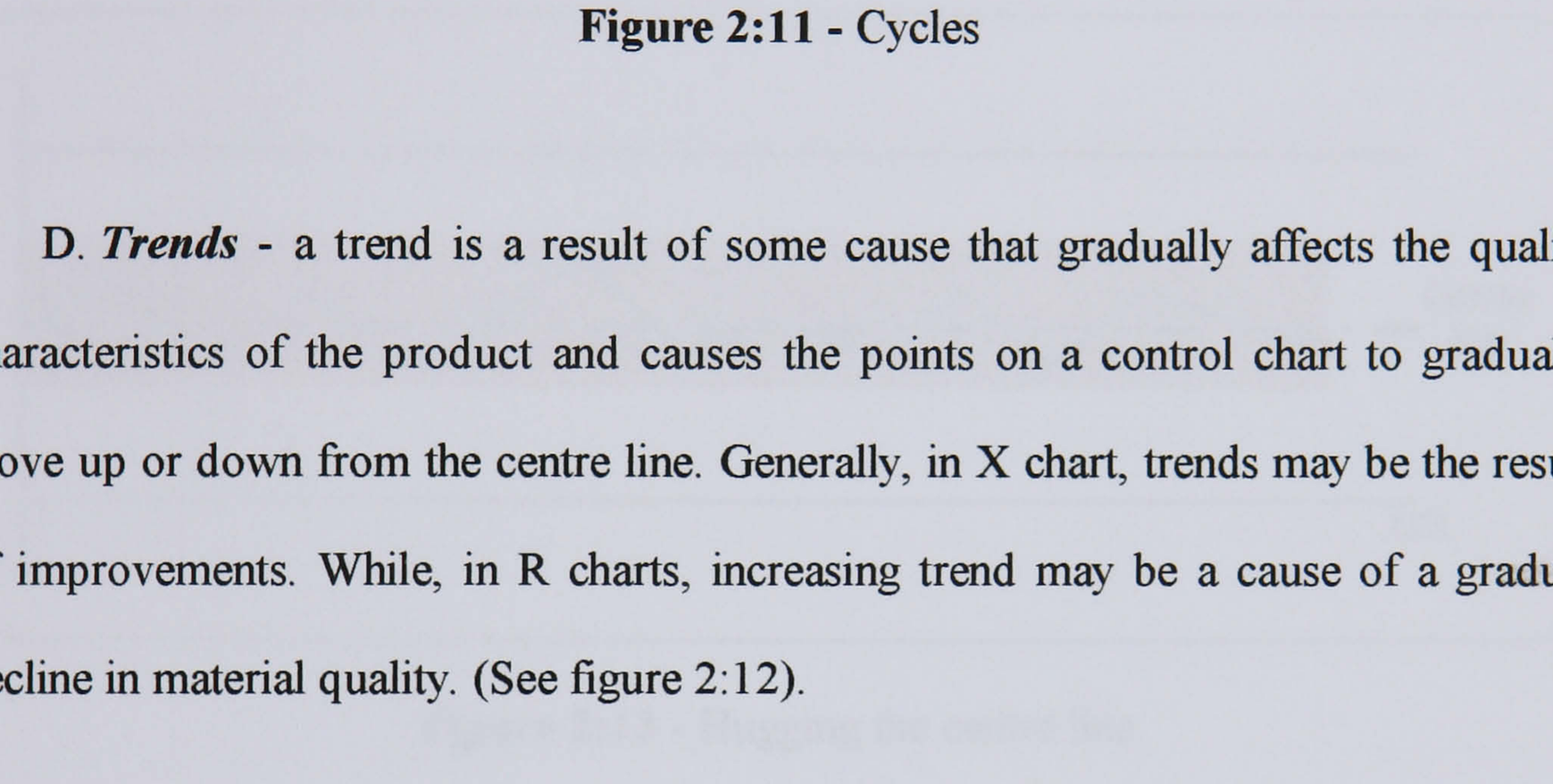

Figure 2:12 - Trend

E.***Hugging the centre line*** - this pattern occurs when nearly all the points fall close to the centre line. In the control chart, it appears that the control limits are too wide. A common cause of hugging the centre line is that the sample indicates one item systematically taken from each of several tests or machines. As well some times, an error with calculating some factors in the control limits may result in such patterns. (See figure 2:13).

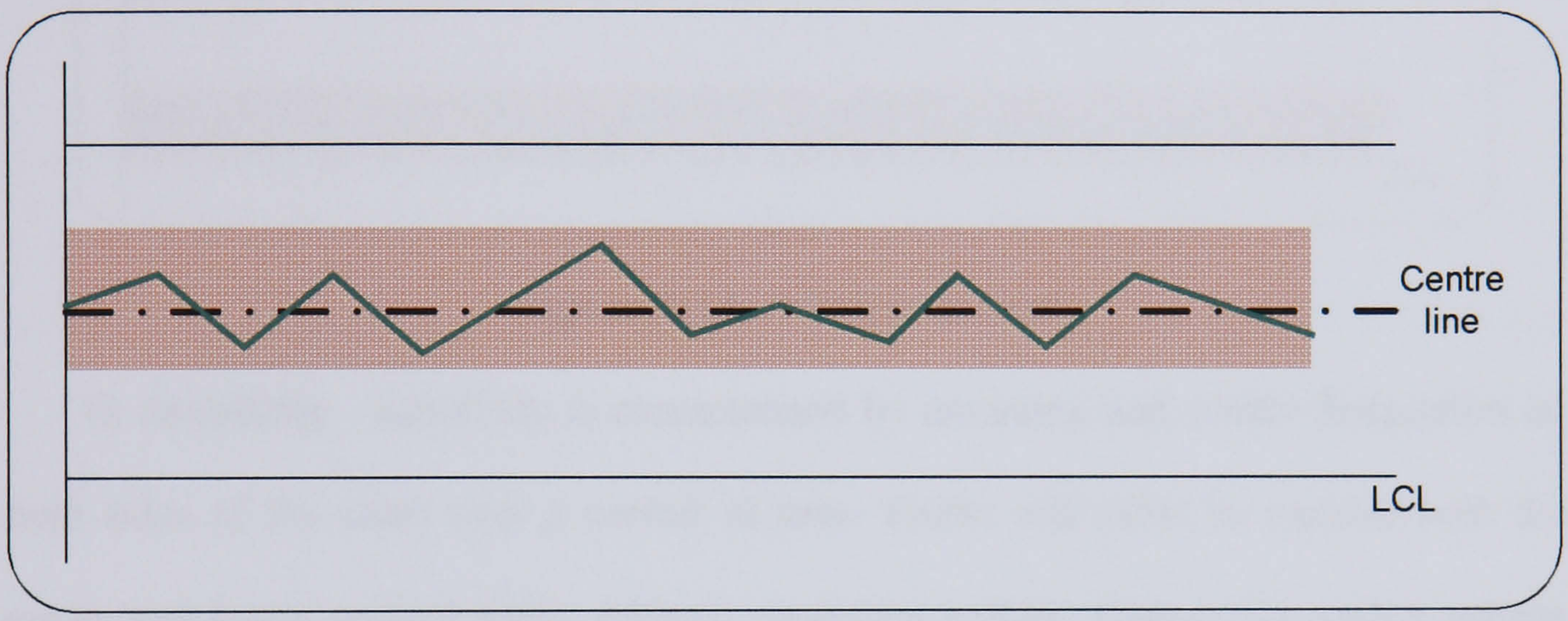

Figure 2:13 - Hugging the centre line

F.***Hugging the control limits*** - in this pattern many points are near the control limits with few points in between. This pattern is often called a mixture, as it is a combination of two different patterns in the same control chart; and a mixture can normally be split into two separate patterns. This pattern normally occurs when two different inputs are used in one process, i.e. the different material supplies. (See figure 2:14).

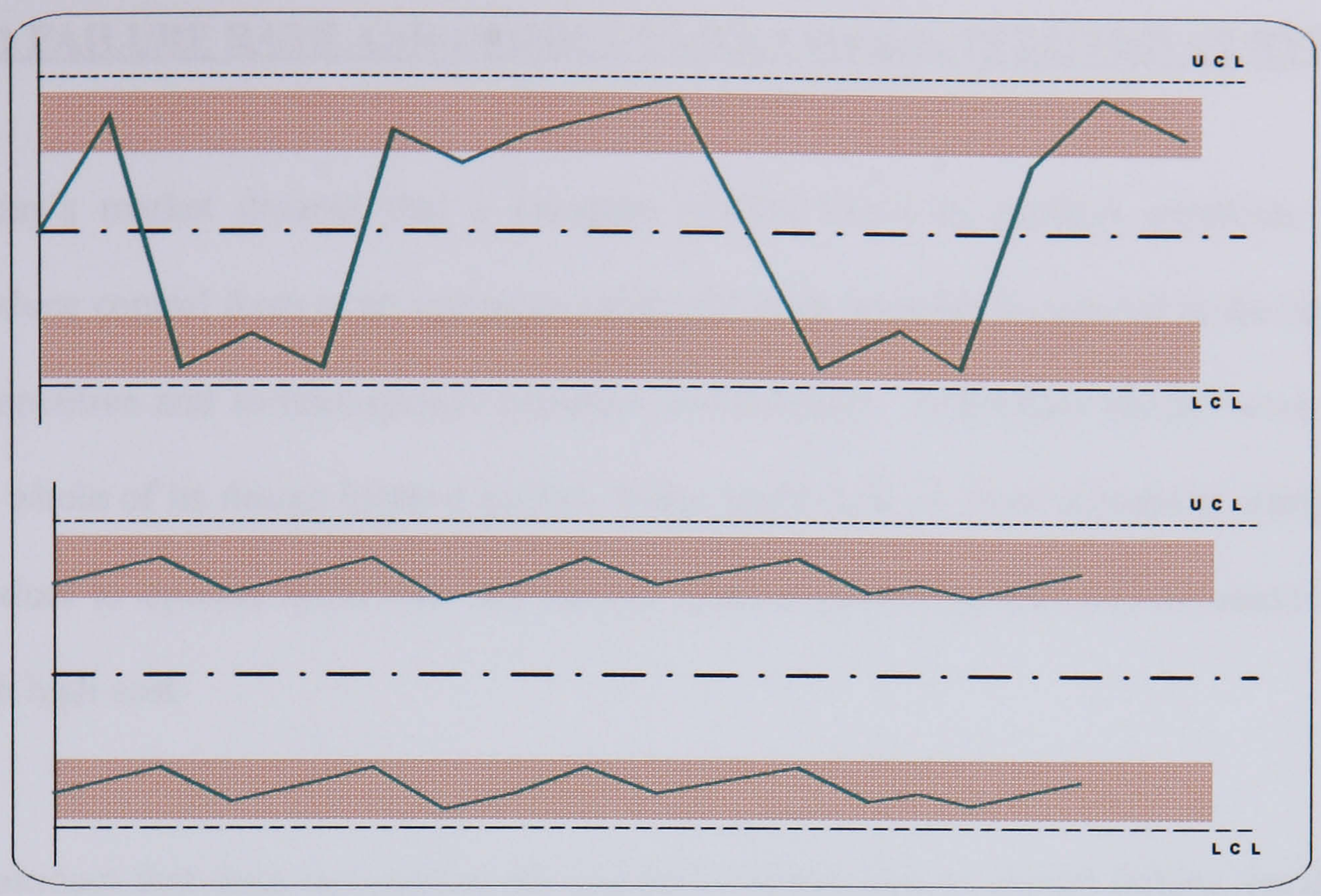

Figure 2:14 - Mixture pattern

G. ***Instability*** - instability is characterised by unnatural and erratic fluctuation on both sides of the chart over a period of time. Points will often lie outside both the upper and lower control limits without consistent pattern. Causes for such a pattern may be difficult to identify. A general cause of instability is over-adjustment of machines. (See figure 2:15).

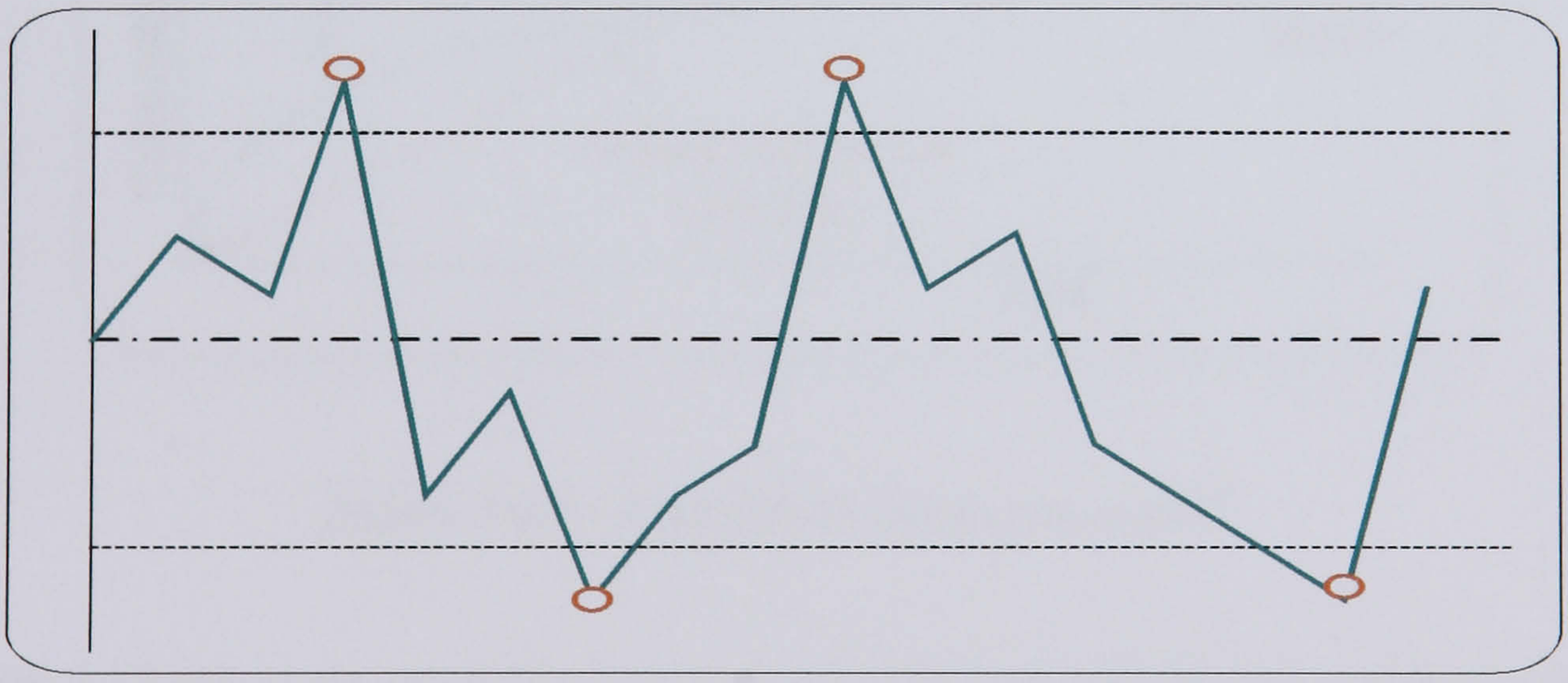

Figure 2:16 - Instability

2.11 FAILURE RATE AND PRODUCT LIFE CHARACTERISTICS CURVE

Today's market dictates that a company should know its product reliability and produce control them in an optimum reliability level, in order to succeed in the highly competitive and technologically complex environment. A product should work for the whole of its design lifetime period. In the same time, it is not advised to design a product to operate more than the desired lifetime period, as this will be associated with high cost.

A product that does not survive its expected lifetime due to certain failure; result in losses to the company profits. Product failures range from minor failure to major failures. Reliability engineering was born out of the necessity to avoid such failures.

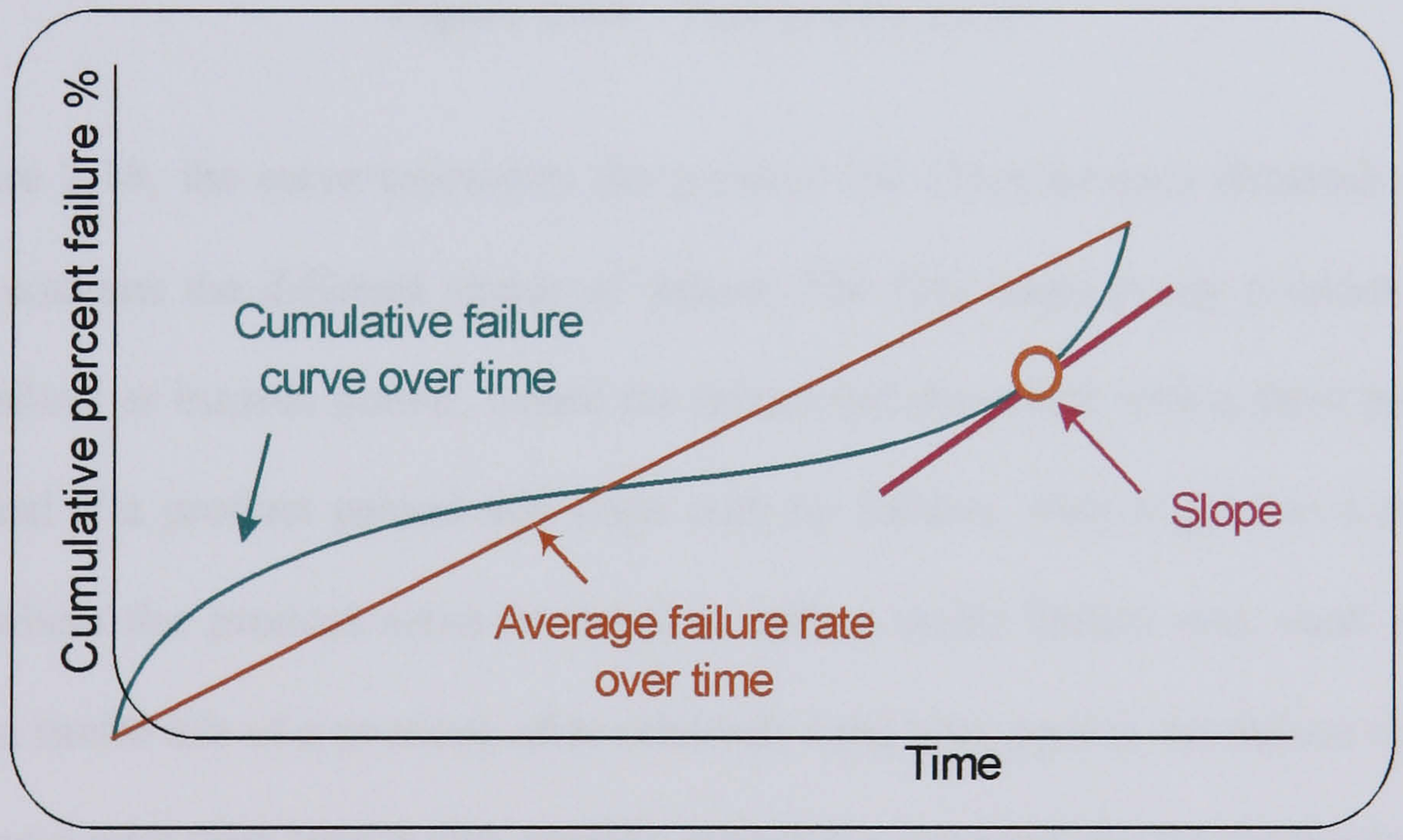

Figure 2:17 – Cumulative failure rate curve[49].

Figure 2:17 shows the cumulative percentage of failures against time, where the slope of the obtained curve at any point (the purple line and star point) represents the

instantaneous failure rate, while the red line represents the average failure rate over this whole time interval.

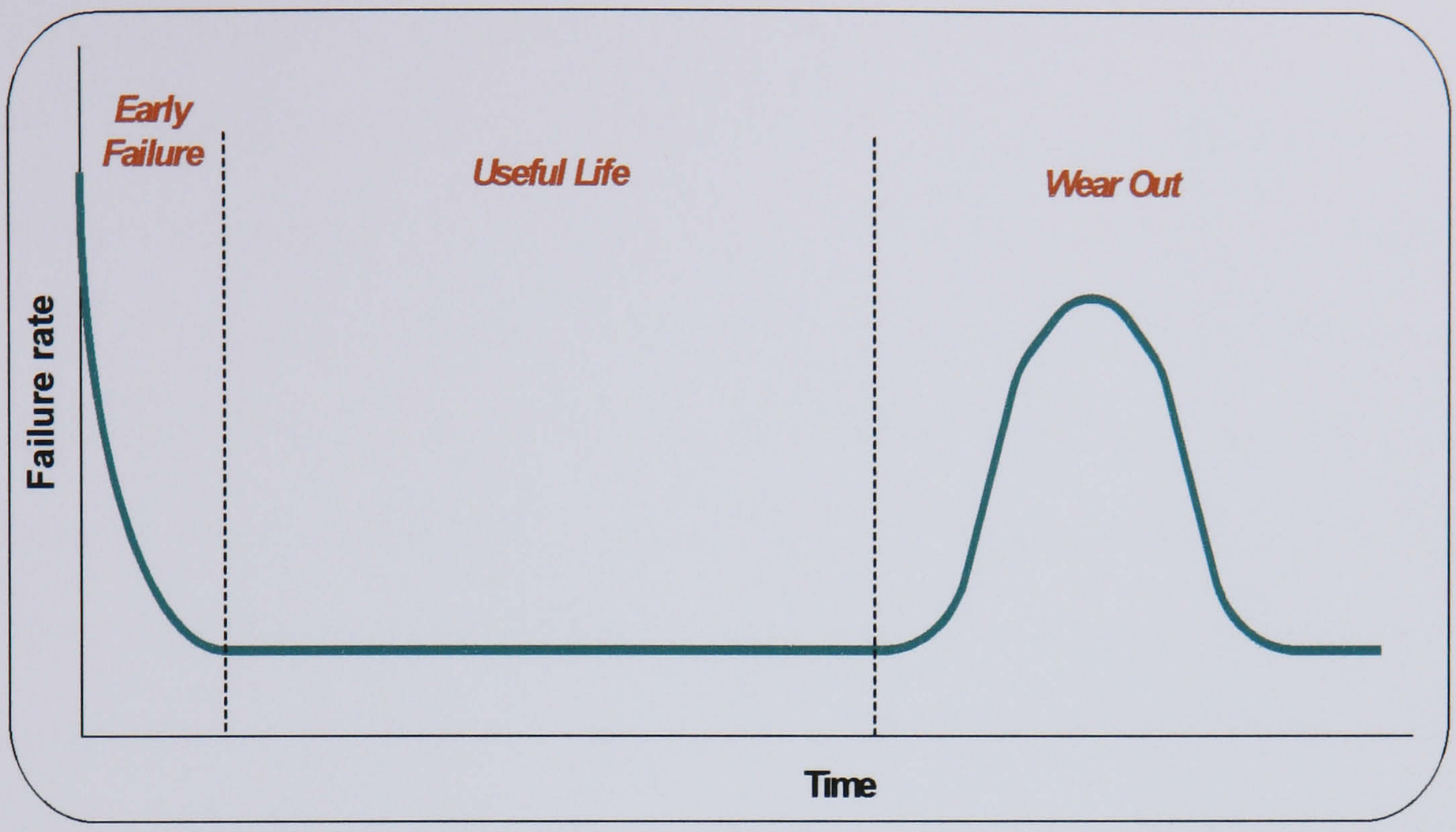

Figure 2:18 – Failure rate curve

In figure 2:18, the carve represents the product life characteristics (Bathtub curve), which contains the different stages of failure. The first stage in any product life, is early failure or burn-in period, where the failure rate decreases with a short period of time, and if a product passed this stage with no failures, then it goes to a constant stage where the product serve its function with a stable failure rate, such stage is called a useful life of a product, after relatively long time period, the failure rate start to increase with time, and in this specific period the assumed life for the product start to decline and failures start to be expected.

As a result, manufacturer should be concerned with the reliability (time of serving or operation of a product) so he can insure a good level of customer satisfaction and gain

their trust, which will end in enforce his market share and gain high reputation and profits.

2.12 WEIBULL MODEL

As Weibull distribution has a flexibility and ability to model a wide range of failure rates, it has been used successfully in many applications as a purely empirical model. The Weibull reliability equation[50] (see eq.2.7 for Weibull cumulative distribution function -CDF) consists of three main parameters, which are shape factor (β), location parameter (γ), and characteristic life factor or scale factor (η). Normally the location parameter (γ) is equal to zero; which means the failures start at the origin. The shape factor in Weibull distribution is related to the behaviour of the hazard function. Therefore, if β equal to 1 that means the hazard function is constant, while if β is greater than 1, that means the hazard function is increasing. When β is less than 1, it indicates that the hazard function is decreasing (see figure 2.19 for Weibull failure rate function). There is a special case, which this research is concerned about, when β is equal to 3.44 then the Weibull distribution is a close approximation to the normal.

$$R(t) = e^{-\left(\frac{t-\gamma}{\eta-\gamma}\right)^{\beta}} \quad \cdots \text{ Equation } 2.7$$

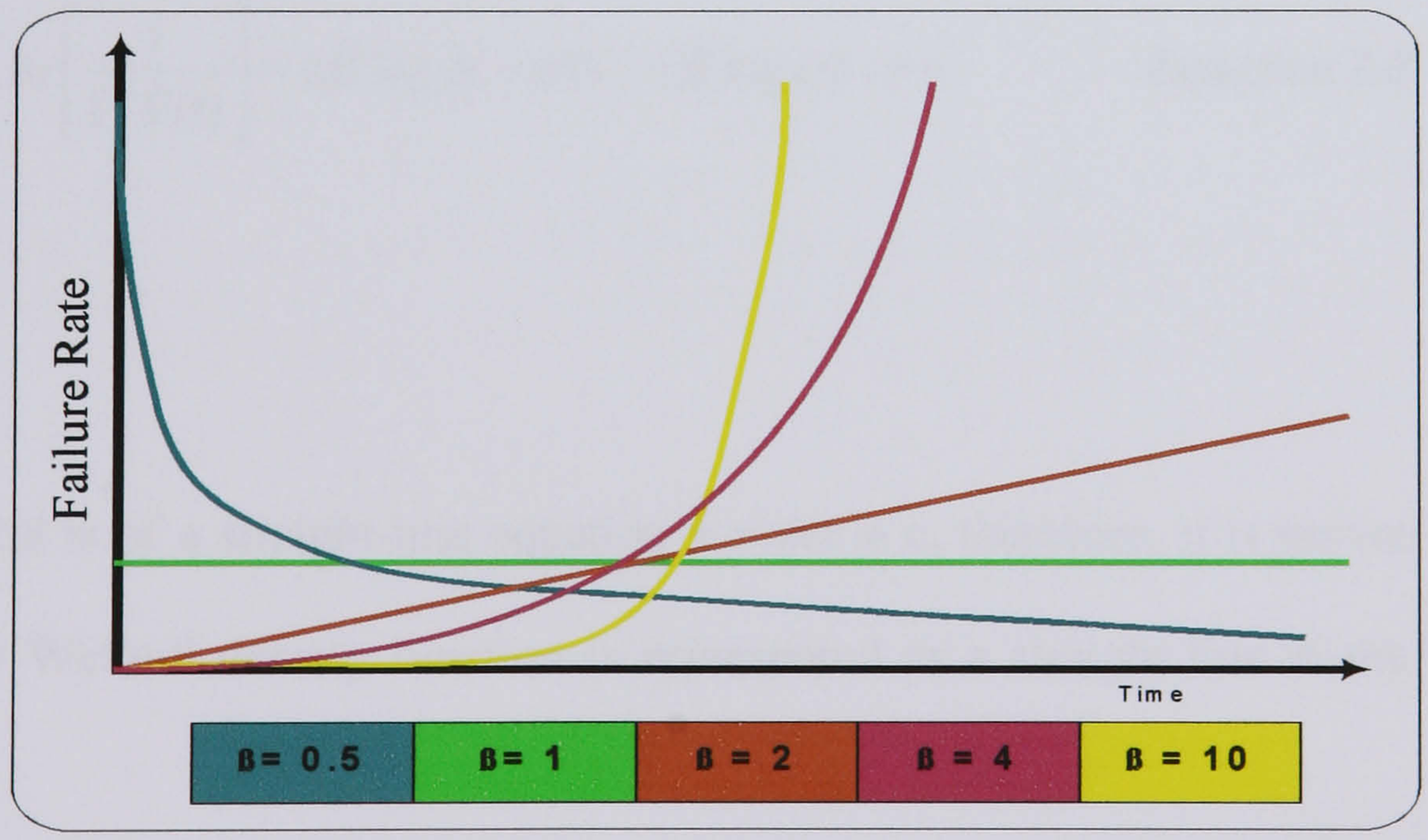

Figure 2:19 - Weibull failure rate function

Weibull plots consist of two axes, Vertical and Horizontal. Horizontal axis is Log of order response, while the Vertical axis is Weibull cumulative probability expressed as a percentage and it is log-log (1-p) where p= (I-0.3)/(n+0.4) and I is the rank of observation and n is the number of observations.

The cumulative Weibull density function is represented as a straight line in the Weibull plot. The following derivation will prove this phenomenon

$$R(t) = 1 - F(t) = e^{-\left(\frac{t-\gamma}{\eta-\gamma}\right)^{\beta}}$$

Inverting both sides, will result in :

$$\left[\frac{1}{1 - F(t)}\right] = e^{+\left(\frac{t-\gamma}{\eta-\gamma}\right)^{\beta}}$$

Taking natural log for each side:

$$\ln\left[\frac{1}{1 - F(t)}\right] = \left(\frac{t-\gamma}{\eta-\gamma}\right)^{\beta}$$

By talking natural log to both sides again will result in:

$$\ln\ln\left[\frac{1}{1 - F(t)}\right] = (\beta \log(t-\gamma)) - (\beta \log(\eta - \gamma)) \qquad \cdots \textit{ Equation 2.8}$$

Equation 2.8 is of a straight-line equation y = mx + c, therefore, it is proven that the cumulative Weibull density function is represented as a straight line in the Weibull plot.

2.18 CHAPTER TWO CONCLUSION

This chapter formed a strong knowledge foundation in order to clearly understand the concept of quality, control charts, limits calculation, reliability, Weibull probability density and Weibull parameter. Also this chapter has showed that the area of research of this current work is a genuine concept to be analyses, as the use of Weibull distribution in control chart as quality tool has not been addressed clearly till the present time.

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CHAPTER 3

METHODOLOGY

3.0 CHAPTER THREE REVIEW

This chapter will introduce the logic, which should be used to establish a acceptable research path to ensure a scientific way to analyse and understand the current problem, to lead to a solution which will over come the problems of small samples size effects.

3.1 CONVENTIONAL PROCESS CONTROL MODELS FOR SMALL SAMPLE INSPECTION

In a small capacity manufacturing process, engineers tend to ensure a 100% inspection strategy to guarantee that all specifications are fulfilled and the level of quality is optimum. However, such application is infrequent in practice, due to the high cost associated with such inspection and the amount of scraps that may produce in the event of destructive testing.

Commonly, using small sample size to generate control charts, which is a subset of quality control methods, implies dealing with samples obtained from a stable process, and these samples are then compared with some functions of the long-term parameters (e.g. mean, variance). If the sample has a very small size (less than six), and the process variation is relatively large, then the results acquired will be very rough. Therefore, the crucial issue in such situations is not the small size of the sample as the large size of the process variance. Normally, it is accepted that the Coeffecient of Variation (CV) can measure such criteria, and it is the percentage of the standard deviation with respect to the mean. Coefficient of variation can show an indication of the variability of the process in terms of its mean[51].

Generally, Shewhart SPC charts can be effectively used with large sample size batches. When using small samples the probability of false notices can increase due to the rise of uncertainty with respect of small samples effect on the theory behind building up such control charts[52]. Small samples can be used for setting up the process, this can be obtained by two ways, firstly by using the known limits of the

process, which has been validated by large sample size testing history or by experienced engineers skills. Secondly, small samples can be the datum for establishing the limits of process control charts, by using each set of samples as a reference point to the next stage of limit calculation. Such method reduce time and money, which by it self a good enhancement of process control[53].

An essential sampling disadvantage of control charts in small sample size methods is the risk of not detecting a non-conformance item[54]. If a sample was deducted from a process and unfortunately, this sample did not contain a failed item (regarding specification), this item will be in the market as a passed item knowing that it is not, despite its high confidence.

Nowadays, conventional SPC chart show a clear lack in complying with the trend of industry to cut its cost specially when using small sample size. SPC philosophy and model is an easy method to be adopted in manufacturing environment, therefore, many researchers are trying to adopt new adjustments to the conventional SPC chart to be used with the association of small sample size inspection and provide reasoning and confident results.

3.2 PRESENT WEIBULL ANALYSIS USAGE IN PROCESS CONTROL WITH SMALL SAMPLE SIZE

Weibull distribution existed due to the unique research delivered by the Swedish Professor Waloddi Weibull. In his paper "A statistical Distribution Function of Wide Application" in 1951, he verified the ability of the Weibull distribution to be used with small sample sizes and to have a good flexibility to establish a good fit to reach reasonable results[55].

The Weibull Density Function is defined as follow[56]:

$$f(t) = \begin{cases} \dfrac{\beta}{\eta^{\beta}} t^{\beta-1} \exp\left[-\left(\dfrac{t}{\eta}\right)^{\beta}\right] & (\textit{for } t \geq 0) \\ 0 & (\textit{for } t < 0) \end{cases} \qquad \textit{... Equation 3.1}$$

Due to the dependency of a Weibull distribution on various parameters, its behaviour is constrained by the values that these Weibull parameters. The location parameter is normally equal to zero at the time of the start of the failure, which begins after initiating the part to operation life. The scale parameter and shape parameter are uncertainly calculated when using small samples, normally their values oscillate around the true unknown value[57]. A true demonstration of this fact is with a shape parameter β=3.44, the Weibull plot approximates to a normal distribution. This is a theoretical value (i.e. a parameter) not an estimation value obtained from a sample. Hence, there is no expectation of an exact value of 3.44 for the shape parameter from a small size sample, which has been drawn from the normal distribution, especially if sample size is small.

In addition, different small samples, regardless of the distribution they come from, may provide widely varying point estimates[58]. This is especially so when variance of parent distribution is large relative to the mean. Therefore, it can be seen that using small sample size is an uncertain method to predict quality and life behaviour for the manufactured product.

The nature of Weibull distribution distinguishes such distribution from others, by having different characteristic due to the altering of the shape parameter. The values of the shape parameter values vary the shape of the Weibull probability density function. As a result, Weibull distribution is a suitable distribution to be employed in various situations, by depending on the value of shape parameter; many distributions can be established (refer to table 3:1)

Beta	**p.d.f. Shape**
$\beta = 1$	Indicates Exponential distribution
$\beta = 2$	Indicates Rayleigh distribution
$\beta = 2.5$	Indicates Lognormal distribution
$\beta = 3.4$	Indicates Normal distribution
$\beta = 5$	Indicates peaked Normal distribution

Table 3:1 – Weibull shape parameter effect on p.d.f.

Weibull is a good model to use, as it is a comprehensive method to cover most of the variation that may be involved in a process.

After the effective use of computers and the efficiency of the modern calculation devices and software, many researchers tried to develop many models (such as Monte Carlo, Maximum Likelihood Estimation MLE, and least square methods) to increase the accuracy of Weibull parameters estimation and to overcome the deficiencies encountered with the use of Weibull in manufacturing environment, specially using small sample size to test the performance of an item. The estimation can be point estimation or range estimation. The main focus in the present work will be on estimating shape parameter as Weibull scale parameter is mostly estimated by MLE method[59], which ensures high confidence level using small sample sizes. On the contrary, Weibull shape parameter show no response with conventional estimation method to comply with these methods and enable an estimation of its value with reasonable confidence level.

3.3 PHD RESEARCH METHODOLOGY

The needs of today's competitive market dictate the essentiality of using small sized samples to test quality and reliability. SPC charts are counted as a simple and effective way to draw conclusion and monitor conformance to specifications and standards.

Having a clear understanding of the available models in the field of Weibull analysis can indicates the lack of research in the small sample sized area regarding Weibull and Shewhart control charts. Aims and objective had been established to ensure the practicability of the PhD research field. In order to recognise such field and ensure a reasonable result of this research a methodology philosophy should be set to establish the guidance path to achieve the aims of the PhD.

The research method consists of many stages to understand problems encountered with small sample size inspection, hence to try to modify new models that will over come the disadvantages with conventional models, to explicitly shows that Weibull analysis is capable to control processes and prove effectiveness in solving existing problems.

Such methodology consists of many pillars, which are essential to success. A thorough background study should be adopted to conventional methods in order to use the disadvantages of such methods, a coverage of existing literature (books, journals, papers) ought to be taken into consideration to know the problem facing industry and

to adopt such problem to be a hot spots in the research and result in a good contribution to knowledge and industry benefit.

Figure 3:1 will illustrate the steps, which will be used to justify the problems of the PhD and achieve the objectives and aims set for the PhD research. The methodology passes through four main stages, which are:

1. Data setting and check.
2. Problem hunting in existing models.
3. Modelling a new method to solve problems and increase accuracy.
4. Validating and testing for the model.

It is proposed that using this methodology will result in a contribution to the variability of Weibull analysis in process manufacturing.

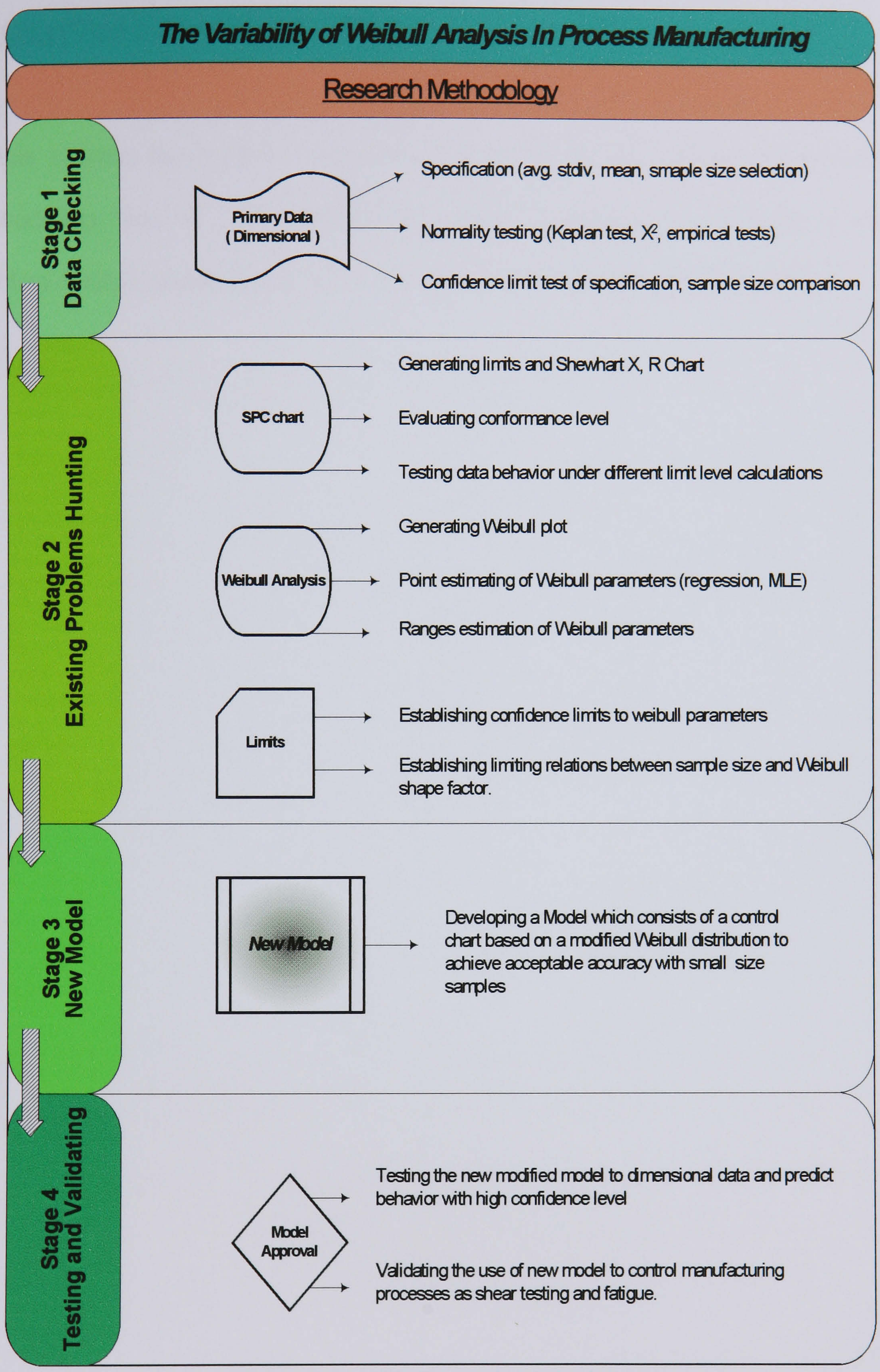


Figure 3:1 – PhD Research Methodology

3.4 CHAPTER THREE CONCLUSION

In this chapter, the problem was clearly understood and a logical scientifically methodology was set. The methodology tackles, understands and diagnose the problem of small sample sizes, and tried to ensure the remedy for such problem.

CHAPTER 4

PRIMARY INVESTIGATION

4.0 CHAPTER FOUR REVIEW

This chapter will be introducing the main key issues when small sample sizes are adopted in quality control analysis. The mathematical behaviour of small sample sizes will be tackled as this may provide a primary idea about the steps, which should be employed to solve the industry problem and provide a effective solution.

4.1 MATHEMATICAL PROPERTIES OF SMALL SAMPLES

Small samples always show an exceptional behaviour when it is adopted in quality and reliability methods. Generally, small samples do not conform to the common knowledge associated with the conventional quality and reliability techniques *(ibid p.2)*. Small sample give widely result different due to the small number of its elements in the individual sub-groups as the small number of elements show difficulty in reflecting the behaviour of the overall universe of data, i.e. degree of freedom. Most of the methods have been approved for large sample size and accurate results are drawn out of such methods, which will enable transparent overview of the process and detect non-conformance with high level of confidence.

Process control charts are based on population, which is normally distributed. Hence, the mechanism of the control chart is concluded through using normally distribution data hypothesis. If two samples $\bar{x}_1$ and $\bar{x}_2$ were selected from the whole population, then the critical shaded regions ($\alpha/2$) are calculated using the sample size and the standard deviation, knowing that α is the risk, which has been accepted to be put up with (Type I error probability). Type I error consists of rejecting the null hypothesis when H_o is actually true, and on the contrary, Type II error consists of not rejecting the null hypothesis when H_o is actually false. In simple words, Type I error can be when somebody is convicted when he is innocent, while Type II error can be when somebody is acquitted when he is guilty. Therefore, it can be seen, as $\bar{x}_1$ falls with in the rejection area then the null hypothesis H_o (the null hypothesis claim that the H_o is initially true, unless proven it is false) would be rejected, while $\bar{x}_2$ does fall within the control limits then H_o cannot be rejected as such sample conforms to

specifications. Therefore, H_o is rejected if and only if the test statistics falls in the rejection area. Generally, it is recommended for a fixed experiment and sample size to decrease the size of the rejection area (decreasing the type I error) and increasing the acceptance area (type II error) for each feasible value in the population characteristics[60]. It is obviously noticed that the effect of small sample size can be apparent when the sample does not represent the actual population and it misjudge the process based on the neglecting of petite variation as small number of items may be not effective to detect such small variation. Figure 4:1 depicts the hypothesis-testing concept.

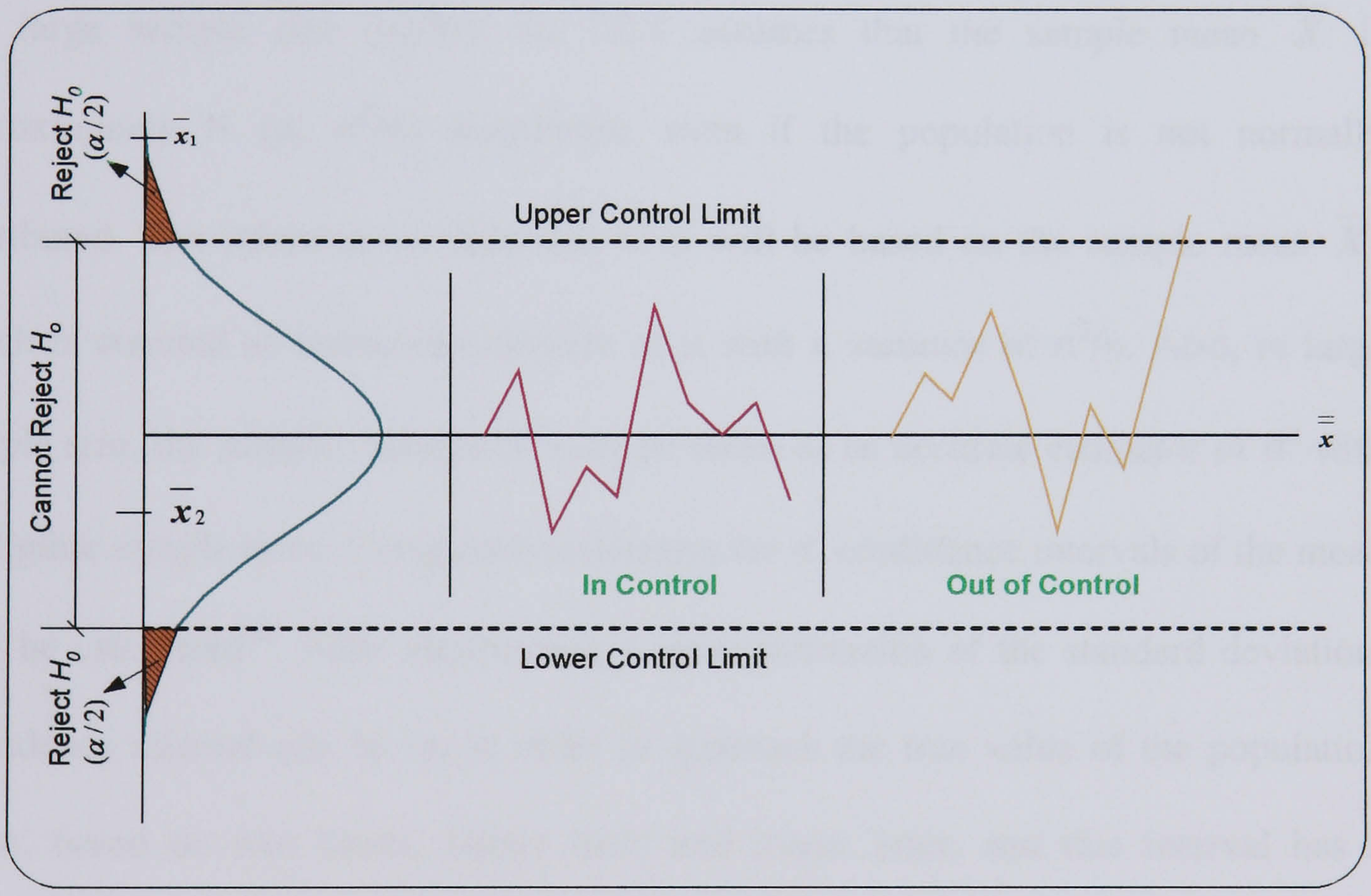


Figure 4:1 – Hypothesis testing in control charts with small sample size.

The Central Limit Theorem (CLT) is directly involved with the analysis of Shewhart control charts. The CLT states that the sum of n independently distributed random variables is approximately normal, regardless of the distributions of the individual

variables (*ibid p.8*). Using such a theorem will facilitate the understanding of small sample problem encountered with control charts.

Controlling a process implies a persistent monitoring of the mean μ, while to insure a valid acceptant precision of the monitoring values; the variance of monitored reading should be taken into consideration. By using the central limit theorem (CLT), inference procedures for the mean of a normal population can be extended to the mean of a non-normal population when enough samples is available[61].

For large sample size (n>30), the CLT assumes that the sample mean $\overline{X}$ is approximately N (μ, σ^2/n) distributed, even if the population is not normally distributed. The inference or detection of μ will be based on the sample mean $\overline{X}$, which is counted as unbiased estimator of μ with a variance of σ^2/n. Also, in large sample size, the sample variance S^2 may be taken as an accurate estimator of σ^2 with negligible sample error. Using such estimation for σ, confidence intervals of the mean may be calculated[62]. After establishing a point estimation of the standard deviation, confidence interval can be set in order to approach the true value of the population mean, based on two limits, Upper limit and lower limit, and this interval has a probability of 1-α (such value is called confidence coefficient) of seizing the true value of the mean parameter. Therefore, the confidence interval of the mean μ is

$$P[\ LCL \leq \mu \leq UCL\ ] \qquad \cdots \textit{Equation}\ 4.1$$

A sample of large size (n>30) is taken from a population of specific mean μ and variance σ^2, and $\overline{X}$ is a point estimator of μ, where the estimator point has a normal

distribution of the mean and variance (based on CLT), then for any value of α, the probability can be (using the standard normal distribution):

$$P\ [\ -Z_{\alpha/2} \leq Z \leq Z_{\alpha/2}\ ] = 1-\alpha \quad \cdots \textbf{Equation } 4.2$$

Using equation 4.2, confidence interval can be derived as follows:

$$1-\alpha = P\ [\ -Z_{\alpha/2} \leq \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} \leq +Z_{\alpha/2}\ ]$$

$$1-\sigma = P\ [-Z_{\sigma/2} \frac{\sigma}{\sqrt{n}} \leq \overline{X}-\mu \leq Z_{\sigma/2} \frac{\sigma}{\sqrt{n}}\ ]$$

$$1-\sigma = P\ [\ \overline{X}-Z_{\sigma/2} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \overline{X} + Z_{\sigma/2} \frac{\sigma}{\sqrt{n}}\ ]$$

Consequently,

$$\mu \textit{ Confidence Interval } = \left(\overline{X} - Z_{\sigma/2} \frac{\sigma}{\sqrt{n}} , \overline{X} + Z_{\sigma/2} \frac{\sigma}{\sqrt{n}} \right) \cdots \textbf{Equation } 4.3$$

Equation 4.3 is valid when large sample is used, on the contrary, when small sample sized is used (n<15) then the standard normal distribution will not variability, and it is appropriate to use the t-distribution. And the confidence interval will be derived as follow,

$$1-\alpha = P\ [\ -t_{n-1,\alpha/2} \leq T \leq t_{n-1,\alpha/2}\ ] \quad \cdots \textbf{Equation } 4.4$$

$$1-\alpha = P[\ -t_{n-1,\alpha/2} \leq \frac{\overline{X}-\mu}{S/\sqrt{n}} \leq t_{n-1,\alpha/2}\ ]$$

$$1-\alpha = P[\ \overline{X} - t_{n-1,\alpha/2} \frac{S}{\sqrt{n}} \leq \mu \leq \overline{X} + t_{n-1,\alpha/2} \frac{S}{\sqrt{n}}\ ]$$

Consequently, for small sample size

$$\mu \textit{ Confidence Interval } = \left(\overline{X} - t_{n-1,\alpha/2} \frac{S}{\sqrt{n}} , \overline{X} + t_{n-1,\alpha/2} \frac{S}{\sqrt{n}} \right) \cdots \textbf{Equation } 4.5$$

It can be clearly seen that using the t-distribution will ensure a good chance of predicting the true value of the mean sample, as the spread of this distribution is bigger and the confidence interval is wider so it will take the variability occurring when using small sample size.

Many other techniques were developed recently due to the huge boost in computer capabilities and the high speed providing easy solutions for complex numerical equations. Some of the recent techniques are the bootstrap technique and the Box-Cox. Generally, the normal theory method, the bootstrap technique and the Box-Cox[63] transformation method can be used to construct the confidence interval of any population, hence the bootstrap technique is accurate methods to be used for predicting the mean and the confidence interval for non-normal population [64]. The basic assumption of the bootstrap techniques are based on the following equations[65]:

$$\overline{\mu}_{bootstrap} = \frac{\sum_{i=1}^{B} \overline{\mu}_{Y(i)}}{B} \qquad \cdots \textit{Equation } 4.6$$

$$S_{bootstrap} = \sqrt{\frac{\sum_{i=1}^{B} \left(Y(i) - \overline{\mu}_{bootstrap}\right)^2}{B-1}} \qquad \cdots \textit{Equation } 4.7$$

where B: is the number of bootstrap samples

Therefore, an approximate (1-α)100% confidence interval for μ_Y by the standard methods is:

$$\overline{\mu}_{bootstrap} \pm Z_{\alpha/2}\, S_{bootstrap} \qquad \cdots \textit{Equation } 4.8$$

Due to the complexity of the bootstrap technique calculations, in this research, the t-distribution technique will be used for small sample mean with confidence interval prediction.

A main point, which should be addressed when using small size samples, is the normality of the sample. A normal distribution is symmetric bell-shaped curved distribution, with a single peak at the mean. This distribution is arguably the most important and used distribution in both the theory and application of statistics. If x is a normal random variable, then the probability distribution of x is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \qquad -\infty < x < \infty \qquad \cdots \textit{Equation } 4.9$$

The parameters of the normal distribution are the mean μ and the standard deviation σ (or the variance σ^2). The normal distribution is a theoretical concept. In reality, almost no data are truly normal (the data do not follow the curve exactly, they are very close to normal). However, many variables are distributed in a nearly normal fashion, so the normal distribution is the basis behind many statistical tests.

There are several tests to check the normality for certain data, such as the Anderson-Darling test, the Ryan-Joiner test, and the Kolmogorov-Smirnow test. In this research, the Ryan-Joiner normality test will be used to examine the collected data. Ryan-Joiner normality test is a correlation-based test (The Anderson-Darling test is an ECDF - empirical cumulative distribution function- based test, and the Kolmogorov-Smirnov test is a chi-square based test). Data, which are plotted in this test, generate a normal probability plot. The grid on the graph resembles the grids found on normal probability paper. The vertical axis has a probability scale; the horizontal axis, a data scale. Ryan-Joiner normality test can help to determine whether the data follow a normal distribution by calculating the p-value (significant factor), The p-value ranges from 0 to 1, and indicates how likely it is that the data follow a normal distribution.

Usually the level of approved significance is 0.1, which equal to α level[66]. A hypothesis test is been used to examine whether or not the observations follow a normal distribution. For the normality test, the hypotheses are:

H_0: data follow a normal distribution, H_1: data do not follow a normal distribution

In some test the hypothesis will be examined with respect to the results of p-value, if the test resulted in a value greater than the common value (usually 0.1) then there is no evidence that the null hypothesis should be rejected, which implies that the data are normally distributed.

4.2.1 Mathematical Behaviour of Small Samples Obtained from Rod Diameter Test

Small sample size show an unforeseen behaviour with respect to the conventional knowledge obtained when using large amount of sample size (n>30). Illustrating the behaviour of small samples could be notice by adopting the rod diameter test. The diameter of a manufactured rod has been measured (see figure 4:2). Fifty reading were collected. Ten samples were taken and each sample has a size of five readings. Using such data may provide a primary understanding to the effect of small samples in conventional quality and reliability techniques. Table 4:1 shows the overall measurements collected by measuring the diameter by using a micrometer.

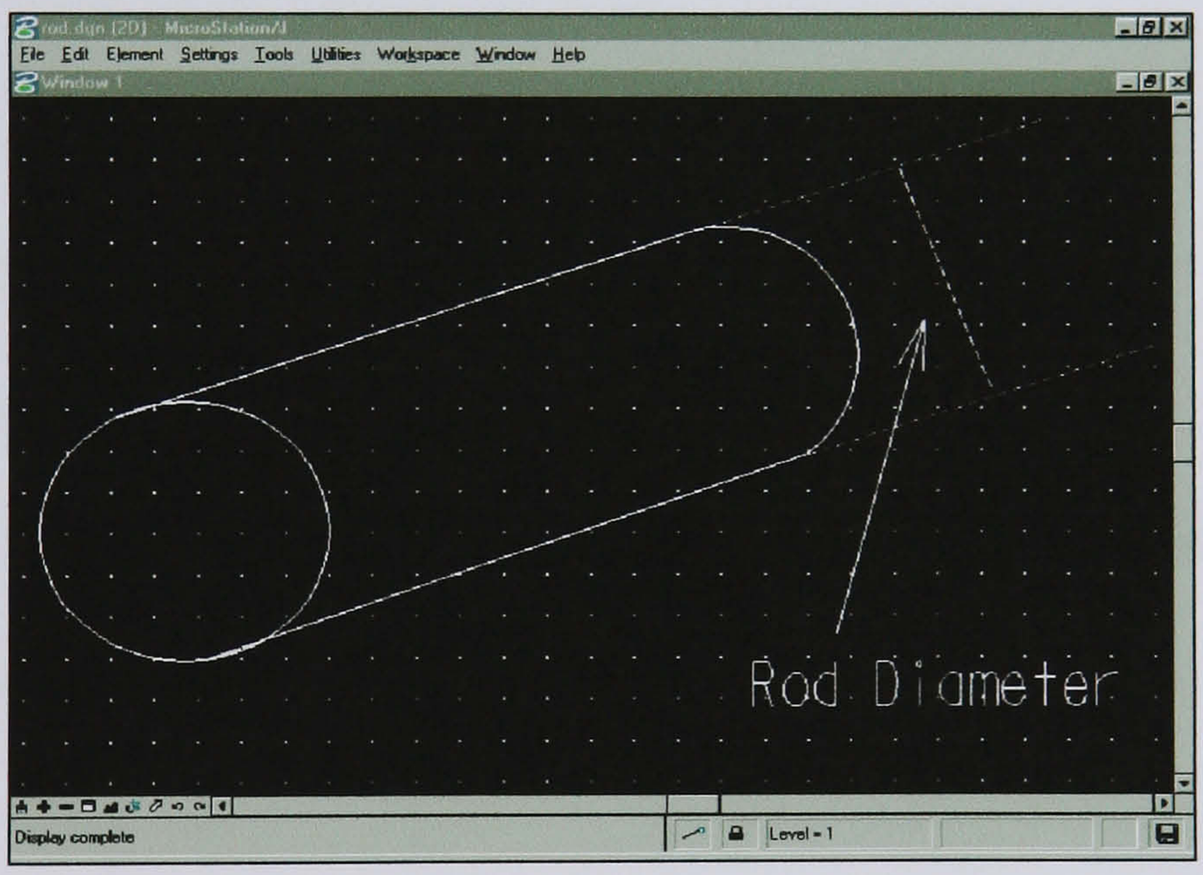


Figure 4:2 – Rod diameter test

	n = 1	n = 2	n = 3	n = 4	n = 5
Sample 1	*0.65*	*0.70*	*0.65*	*0.65*	*0.85*
Sample 2	*0.75*	*0.85*	*0.75*	*0.85*	*0.65*
Sample 3	*0.75*	*0.80*	*0.80*	*0.70*	*0.75*

Sample 4	*0.60*	*0.70*	*0.70*	*0.75*	*0.65*
Sample 5	*0.90*	*0.75*	*0.65*	*0.85*	*0.80*
Sample 6	*0.60*	*0.75*	*0.75*	*0.85*	*0.70*
Sample 7	*0.75*	*0.80*	*0.65*	*0.75*	*0.70*
Sample 8	*0.60*	*0.70*	*0.80*	*0.75*	*0.75*
Sample 9	*0.65*	*0.80*	*0.85*	*0.85*	*0.75*
Sample 10	*0.60*	*0.70*	*0.80*	*0.80*	*0.65*

Table 4:1 – Rod diameter

Using MINITAB, these data will be used to run a few test in order to understand their statistical behaviour. Figure 4:5 shows a descriptive statistics for the overall data for rod diameter test. This figure shows the following details:

- Histogram of data with normal curve fit.
- 95% confidence interval graph for σ (Sigma).
- 95% confidence interval graph for μ (Mu).
- 95% confidence interval graph for the Median.
- Basic statistical values such as Mean, Standard Deviation, Variance, Skewness, and Kurtosis.

The mean of an average value for the overall data, is computed by dividing the summation of the measurements and the number of measurements. The variance (σ^2) is a measure of how spread out a distribution is. It is computed as the average squared deviation of each number from its mean (See eq.4.10). And the standard deviation (s or some use σ) is the square root of the variance (see eq.4.11).

$$Variance = \sigma^2 = \frac{(x-\mu)^2}{n} \qquad \cdots \text{ Equation } 4.10$$

$$S\,\text{tan}\,dard\ Deviation = s = \sqrt{\sigma^2} \qquad \cdots \text{ Equation } 4.11$$

Skewness is a measure of the asymmetry of the data around the sample mean. If skewness is negative, the data are spread out more to the left of the mean than to the right. If skewness is positive, the data are spread out more to the right (see figure 4:3). The skewness of the normal distribution (or any perfectly symmetric distribution) is zero.

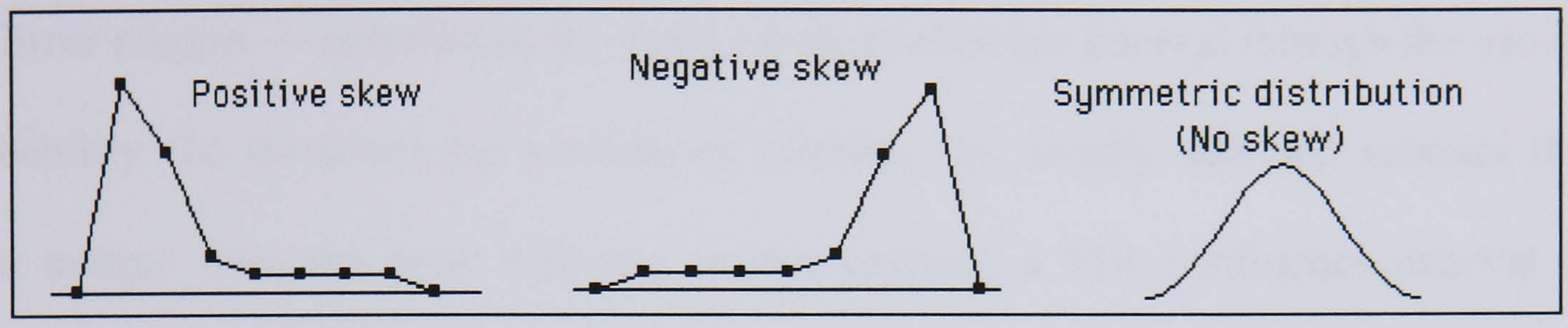


Figure 4:3 - Skewness

Kurtosis is a measure of how outlier-pronehow (sharply peaked) a distribution is (see figure 4:4). A flat-topped distribution tends to have a low value of Kurtosis and is called *platykurtic* (flat bulging). A sharp-peaked distribution will tend to have a high value of kurtosis and is called *leptokurtic* (thin bulging).

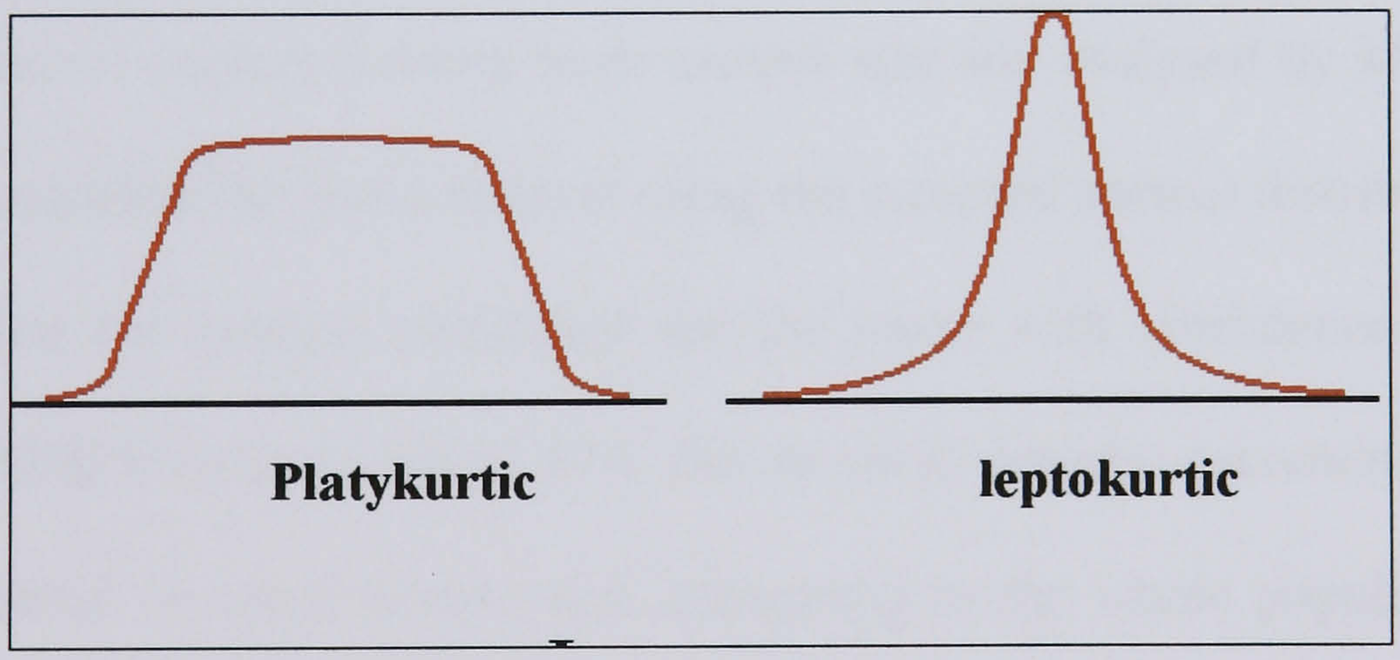


Figure 4:4 – Kurtosis

A confidence interval is an interval used to estimate a population parameter from sample data. The upper and lower bounds of the confidence intervals for μ (Mu), σ (sigma), and the median are displayed in the graphical summary. Confidence intervals

are composed of two basic parts: (1) point estimate — a single value computed from the sample data. This value is considered to be an estimate of the parameter of interest, however it is unlikely that the point estimate is equal to the parameter. Therefore, to account for the possibility of estimation error, the error margin is included in the confidence interval to provide a range of possible parameter values. (2) Error margin — determines the width of the confidence interval through the use of probability. To construct the confidence interval, you simply add and subtract the error margin from the point estimate. In this analysis, a 95% confidence interval is selected; the method used to construct the interval has a probability of 0.95 of producing an interval containing the parameter of interest. In other words, you can be 95% confident that the true value of the parameter is within the interval. Thus, if one hundred 95% confidence intervals were constructed, you would expect around 95 of the intervals to contain the parameter.

As usual practice of engineers using large sample size and analysed by MINITAB, it is designed to calculate the mean interval using the standard normal distribution and it can be seen that the interval calculated for the mean with confidence of 95% is [0.691995, 0732005] (refer to figure 4:5). But as small samples (assuming that these data are considered as small sample size comparing to the whole population) mean interval is better calculated with t distribution. Using equation 4.5 will result in more accurate prediction of the mean value.

$$\mu \text{ Confidence Interval} = \left(\overline{X} - t_{n-1,\alpha/2} \frac{S}{\sqrt{n}} , \overline{X} + t_{n-1,\alpha/2} \frac{S}{\sqrt{n}} \right)$$

Substituting the result of the MINITAB analyses will result in

$$\mu \text{ Confidence Interval} = \left(0.712 - t_{49,0.025} \frac{0.70392}{\sqrt{50}} ,.712 + t_{49,0.025} \frac{0.70392}{\sqrt{50}} \right)$$

Noting that $t_{49,0.025} = 02.011$

Then the μ Confidence Interval is equal to

$$C.I._{\mu} = [0.691981, 0.732010]$$

Clearly the C.I. resulting from the use of t distribution is wider that the C.I. resulting from using the standard normal distribution. In other words, the probability of predicting a true value in the confidence interval obtained from t distribution is higher that the confidence interval of standard normal distribution.

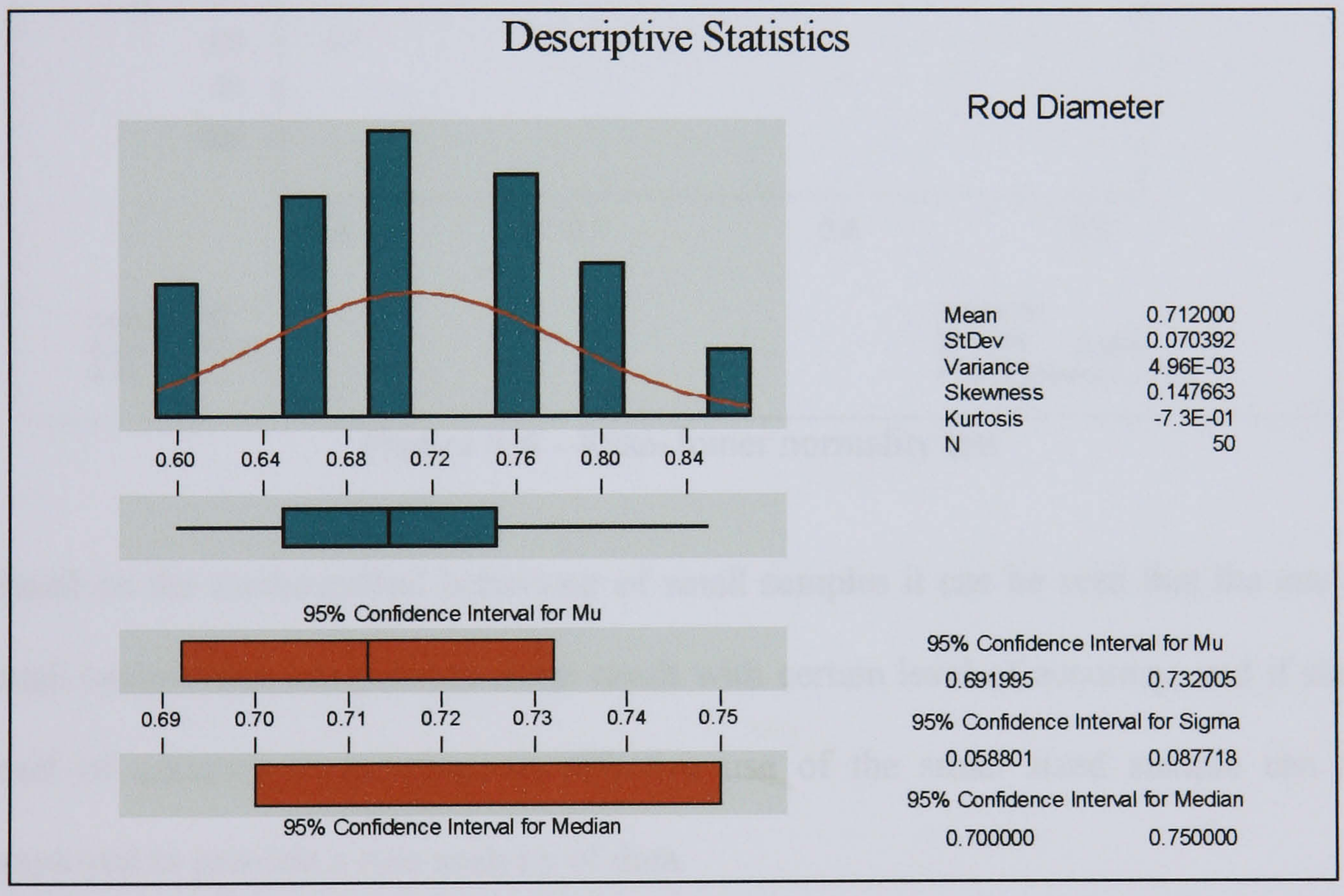


Figure 4:5- Descriptive statistics for rod test (overall data)

Moreover, applying the normality facts and using a Ryan-Joiner test on the rod diameter figure 4:6 is obtained. This figure shows that the points almost falling on the lines, and the calculation shows that the p-value is greater than the specified α-level which is 0.1. Therefore, the dietician will not reject H_0 as there is not enough evidence

to suggest that the data are not normally distributed. As well, the Skewness and Kurtosis factors are 0.147663 and -0.73 respectively, which are near zero in values. Consequently, it can be concluded that these data are normally distributed.

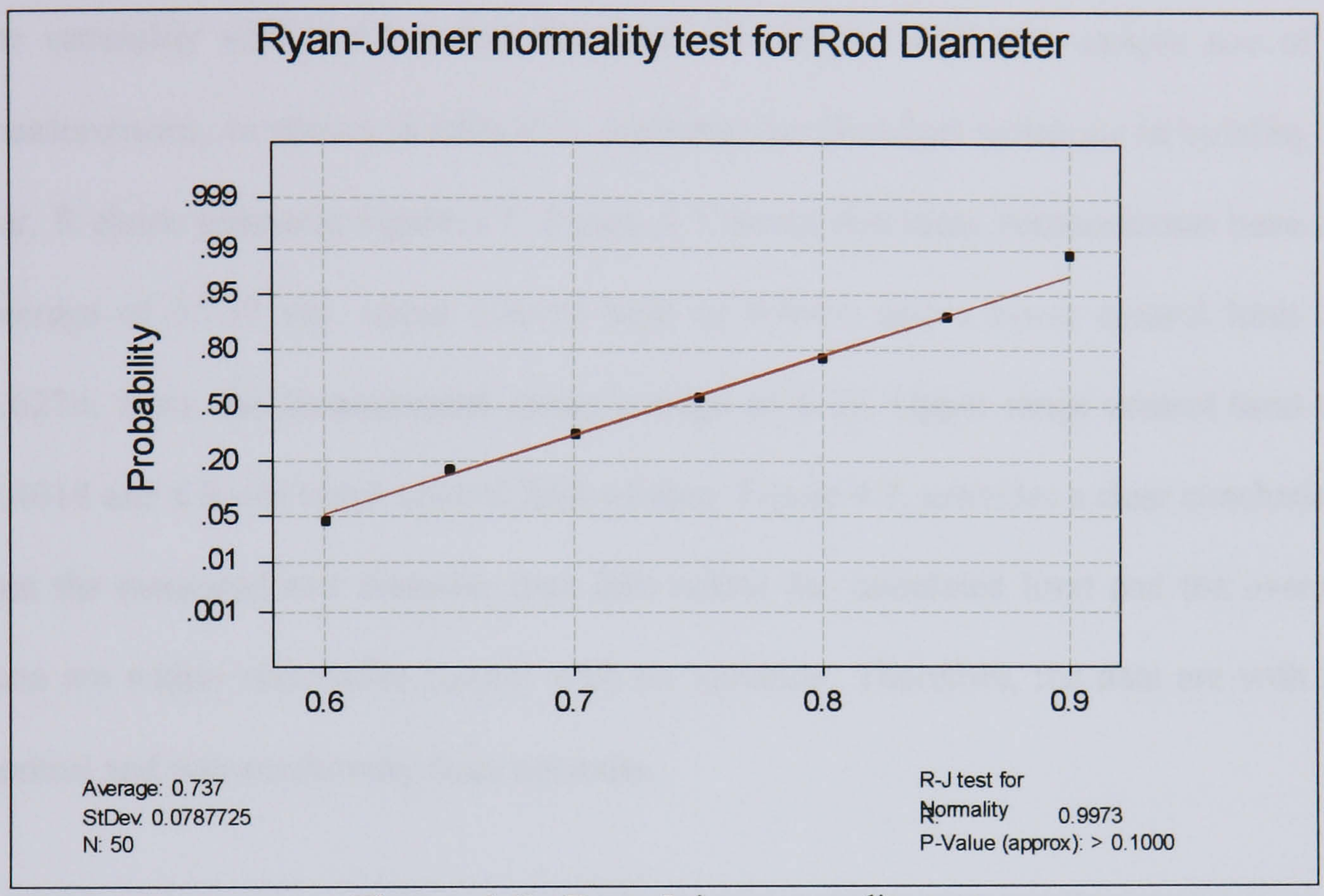


Figure 4:6 - Ryan-Joiner normality test

Based on the mathematical behaviour of small samples it can be seen that the use of small sample size can provide some result with certain level of accuracy, and if such level of accuracy is increase, an effective use of the small sized sample can be employed to provide a true analysis of data.

4.2.2 Quality Control Charts of Small Samples Obtained from Rod Diameter Test

X Bar, R charts will be used to study the behaviour of the measurements and to know the variability with in these data based on 10 samples with each sample size of 5 measurements, as shown in table 4:1. Applying the Shewhart technique in building X bar, R charts generates figure 4:7. Figure 4:7 shows that these measurements have an average of 0.737 cm, upper control limit of 0.8466 and a lower control limit of 0.6274. Also, the measurement range average of 0.19, Upper range control limit of 0.4018 and a lower range control limit of zero. Figure 4:7, provides a clear conclusion that the measured rod diameter data falls within the calculated limit and the overall data are within reasonable control with no variation. Therefore, the data are with in control and non-conformity does not exist.

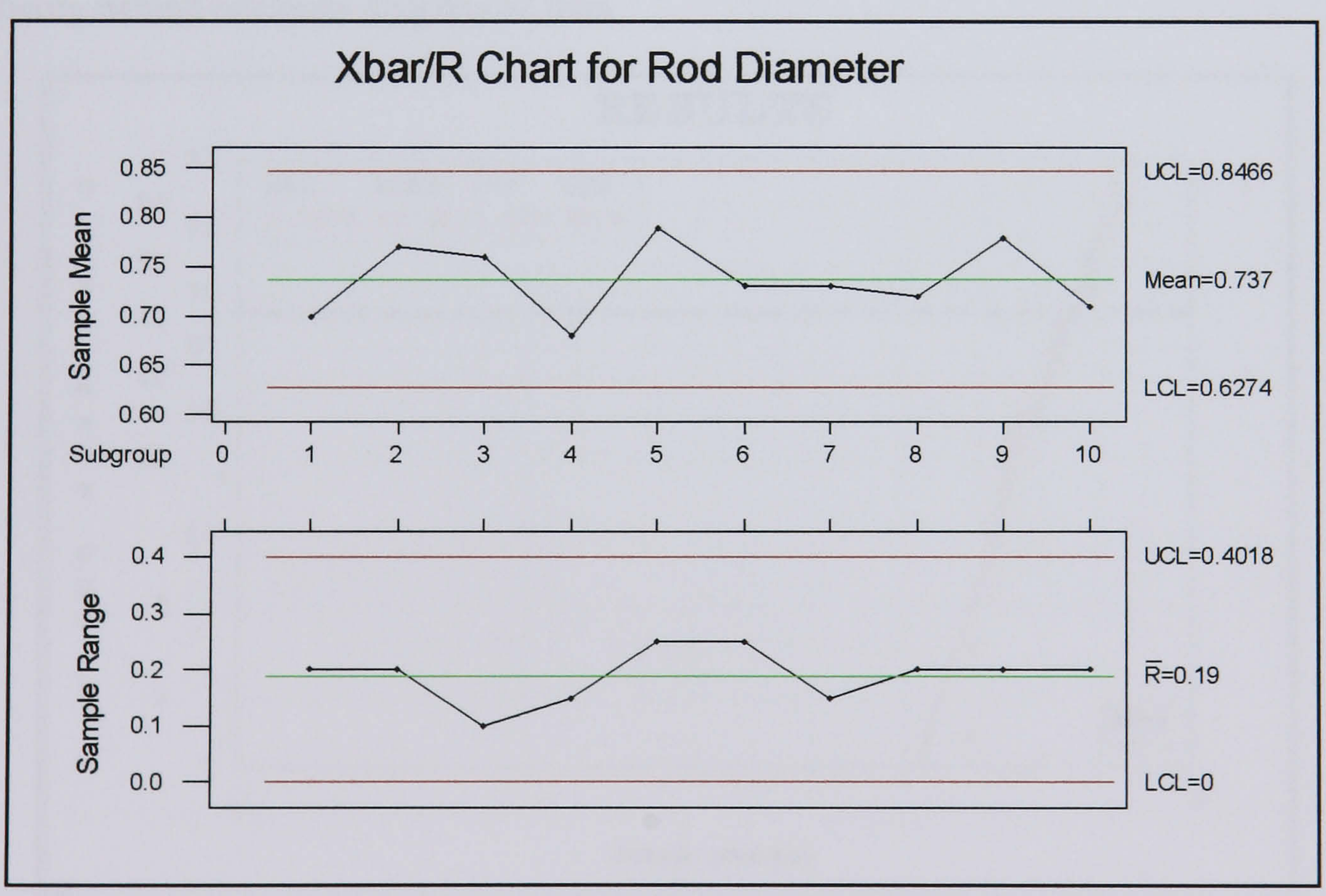


Figure 4:7 - Rod diameter control charts

4.2.3 Weibull Plot of Small Samples Obtained from Rod Diameter Test

Using the Weibull distribution on the measure data and using WINSMITH software, a Weibull plot is generated as seen in figure 4:8. This figure indicates that the 50 measurements taken have a Beta value (shape parameter) of 12.18 and Eta (scale parameter) value of 0.7473. This plot shows a contradiction with the fact that the normal distribution should give a Shape parameter of 3.44[67]. The margin of variation (Error) of the calculated value is:

$$Error = \frac{(Real\ Value - Theoretical\ value)}{Theoretical\ value} * 100\% \qquad \cdots Equation\ 4.12$$

$$= \frac{(12.18 - 3.44)}{3.44} * 100\% = 254.07\ \%$$

Equation 4.12 shows the real value of the shape parameter is almost 2.5 times the theoretical value of the shape parameter for normally distributed measurements. Therefore, it can be concluded that there is a difference between the reality and the theory behind normally distributed data.

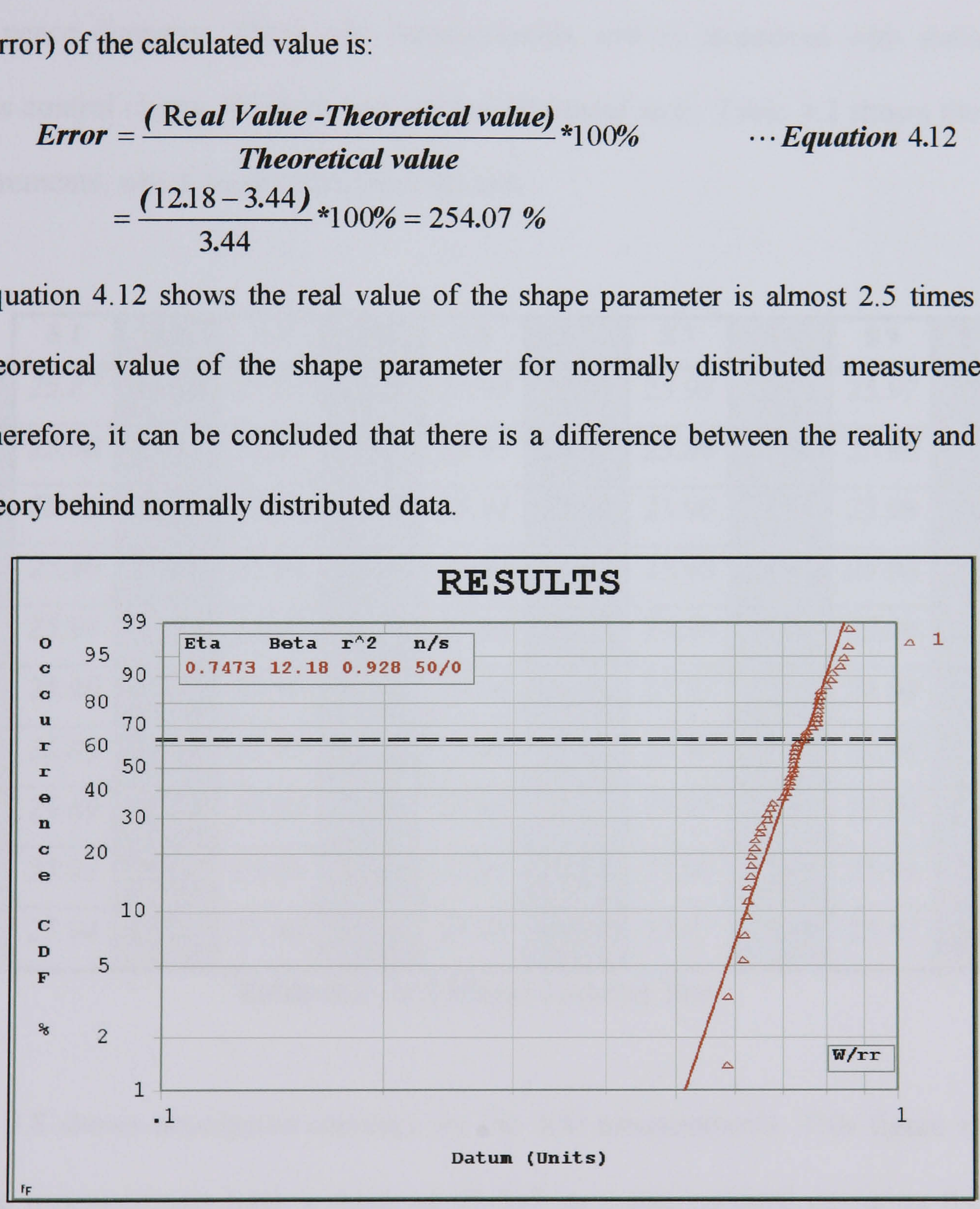


Figure 4:8 - Rod diameter Weibull plot

4.3 ANALYSIS OF A 2 PENCE DIAMETER TEST

In order to prove the diversity between the theory and the practical application of small sample usage in quality and reliability another test is carried out to show the deviation between predictive values and true values. In this test, a 2 pence coin diameter is been measured. 100 coins are been used to collect 100 measurements for the 2 pence diameter. These 100 measurements will be examined with statistical process control charts, Weibull, and normal statistical tests. Table 4:2 shows the 100 measurements, which were taken from the test.

	S 1	S 2	S 3	S 4	S 5	S 6	S 7	S 8	S 9	S 10
n=1	*25.87*	*25.96*	*25.91*	*25.93*	*25.93*	*25.97*	*25.93*	*25.91*	*25.97*	*25.95*
n=2	*25.98*	*25.91*	*25.97*	*25.90*	*25.93*	*25.97*	*25.89*	*25.88*	*25.90*	*25.94*
n=3	*25.92*	*25.90*	*25.95*	*25.91*	*25.91*	*25.90*	*25.96*	*25.98*	*25.98*	*25.99*
n=4	*25.90*	*25.92*	*25.94*	*25.94*	*25.94*	*25.91*	*25.98*	*25.95*	*25.98*	*25.94*
n=5	*25.94*	*25.94*	*25.97*	*25.93*	*25.95*	*25.95*	*25.89*	*25.96*	*25.96*	*25.95*
n=6	*25.90*	*25.97*	*25.90*	*25.94*	*25.96*	*25.96*	*25.93*	*25.93*	*25.94*	*25.93*
n=7	*26.03*	*25.97*	*25.97*	*25.89*	*25.95*	*25.95*	*25.96*	*25.98*	*25.98*	*25.97*
n=8	*25.89*	*25.92*	*25.89*	*25.93*	*25.86*	*25.93*	*25.97*	*25.97*	*25.99*	*25.95*
n=9	*25.93*	*26.01*	*25.96*	*25.88*	*25.91*	*25.94*	*25.90*	*25.94*	*25.94*	*25.87*
n=10	*25.94*	*25.95*	*25.96*	*25.91*	*25.95*	*26.01*	*25.87*	*25.89*	*25.87*	*25.92*

Table 4:2 – a 2 Pence Diameter Test

Figure 3.8 shows descriptive statistics for the 100 measurements. This figure shows that the measurements have a mean of 25.917 mm and standard deviation 0.0346. Skewness and Kurtosis are low in value; they are 0.068 and 0.31 respectively. Also the 95% confidence limits for μ, σ, and median are [25.93,25.94], [0.03,0.04] and [25.91,25.95] respectively. These confidence limits are based on standard normal

distribution. But using small sample size ought to be associated with C.I. based on t distribution, which equal to [25.9301319, 25.9438681]. It is a fact that when the sample size increase the accuracy of standard normal distribution will provide a similar mean confidence interval as the t distribution, and this test illustrate such fact clearly.

Figure 4:9 - 2 pence diameter test

Ryan-Joiner normality is been used to provide figure 4:10, which shows that the points are on the line, and the differences in distances almost negligible. Such a graph gives a clear indication that the 100 measurements are distributed in a normal distribution, as p-value is greater than 0.1. Therefore, there is no proof that these data are not distributed normally.

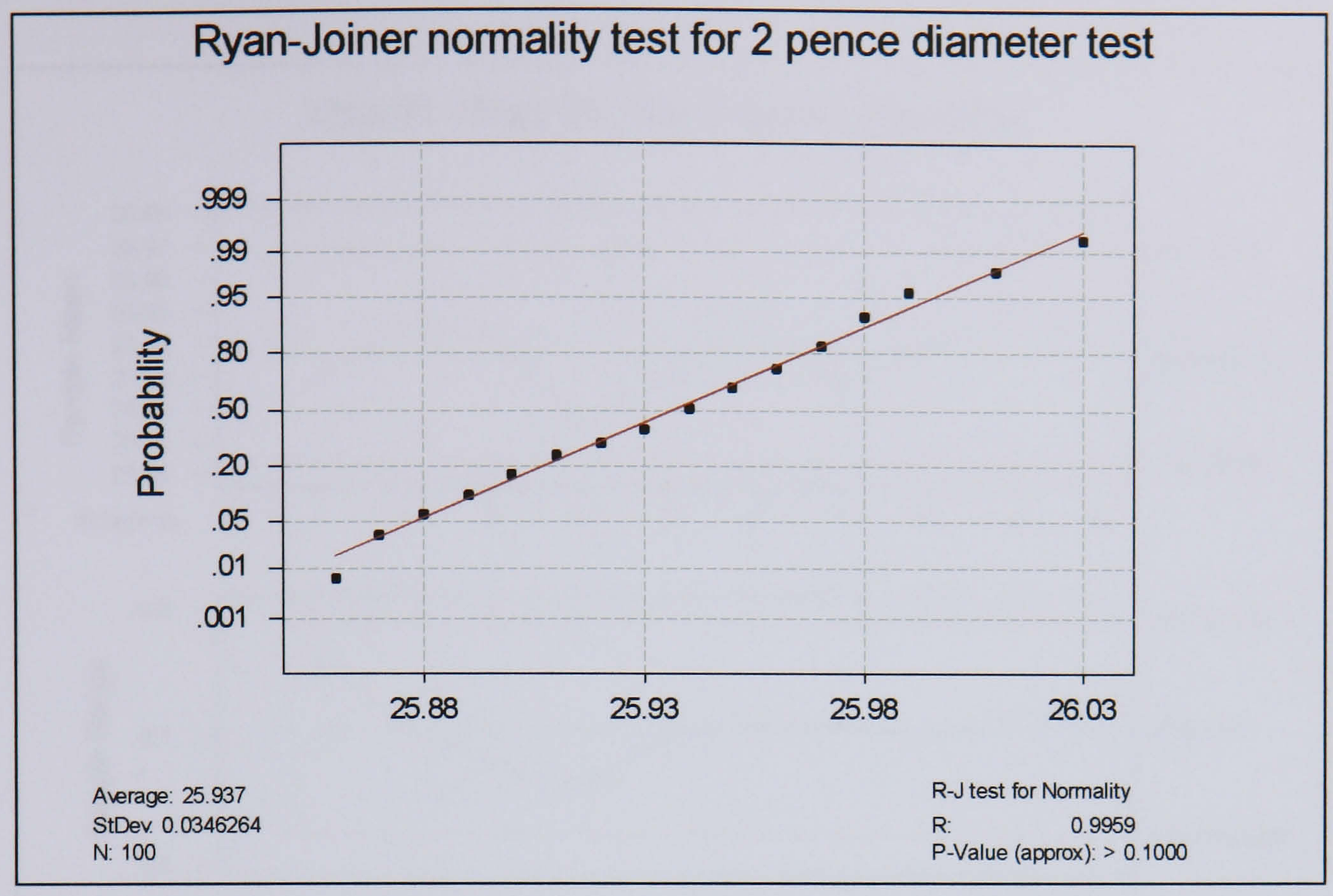


Figure 4:10- a 2 pence diameter normality test

Figure 4:11, shows control charts for 100 measurements (based on 10 samples; each sample with 10 sample size). The calculation shows that the control upper limit for the 2 pence diameter is 25.97mm, while the lower control limit for the 2 pence diameter is 25.90 mm, and the average of the mean in is 25.94 mm. The 2 Pence test shows that the range of diameter has an upper range limit of 0.1901 mm, lower diameter range limit of 0.02387 mm, and an average for the ranges equal to 0.107 mm.

Based on figure 4:11, it can be noticed that the measurements of the 2 pence diameter are within the control limits. Also no variability can happen in such data sets. Therefore, it can be concluded that the measurement are conforming to the limits, and no variability is occurring.

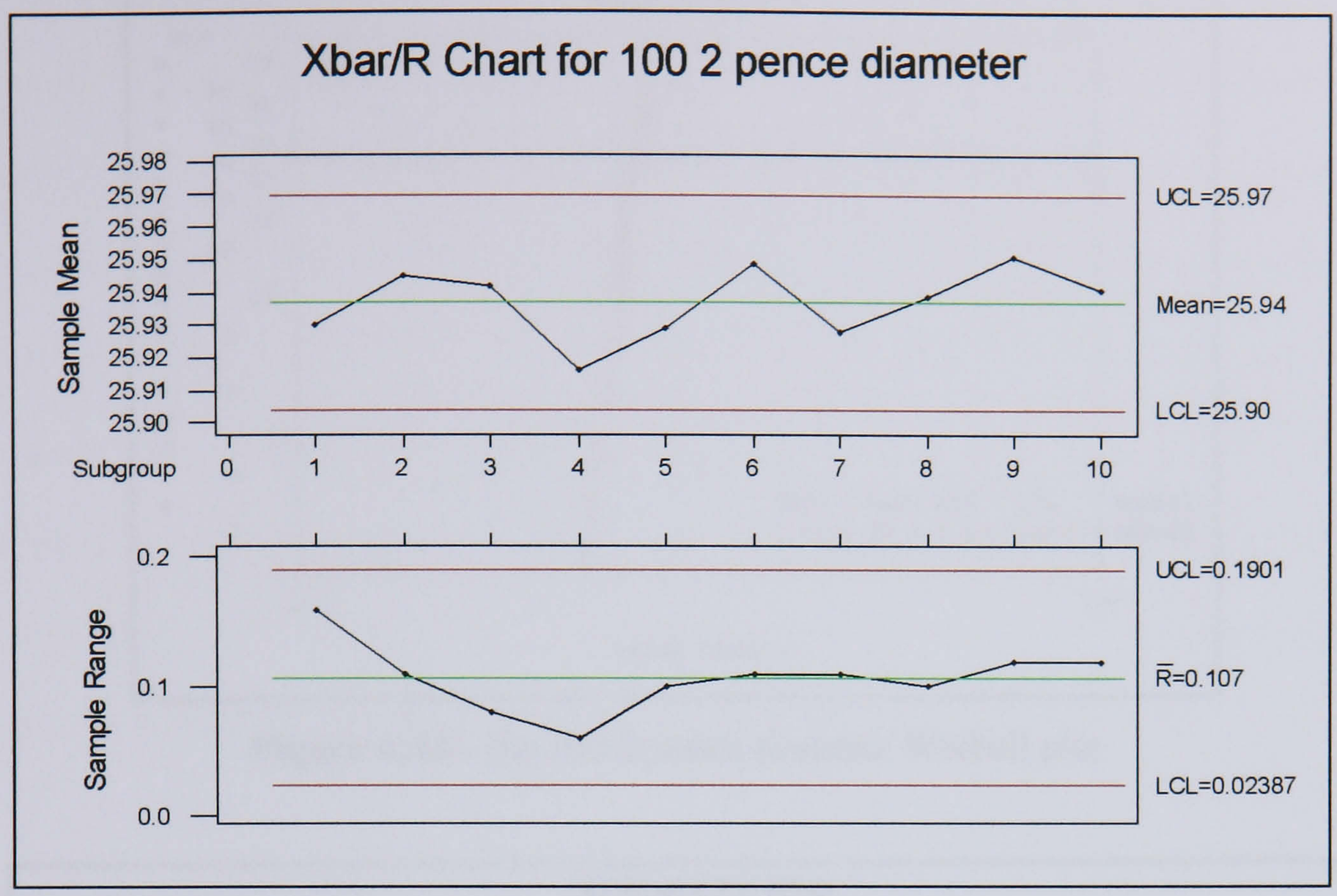


Figure 4:11 – a 2 pence X Bar, R control charts

Using WINSMITH to analysis the 2 pence diameter measurements generates figure 4:12. Weibull calculation gives a Shape parameter (Beta value) of 44.9, and a scale parameter (Eta value) of 26.35. This figure gives another proof of the contradiction that normally distributed data have a shape parameter of 3.44. Using such facts may help in predicting a new method for a Weibull plot.

In order to achieve a clear understanding for the contradiction associated with the use of small sample size, a sample size of 10 measurements will be taken. Such samples will be plotted on Weibull, and then a shape parameter will be calculated. Figure 4:13, shows 10 samples (each with a size of ten) gives a Weibull plot with a slope (Beta) in a range of [31.54, 54.34]. The calculated values show a difference between theory and practice.

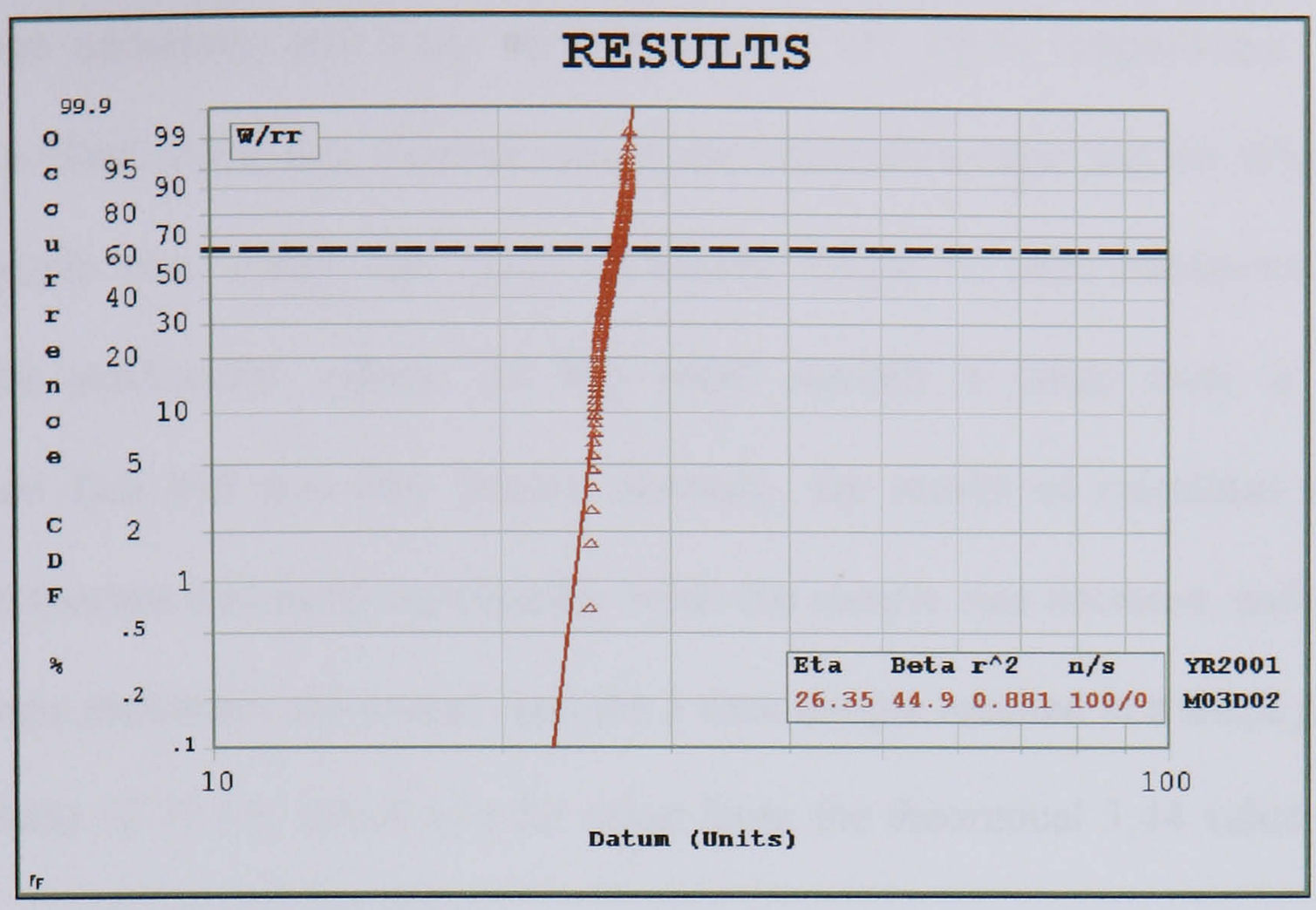


Figure 4:12 - the 100 2 pence diameter Weibull plot

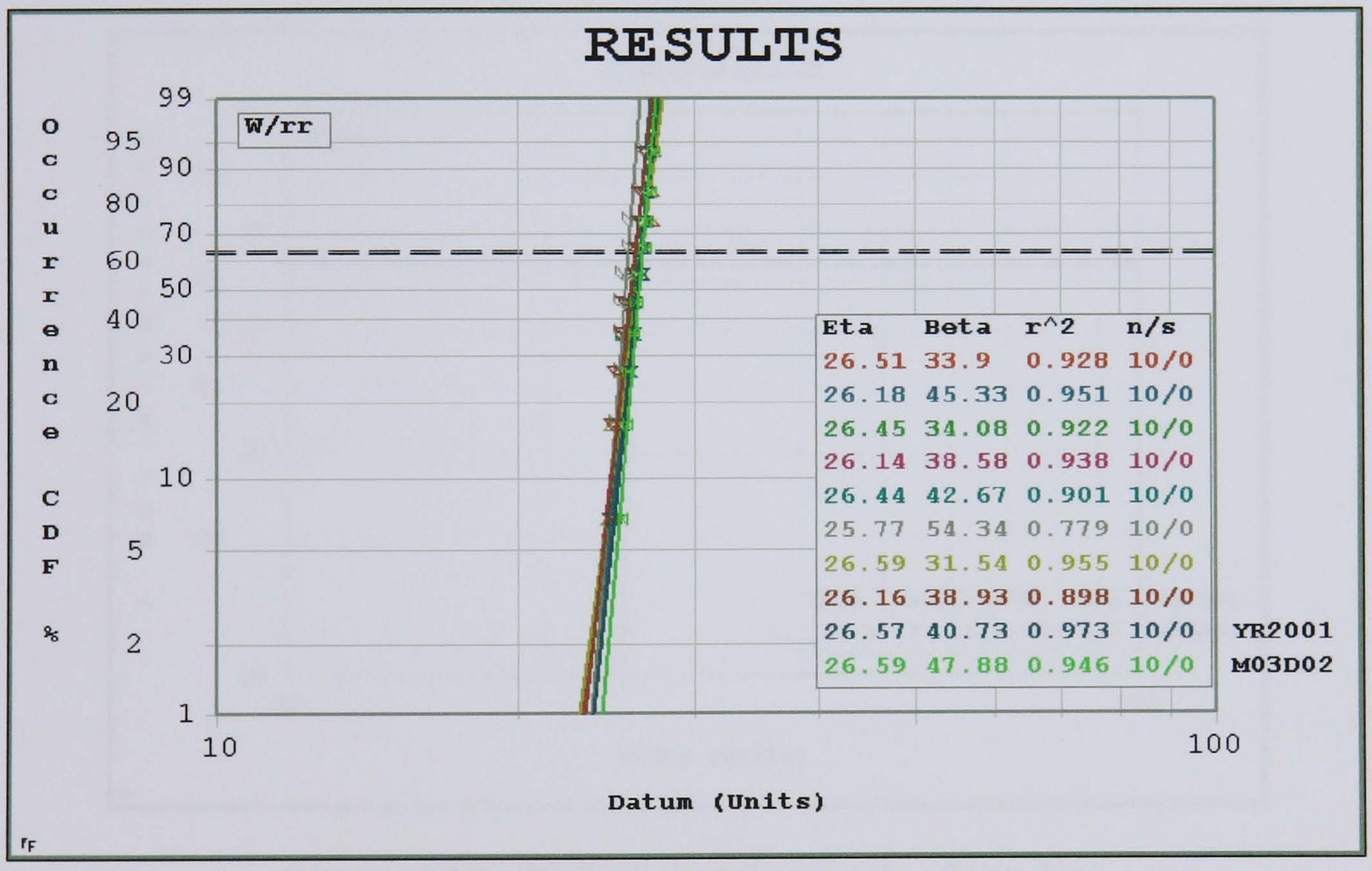


Figure 4:13 - Weibull plot for samples with size = 10

To make the inconsistency of results associated with the sample size obvious, a sample of a size equal to five will be taken randomly from the overall 100 measurements. This sample will consist of the following elements (*25.95, 25.90, 25.92, 26.01, 25.87)*. Figure 4:14 shows the Weibull plot for such random a sample

with shape parameter and scale parameter of 77.19, 25.31 respectively. This can provide a clear proof that Weibull shape and scale parameter can be affected with small sample size, which can result in altered values of such parameter than the theoretical predictable values. As this small sample is taken from a normally distributed data and also they behave normally, the results of calculated scale and shape parameters increased significantly when the sample size decrease, and the value of the shape parameter associated with the 5 data sample resulted in a shape parameter value (Beta) of 77.19, which is a far value from the theoretical 3.44 value expected from a normally distributed data.

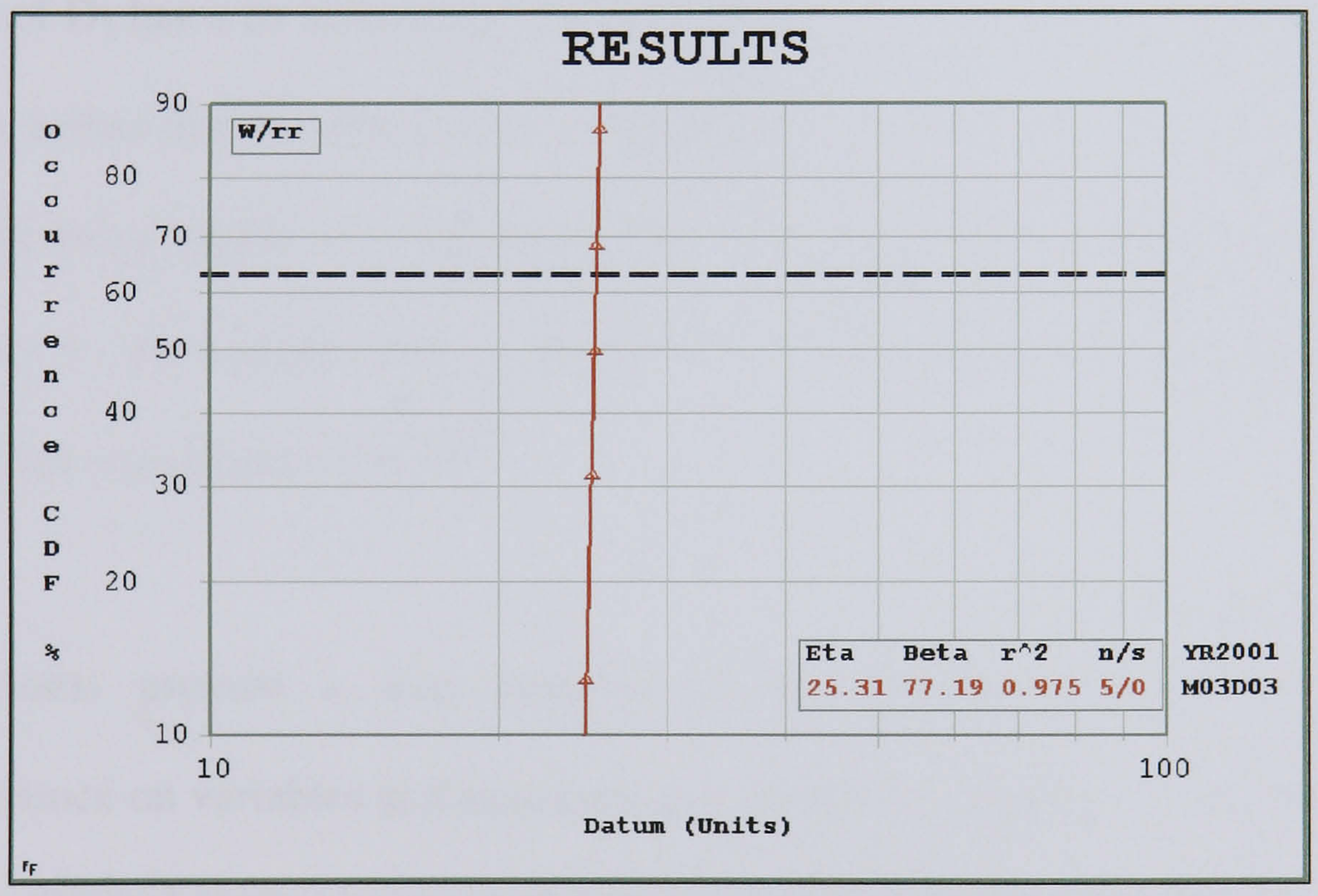


Figure 4:14 - Weibull plot for sample of size five

4.4 SAMPLE SIZE EFFECT ON CONTROL CHARTS

Noting from the equation of UCL_R and LCL_R that both limits depend on D_3, D_4 factors (refer to Equation 2.5 & 2.6). D_3, D_4 have various numerical values depending on the sample size[68], which can be used to generate a plot with extrapolating D_3 and D_4 values. It is seen in figure 4:15, that D_4 and D_3 converge to specified numerical value at large sample size. Therefore, the result of UCL_R and LCL_R calculations will be achieved with high confidence and approved certainty.

The value of D_3 has a an increasing trend, it start increasing after the value of n equal to 6, while before that the effect of D_3 is negligible as D_3 has the value of zero then it increase till being steady at n more than 25. On the contrary, D_4 value is inversely proportional to the sample size, it decrease when the value of n increase, and it stabilise when n is greater than 25[69].

Control charts provide a true analysis of the process or system. It keeps superintendence on variables and acquaints any variability of specified variable within the system. Control charts are successful tools to respond to any fluctuation within the system parameter. The disadvantages of Control chart are: control charts effectively operate with large sample size not on small sample size bases. Therefore, this fact makes control charts not an efficient tool; especially when they are used in high cost manufacturing product environment and small batches; also control charts do not provide a prediction on system failures.

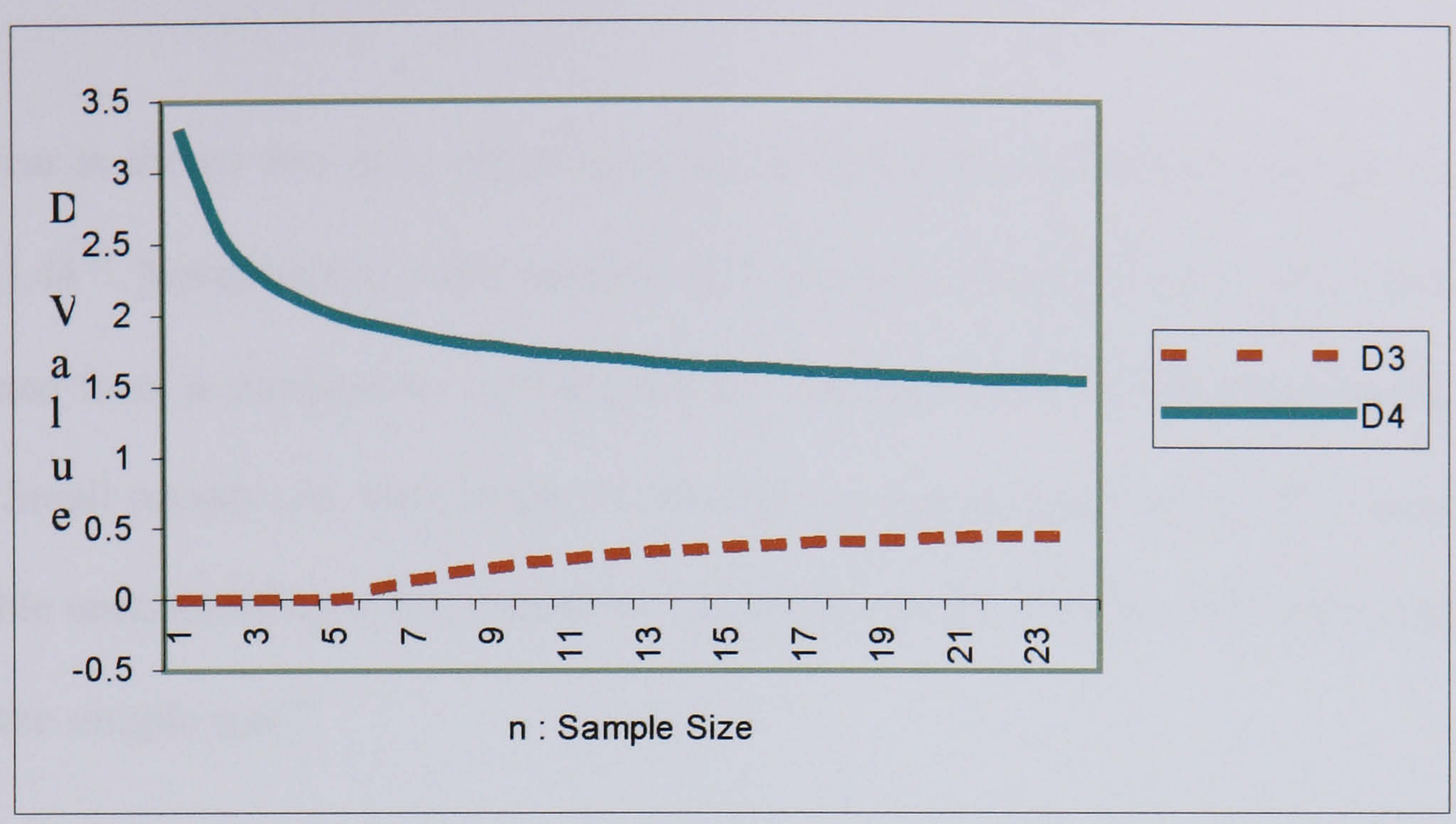


Figure 4:15 – D_3 and D_4 curves

As seen from figure 4:14, the accuracy established by using the conventional control charts, which is based on normal distribution, increase only at sample size greater than 30[70]. Many researchers in the field of statistical quality control have agreed such fact, which has been exposed in the analysis of figure 4:15.

In attribute charts, it is preferable to use samples size greater than 30[71], as the accuracy of the results in attribute charts analysis increase significantly with the increase of the sample size. Therefore, it is generally known that attribute charts need double or more the sample size that of the variable charts to obtain accurate result with acceptable level of confidence[72], which is able to detect variability and non-conformance.

4.5 SAMPLE SIZE EFFECT ON WEIBULL ANALYSIS

It is clear in theory that data, which is normally distributed, will have a shape factor (β) of 3.44[73]. Nevertheless, small sample sizes produce various β values, which can be explained from a comparison with D_3 and D_4 constants used in SPC (refer to figure 4:14). Small sample size with respect to Weibull techniques had been used to achieve a durable understanding of the behaviour, and relationship of the Weibull shape factor β and the sample size[74].

An analysis based on the median rank of n=5 will be discussed in order to contribute to the understanding of the problem of this research. In fig. 4:16, a Weibull line with a β =3.44 is plotted, which represent the Weibull plot of normally distributed data in theory. The corresponding age of failure values of the median rank values can be known using β=3.44 line (refer to figure 4:16 & table 4:3). Subsequently, further mathematical calculation will be used to show the behaviour of β in small sample sizes (n=5)[76].

	Median Rank	**Age Failure**
n=1	*12.945*	*113*
n=2	*31.381*	*150*
n=3	*50.000*	*180*
n=4	*68.619*	*208*
n=5	*87.055*	*245*
Mean of X		179.2
σ		50.97745

Table 4:3 - Median rank Vs. Age failure for n=5

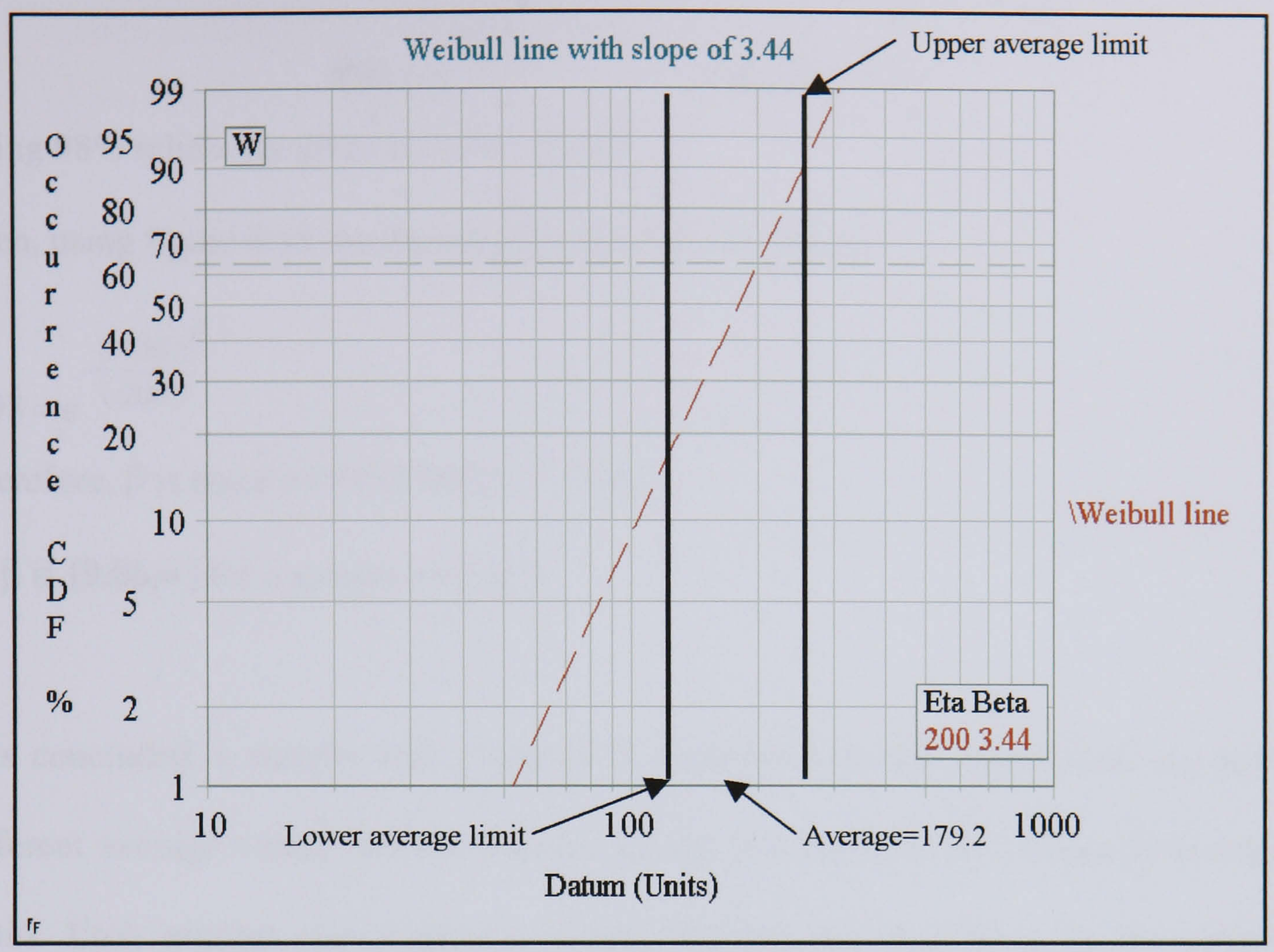


Figure 4:16 - Weibull plot

For a given sample of size 5 and known Age failure average, the range can be calculated as follow:

$$\sigma = \frac{\overline{R}}{d_2} \qquad \cdots \textit{Equation } 4.13$$

$$\Rightarrow \overline{R} = d_2 * \sigma = 50.977 * 2.326$$

$$\overline{R} = 118.573$$

Having a range average of 118.573 can be used to produce average control limits; these control limits are calculated as follow;

$$\textit{Range Control Limits for } \overline{X} = \overline{X} \pm \frac{\overline{R}}{2} \qquad \textit{Equation } 4.14$$

$$= 179.2 \pm \frac{118.573}{2}$$

$$\textit{Therefore } UCL_{\overline{X}} = 179.2 + 59.2865 = 238.7865 \cong 239$$

$$\textit{And} \qquad LCL_{\overline{x}} = 179.2 - 59.2865 = 120.2135 \cong 120$$

Using Weibull reliability function

$$R(t) = e^{-\left(\frac{t}{\eta}\right)^{\beta}} \qquad \cdots \textit{Equation } 4.15$$

Using 98% reliability gives a t value of 130

Then, using figure 4:15 the limiting β value for n=5 will be

$$0.98 = e^{-\left(\frac{130}{200}\right)^{\beta}}$$

Therefore, β is equal to $9.057784124 \approx 9.06$

$\Rightarrow \beta \in$ **[9.06,∝]** for a sample size of 5

It is concluded, a sample with a size of 5, which is normally distributed, can have different average values, but the average should fall between the calculated average limits. Each average can produce a certain Weibull line depending on the sample standard deviation. The Weibull line has a Beta value constrained within the calculated interval. Therefore, for a certain average; an infinite number of Weibull lines and each of them has a different beta value.

The previous analysis draws a result, which is considered a primary finding in the field of Weibull parameter prediction. The common knowledge in small sample size use with Weibull distribution revolves around the idea, which small samples from normally distributed data generates any value of shape parameter[57]. But, such an idea can be considered as a general piece of evidence, and the pervious analysis constrained the validity of such statement by verification the limits of the shape parameter when using small sample size. Thus, it is shown that when using a sample in Weibull analysis, predicted value of the shape parameter has a lower limit, which depend on the sample size, in other words, the shape factor is impeded in an interval, and it can not has any value as it was known by Weibull conventional knowledge.

4.6 SMALL SAMPLES SIZE CONSEQUENCES ON WEIBULL SHAPE PARAMETER PREDICTION

Weibull distribution is a useful distribution to be used with small sample size[57]. Weibull distribution consists of three main parameters (shape, scale and location parameter), which are the pivots to the success of Weibull analysis. The role of these parameters can be seen from the equation 4.16, which represent the probability density function of Weibull distribution[75].

$$f(t) = \frac{\beta\,(t-\gamma)^{\beta-1}}{\eta^{\beta}} \exp\left[-\left(\frac{t-\gamma}{\eta}\right)^{\beta}\right], t \geq \gamma. \qquad \cdots \textit{Equation } 4.16$$

Where : β = Shape parameter

γ = Location Parameter

η = Scale parameter

The success of the Weibull analysis significantly depends on the accuracy of the parameters used. Many techniques have developed in recent years to study the behaviour of such parameter, and try to establish accurate point estimation for Weibull parameter. In this research, the estimation of shape parameter will be dealt with in details as such parameter is important in Weibull analysis, and the difference between theory and practice occur when using small sample size. Three main techniques will be discussed, these techniques are:

1. Weibull probability plot.
2. Least square technique (Regression analysis).
3. Maximum Likelihood Estimation.

4.6.1 Probability Plotting to Predict Weibull Shape Parameter

A particular method of calculating the shape parameter of the Weibull distribution is by using probability plotting. As shown in equation 2.8, the Weibull line can be plotted, and such plot will facilitate the analyses of data. This plot is designed to predict a value for Weibull shape parameter by using specially designed Weibull plot graph paper. This paper is constructed from to main coordinates axes; the Y coordinate axis is median rank values, and the X coordinate axis is the time to failure or data axis. Both of the X and Y axes are log-log axes.

The median rank is a non-parametric estimate of the cumulative distribution based on ordered failures or data in a sample[77]. Such estimation is hard to be developed without the use of modern computer technology, however, an approximate expression (Bernard's method) provide an acceptable values to the real median rank cumulative sum.

$$\%\, \boldsymbol{MR}_{\boldsymbol{Bernard}} = \frac{\boldsymbol{j} - 0.3}{\boldsymbol{N} + 0.4} \times 100\% \qquad \cdots \textbf{\textit{Equation}}\ 4.17$$

Equation 4.17, shown Bernard's approximation, where j is the Failure or Data order, and N is the total sample size. Bernard's approximation can be used to achieve acceptable Weibull plot using special Weibull graph papers (refer to Appendix). In this research computer software (WEIBULL++) will be used to achieve high accuracy in Weibull probability plot and the exact median rank will be calculated by equation 4.18.

$$\sum_{k=i}^{N} \binom{\boldsymbol{N}}{\boldsymbol{k}} (\boldsymbol{MR}_{\boldsymbol{Exact}})^{k} (1 - \boldsymbol{MR}_{\boldsymbol{Exact}})^{N-k} = 0.50 = 50\% \qquad \cdots \textbf{\textit{Equation}}\ 4.18$$

4.6.2 Regression Analysis to Predict Weibull Shape Parameter

Least square method (regression analysis) is another way to establish a point estimation for Weibull shape parameter. Using the idea of probability plotting discussed previously, regression analysis mathematically fits the best straight line to a set of points, in an attempt to estimate the value of shape parameter accurately. The term rank regression is used in this research instead of least squares, or linear regression, because the regression is performed on the rank values, more specifically, the median rank values (represented on the Y-axis of the Weibull plot).

The method of least squares requires that a Weibull straight line is fitted to a set of data points such that the sum of the squares of the distance of the points to the fitted line is minimized. This minimization can be performed in either the vertical or the horizontal direction. If the regression is on X, then the line is fitted so that the horizontal deviations from the points to the line are minimized. If the regression is on Y, then this means that the distance of the vertical deviations from the points to the line is minimized. This is illustrated in figure 4:17.

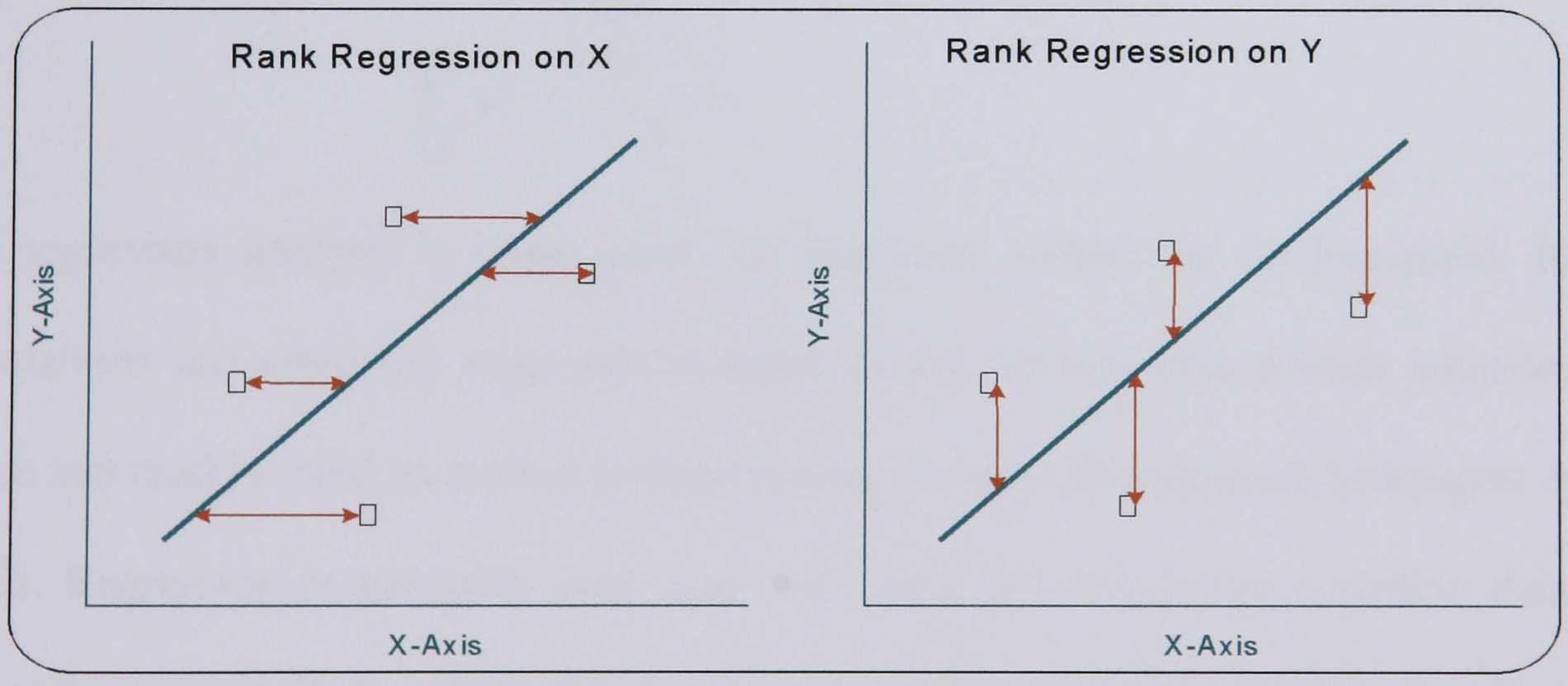


Figure 4:17 – Regression analysis

When a set of data pairs (x_1, y_1), (x_2, y_2),.., (x_N, y_N) is obtained and plotted, and that the y-values are known exactly. Then, according to the least squares principle, which minimizes the horizontal distance between the data points and the straight line fitted to the data, the best fitting straight line to these data is the straight line $x = \hat{a} + \hat{b}y$ (where the recently introduced (^) symbol indicates that this value is an estimate) such that[78]:

$$\sum_{i=1}^{N}(\hat{a}+\hat{b}y_i - x_i)^2 = \min(a.b)\sum_{i=1}^{N}(a+bx_i - y_i)^2 \qquad \cdots \textbf{\textit{Equation}}\ 4.19$$

Where : $\hat{a}$, $\hat{b}$ are the least square estimates for a and b. N = total number of data points

These equations are minimized by estimates of $\hat{a}$ and $\hat{b}$ such that,

$$\hat{a} = \frac{\sum_{i=1}^{N} x_i}{N} - \hat{b}\frac{\sum_{i=1}^{N} y_i}{N} = \bar{x} - \bar{b}\,\bar{y} \qquad \cdots \textbf{\textit{Equation}}\ 4.20$$

and

$$\hat{b} = \frac{\sum_{i=1}^{N} x_i y_i - \frac{\sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i}{N}}{\sum_{i=1}^{N} y_i^2 - \frac{\left(\sum_{i=1}^{N} y_i\right)^2}{N}} \qquad \cdots \textbf{\textit{Equation}}\ 4.21$$

The regression analysis is quite good for functions, which can be linearized. Its calculations are relatively easy and straightforward, having closed-form solutions which can readily yield an answer without having to resort to numerical techniques or tables. Regression is generally best used with data sets containing complete data. Normally in Weibull, X variable has much more scatter and statistical error than Y. It is recommended to select the scale with largest error as the dependent variable[79]. Therefore, in this research X rank regression will be adopted to ensure accurate calculation and analysis results.

4.6.3 Maximum Likelihood Analysis to Predict Weibull Shape Parameter

Maximum likelihood estimation is considered to be the most robust and complicated of the parameter estimation techniques. The basic idea behind MLE is to obtain the most likely values of the parameters, for a given distribution, that will best describe the data. Ideally, Maximum likelihood estimation works by developing a likelihood function based on the available data and finding the values of the parameter estimates that maximize the likelihood function. This can be achieved by using iterative methods to determine the parameter estimate values that maximize the likelihood function, but this can be rather difficult and time-consuming, particularly when dealing with the three-parameter distribution. Therefore, another method of finding the parameter estimates involves taking the partial derivatives of the likelihood function with respect to the parameters, setting the resulting equations equal to zero, and solving simultaneously to determine the values of the parameter estimates. The log-likelihood functions and associated partial derivatives used to determine maximum likelihood estimates for the Weibull distribution.

The likelihood function is a function of the data. It is the product of the probability density function, for each data point, with the distribution parameter unidentified. If x is a continuous random variable with a probability density function

$$f(x;\theta_1,\theta_2,\cdots,\theta_k)$$

Where θ1, θ2,…, θk are unidentified parameters, which need to be estimated, where R independent observations, x_1, x_2,…, x_k , which correspond data analysis. The likelihood function is given by[80]:

$$L(\theta_1,\theta_2,\cdots,\theta_k \mid x_1,x_2,\cdots,x_R) = L = \prod_{i=1}^{R} f(x_i,\theta_1,\theta_2,\cdots,\theta_k) \qquad \cdots \textbf{\textit{Equation}}\ 4.22$$

Where i = 1, 2, 3, …, R.

The logarithmic likelihood function is,

$$\Lambda = \ln L = \sum_{i=1}^{R} \mathrm{Ln}\ f(x_i,\theta_1,\theta_2,\cdots,\theta_k) \qquad \cdots \textbf{\textit{Equation}}\ 4.23$$

The maximum likelihood estimators (or parameter values) are obtained by maximizing L or Λ. By maximizing Λ, which is much easier to work with than L, the maximum likelihood estimators (MLE) are the simultaneous solutions of k equations such that:

$$\frac{\partial \Lambda}{\partial \theta_j} = 0 \quad , \quad j = 1,2,\cdots,k \qquad \cdots \textbf{\textit{Equation}}\ 4.24$$

Log Likelihood function is used to predict Weibull parameters, as this research is only concerned with 2-parameter Weibull, the 2-weibull log likelihood function will be discussed. The 2 parameter Weibull log-likelihood function is composed of three-summation portions[81]:

$$\ln(L) = \Lambda = \sum_{i=1}^{F_e} N_i \ln\left[\frac{\beta}{\eta}\left(\frac{T_i}{\eta}\right)^{\beta-1} e^{-\left(\frac{T_{ii}}{\eta}\right)^{\beta}}\right] - \sum_{i=1}^{S} N_i'\left(\frac{T_i'}{\eta}\right)^{\beta}$$
$$+ \sum_{i=1}^{FI} N_i'' \ln\left[e^{-\left(\frac{T_{Li}''}{\eta}\right)^{\beta}} - e^{-\left(\frac{T_{Ri}''}{\eta}\right)^{\beta}}\right] \qquad \cdots \textbf{\textit{Equation}}\ 4.25$$

Where,

F_e= is the number of groups of data points,
N_i= is the number of data in the ith data group,
β = is the Weibull shape parameter (unknown a priori),
η = is the Weibull scale parameter (unknown a priori),
T_i= is the time of the group i^{th} of data,
S = is the number of groups of data points,
N'_I=is the number of data in i^{th} group of data points,
T_i'= is the data of the ith data group,

FI= is the number of interval data groups,
N''_i =is the number of intervals in i^{th} group of data intervals
T''_{Li} = is the beginning of the ith interval,
T''_{Ri} = is the ending of the ith interval.

For the purposes of MLE, data will be considered to be intervals with $T''_{Li} = 0$. The solution will be found by solving for a pair of parameters $(\hat{\beta},\hat{\eta})$ so that $\frac{\partial \Lambda}{\partial \beta} = 0$ and $\frac{\partial \Lambda}{\partial \eta} = 0$. It should be noted that other methods could also be used, such as direct maximization of the likelihood function, without having to compute the derivatives.

$$\frac{\partial \Lambda}{\partial \beta} = \frac{1}{\beta}\sum_{i=1}^{F_c} N_i + \sum_{i=1}^{F_c} N_i \ln\left(\frac{T_i}{\eta}\right) - \sum_{i=1}^{F_c} N_i \left(\frac{T_i}{\eta}\right)^{\beta} \ln\left(\frac{T_i}{\eta}\right) - \sum_{i=1}^{S} N'_i \left(\frac{T'_i}{\eta}\right)^{\beta} \ln\left(\frac{T'_i}{\eta}\right) + \sum_{i=1}^{FI} N''_i \frac{-\left(\frac{T_{Li}}{\eta}\right)^{\beta} \ln\left(\frac{T_{Li}}{\eta}\right) e^{-\left(\frac{T_{Li}}{\eta}\right)^{\beta}} + \left(\frac{T_{Ri}}{\eta}\right)^{\beta} \ln\left(\frac{T_{Ri}}{\eta}\right) e^{-\left(\frac{T_{Ri}}{\eta}\right)^{\beta}}}{e^{-\left(\frac{T_{Li}}{\eta}\right)^{\beta}} - e^{-\left(\frac{T_{Ri}}{\eta}\right)^{\beta}}} \quad \text{... } \textbf{\textit{Equation}} \text{ 4.26}$$

$$\frac{\partial \Lambda}{\partial \eta} = \frac{-\beta}{\eta}\sum_{i=1}^{F_c} N_i + \frac{\beta}{\eta}\sum_{i=1}^{F_c} N_i \left(\frac{T_i}{\eta}\right)^{\beta} + \frac{\beta}{\eta}\sum_{i=1}^{S} N'_i \left(\frac{T'_i}{\eta}\right)^{\beta} + \sum_{i=1}^{FI} N''_i \frac{\left(\frac{\beta}{\eta}\right)\left(\frac{T_{Li}}{\eta}\right)^{\beta} e^{-\left(\frac{T_{Li}}{\eta}\right)^{\beta}} - \left(\frac{\beta}{\eta}\right)\left(\frac{T_{Ri}}{\eta}\right)^{\beta} e^{-\left(\frac{T_{Ri}}{\eta}\right)^{\beta}}}{e^{-\left(\frac{T_{Li}}{\eta}\right)^{\beta}} - e^{-\left(\frac{T_{Ri}}{\eta}\right)^{\beta}}} \quad \text{... } \textbf{\textit{Equation}} \text{ 4.27}$$

It is observed that when the log likelihood is differentiated with respect to the parameters, and the resulting equation is set to equal to zero. The resulting equations are then solved simultaneously to obtain the best estimates of the parameters that maximize the likelihood function and such estimate is called the Maximum Likelihood Estimate (MLE).

The MLE method has many large sample properties that make it attractive for use. It is asymptotically consistent, which means that as the sample size gets larger, the estimates converge to the right values. It is asymptotically efficient, which means that for large samples it produces the most precise estimates. It is asymptotically unbiased, which means that for large samples one expects to get the right value on average. The distribution of the estimates themselves is normal, if the sample is large enough.

Unfortunately, the size of the sample necessary to achieve these properties can be quite large, thirty, fifty to more than a hundred exact data points, depending on the application. With fewer points, the methods can be badly biased[82]. It is known, for example, that MLE estimates of the shape parameter for the Weibull distribution are badly biased for small sample sizes, and the effect can be increased depending on the amount of censoring. This bias can cause major discrepancies in analysis.

4.6.4 Unbiased Maximum Likelihood Estimation of Weibull Shape Parameter

The likelihood function usually has a maximum at specific values of the distribution parameters. These values of parameters are more likely to give rise to the data that other values. Therefore, using a maximum likelihood method will provide a best single point estimate in predicting a parameter of the needed function.

The MLE analysis provides a point estimate of beta, but this calculated value is biased for a small *n*. Bain and Engelhardt suggest the use of an unbiased factor G_n. Using such factor, the unbiased estimation of the shape parameter is[83]

$$\hat{\beta} = G_n \times \hat{\beta}_{MLE} \qquad \cdots \textbf{\textit{Equation}}\ 4.28$$

G_n is calculated using the approximation:

$$G_n = 1.0 - \frac{1.346}{n} - \frac{0.8334}{n^2} \qquad \cdots \textbf{\textit{Equation}}\ 4.29$$

Where n is the sample size.

Therefore, the unbiased Shape parameter β is the multiplication of the unbiased factor by the shape parameter derived from the Maximum Likelihood Estimation (MLE). Using Bain and Engelhardt technique gives us the following approximation for the upper and lower limit for the estimated unbiased Weibull shape factor[62].

$$\hat{\beta}_L = \hat{\beta}\left[\frac{X_{(1-\alpha),df}^{\ 2}}{c\ n}\right]^{\frac{1}{1+p^2}} \qquad \cdots \textbf{\textit{Equation}}\ 4.30$$

$$\hat{\beta}_U = \hat{\beta}\left[\frac{X_{(\alpha),df}^{\ 2}}{c\ n}\right]^{\frac{1}{1+p^2}} \qquad \cdots \textbf{\textit{Equation}}\ 4.31$$

Where c is the chi - squared factor = $2/[(1+p^2)^2\ p\ c_{22}]$ where c_{22} is asymptotic values for MLE and c= 0.822 for p equal to 1.

4.7 WEIBULL WIRE TEST ANALYSIS

A wire diameter test has been set to illustrate the methods of estimating Weibull parameters. In this test 16 samples were collected, where each sample has a size of 7 diameter values (refer to table 4:4). These collected data will be used to estimate Weibull parameters in order to understand the behaviour of such parameter when linked with small sample size. The parameters will be estimated using three main techniques, which are: Weibull Rank Regression on X (RRX), Maximum Likelihood estimation (MLE), and the unbiased shape parameter estimation. The probability plotting technique was omitted from the analysis due to the need of high in accuracy in plotting the data by hand.

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
1	*5.6*	*6.1*	*6.0*	*5.8*	*5.9*	*6.0*	*6.0*	*5.9*
2	*6.2*	*5.4*	*6.2*	*6.3*	*5.7*	*6.1*	*6.3*	*5.8*
3	*5.9*	*5.9*	*5.8*	*6.0*	*6.3*	*5.8*	*5.9*	*6.1*
4	*6.1*	*6.0*	*6.1*	*5.9*	*6.1*	*6*	*5.8*	*6.0*
5	*5.7*	*5.8*	*6.0*	*6.2*	*5.6*	*6.2*	*6.1*	*6.2*
6	*6.0*	*6.2*	*5.9*	*6.1*	*6.2*	*5.9*	*6.2*	*5.9*
7	*5.8*	*6.1*	*5.7*	*6.0*	*6.0*	*5.7*	*6.0*	*6.3*

Continued …

	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}
1	*6.0*	*5.7*	*6.2*	*5.9*	*5.9*	*5.8*	*6.3*	*5.9*
2	*6.1*	*6.2*	*5.7*	*6.1*	*6.5*	*6.0*	*5.6*	*6.2*
3	*6.0*	*6.4*	*5.9*	*6.5*	*5.5*	*6.3*	*6.0*	*5.8*
4	*6.2*	*5.9*	*6.1*	*5.8*	*6.0*	*5.9*	*6.1*	*6.3*
5	*5.5*	*6.3*	*6.1*	*6.2*	*6.4*	*5.7*	*6.0*	*5.9*
6	*5.9*	*5.8*	*6.0*	*6.0*	*6.1*	*6.2*	*6.4*	*6.1*
7	*5.8*	*6.1*	*5.8*	*6.6*	*5.8*	*6.1*	*5.9*	*5.7*

Table 4:4 – Wire Diameter (mm)

By using simple statistical calculation by MINITAB figure 4:18 is generated, from the descriptive statistics it can be seen that overall wire diameter data have a mean of 6 and a standard deviation of 0.22461.

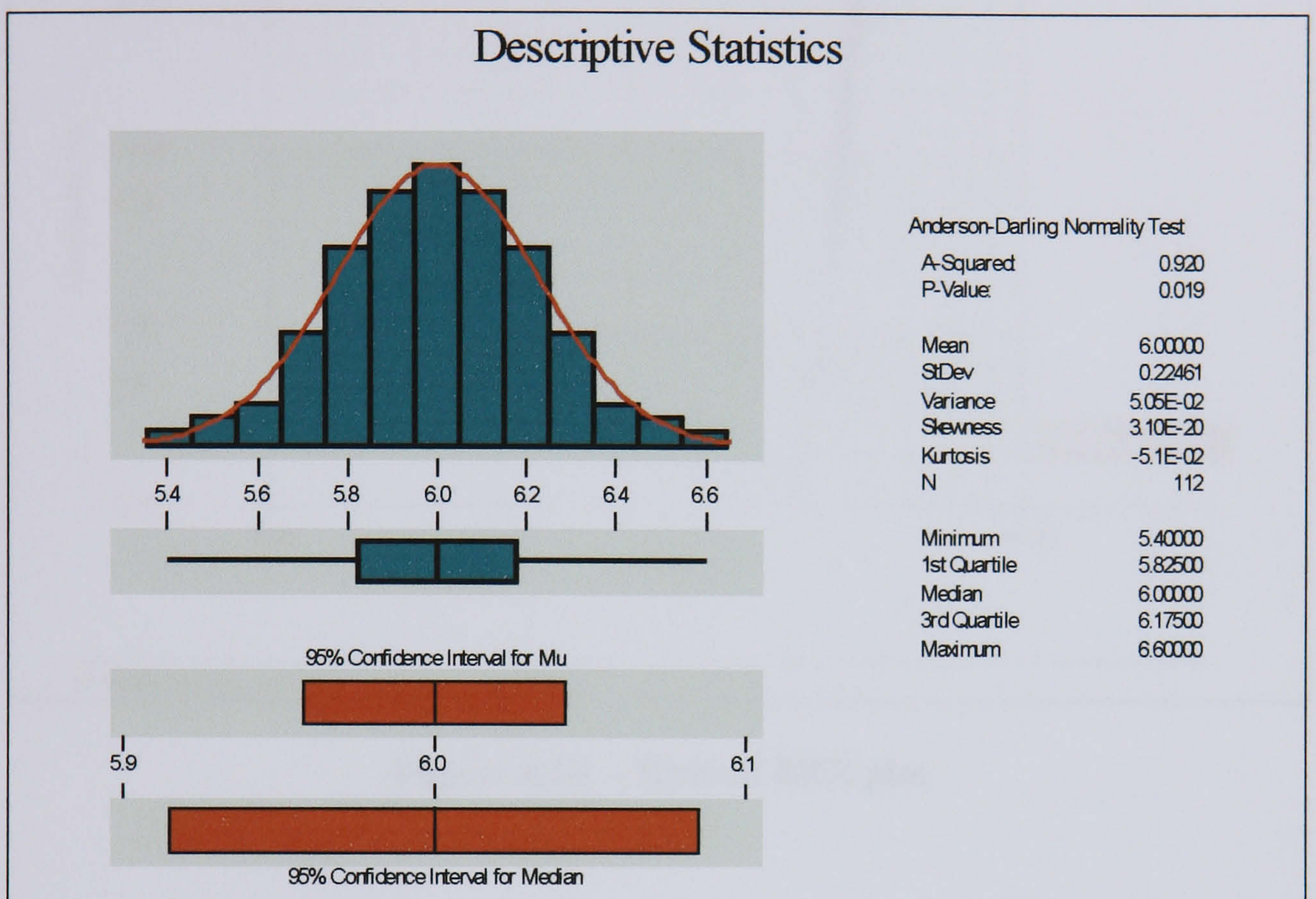


Figure 4:18 – Descriptive statistics of wire diameter test

Founded on the wire diameter data, Weibull++ is used to calculate the parameter of Weibull based on rank regression on X and Weibull logarithmic maximum likelihood (MLE). Figure 4:19 shows a RRX Weibull plot, from this method, the estimated Weibull parameters are: Shape parameter of 35.2530 and Scale parameter of 6.0963. It is noticed that the shape parameter is extremely far away value than expected (3.44). While The estimated MLE parameters are: Shape parameter of 21.77 and Scale parameter of 6.12 (refer to figure 4:20). Clearly, the MLE provide much closer value to the theoretical value than RRX method, hence it is a poor estimation.

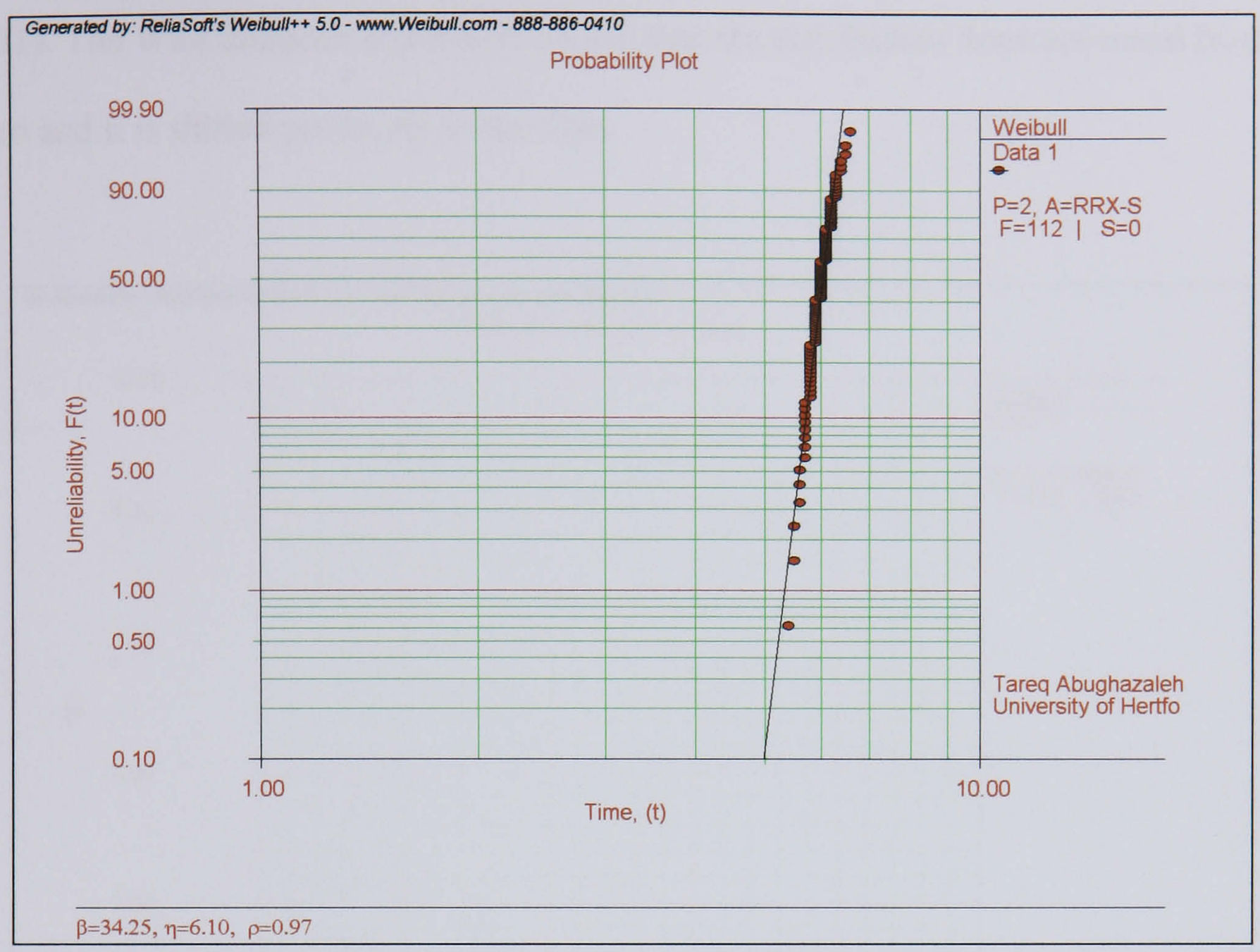


Figure 4:19 – Weibull RRX plot

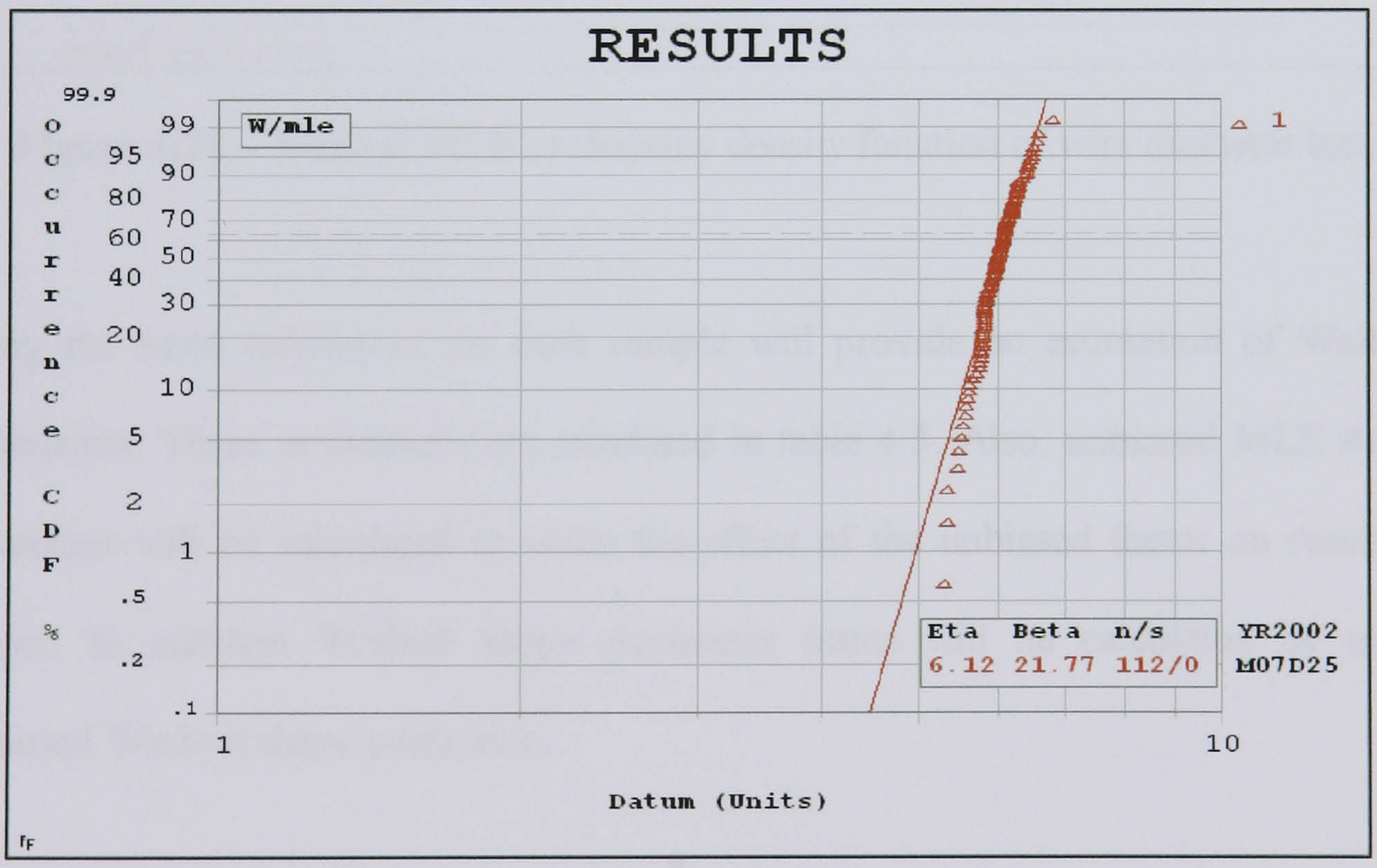


Figure 4:20 – Weibull MLE plot

Using the MLE estimation, a probability density function is drawn (refer to figure 4:21). The Wire diameter test P.D.F. shows that the distribution does not initial from zero and it is shifted positively to the right.

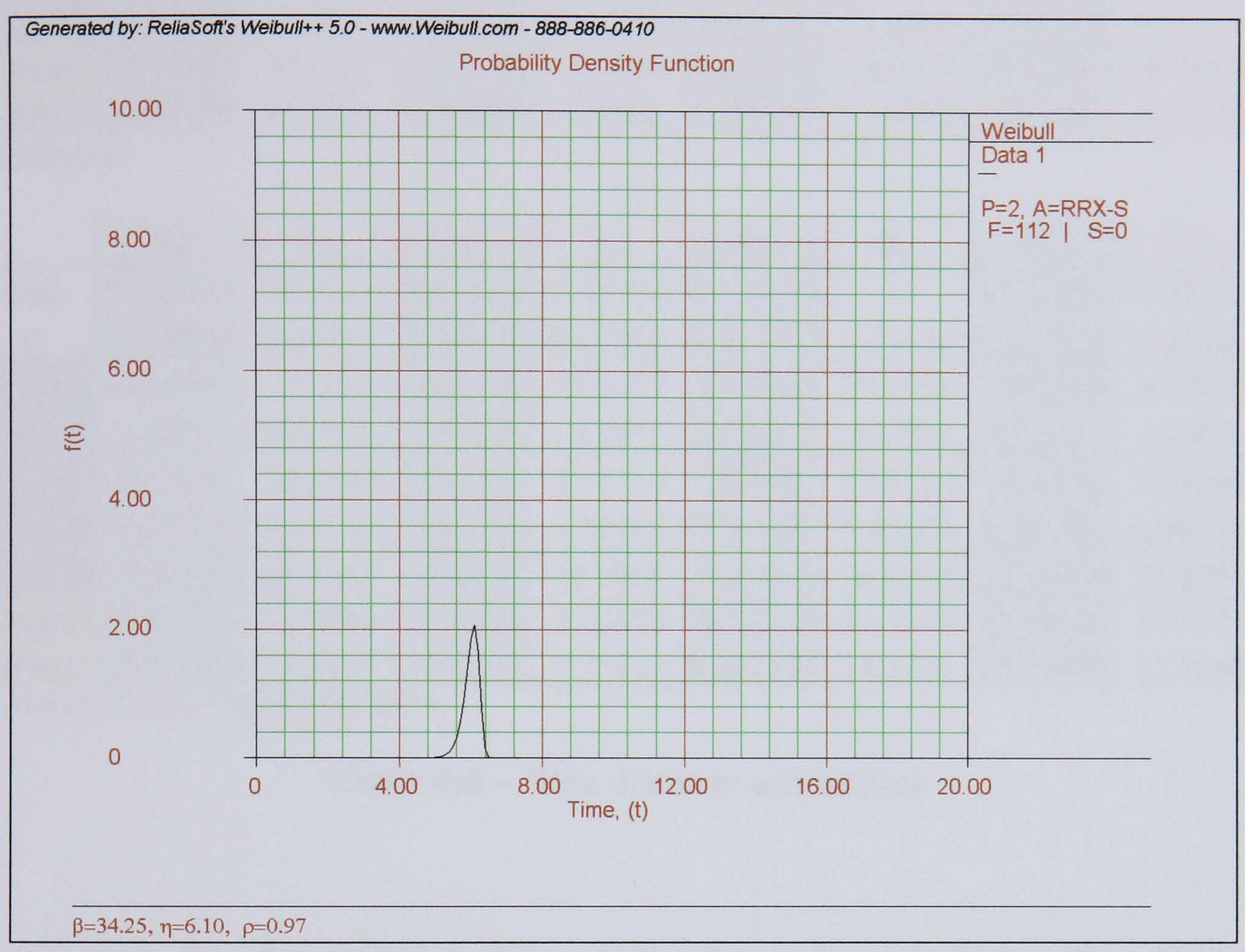


Figure 4:21 – Weibull MLE probability density function of wire diameter test

Using the same techniques on each sample will provide an estimation of Weibull parameters. These estimations are tabulated in table 4:5. Also, unbiased MLE shape parameter will be calculated to study the effect of the unbiased factor on resulting values. In addition Weibull shape parameter limits will be calculated by using unbiased Weibull shape parameter.

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
Avg.	5.9	5.92857	5.95714	6.04286	5.971429	5.95714	6.042857	6.02857
σ	0.216025	0.26904	0.17183	0.17183	0.256348	0.17182	0.171825	0.17995
β_{RRX}	29.8885	24.0358	37.784	38.9227	25.2915	37.784	38.9227	37.9189
η_{RRX}	5.9978	6.0499	6.0357	6.1203	6.0879	6.0357	6.1203	6.1077
β_{MLE}	33.0637	34.4981	42.353	40.6887	29.8918	42.353	40.6887	38.136
η_{MLE}	5.99658	6.03455	6.03358	6.12119	6.08293	6.03358	6.12119	6.1149
β_{Unbias}^{*}	26.14367	27.2779	33.4888	32.1728	23.63563	33.4888	32.1728	30.1544
Upper	38.67427	40.3521	49.5399	47.5932	34.96413	49.5399	47.59315	44.6073
Lower	13.93774	14.5424	17.8536	17.152	12.60065	17.8536	17.152	16.0759

Continued …

	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}
Avg.	5.928571	6.05714	5.97143	6.15714	6.028571	6	6.042857	5.98571
σ	0.228869	0.26367	0.17995	0.29921	0.345033	0.21603	0.263674	0.21931
β_{RRX}	28.044	25.276	36.1136	23.6017	19.1349	30.3969	25.0954	30.7155
η_{RRX}	6.0332	6.1753	6.0538	6.285	6.1823	6.0979	6.1616	6.0822
β_{MLE}	36.7344	28.8187	43.116	23.1322	21.047	33.6196	28.4529	31.9205
η_{MLE}	6.02264	6.17335	6.05006	6.29406	6.18019	6.09663	6.15766	6.08543
β_{Unbias}^{*}	29.04612	22.7872	34.0921	18.2908	16.64199	26.5832	22.49788	25.2397
Upper	42.96785	33.7089	50.4324	27.0575	24.61846	39.3245	33.28106	37.3371
Lower	15.48509	12.1483	18.1752	9.75119	8.872194	14.1721	11.99409	13.4558

* Unbiased factor = $G_n = 0.79070614$

Table 4:5 – Wire diameter estimations

It can clear that the unbiased shape parameter is the minimum value obtained compared with MLE and Weibull RRX. (See figure 4:22)

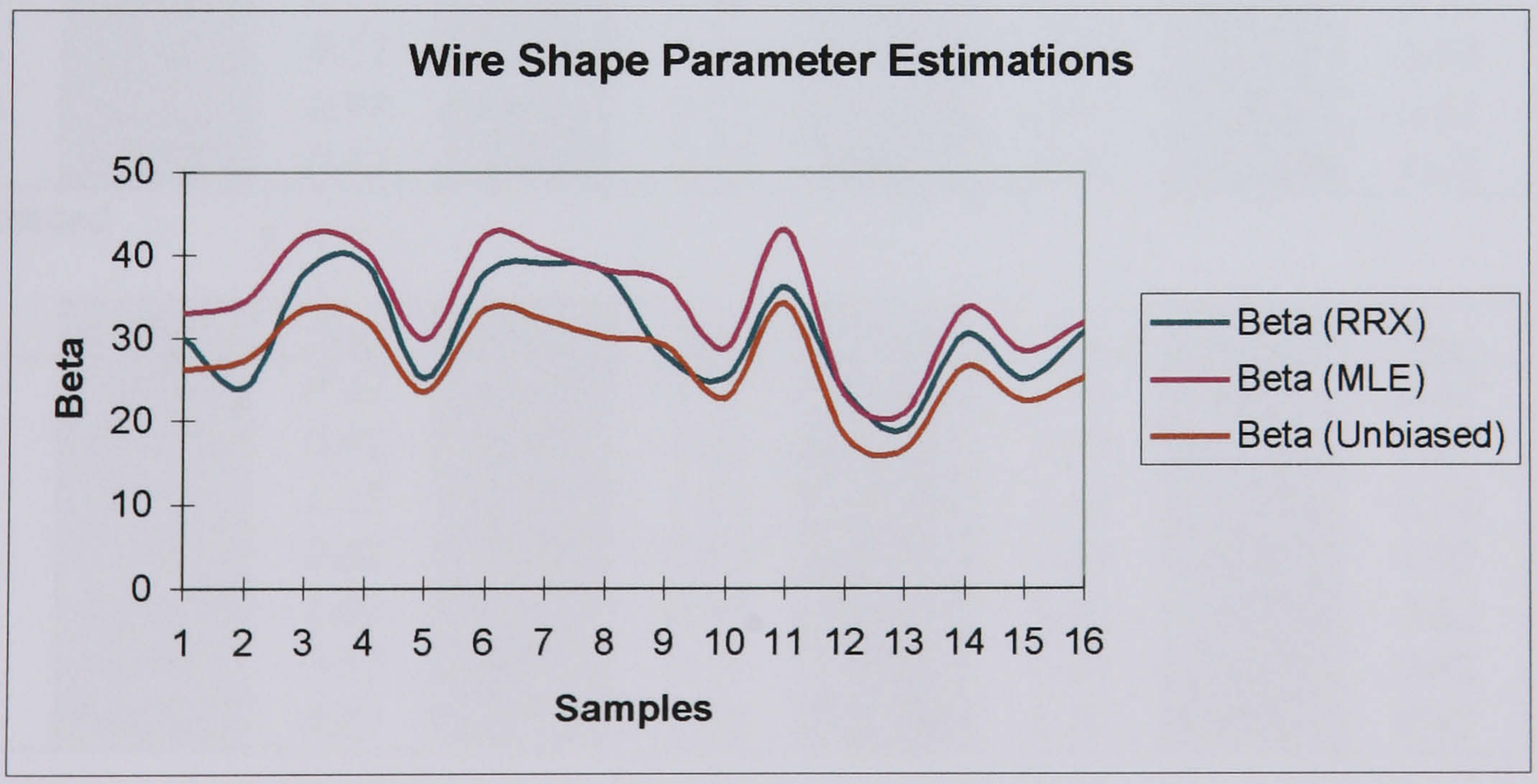


Figure 4:22 – Wire shape parameter estimations

It is noticed from figure 4:21, that Weibull p.d.f. initiates from a positive value, and it is an area of concern to be studied. After little mathematical and empirical calculation, it has been observed that deducting a numerical value of 5.28 will help achieving shape parameter, which are expected to occur based on the theory that normally distributed data will have a shape parameter of 3.44.

To achieve understanding of the small sample behaviour with this deduction criterion, each sample of the 16 samples obtained in the wire test will be analysed individually. Each sample will be used to calculate its average and standard deviation values (after the deduction); also Weibull parameters will be obtained by different estimation methods. Table 4:6 is showing the new data after the deduction and table 4:7 will illustrate results of this analysis.

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
1	*0.32*	*0.82*	*0.72*	*0.52*	*0.62*	*0.72*	*0.72*	*0.62*
2	*0.92*	*0.12*	*0.92*	*1.02*	*0.42*	*0.82*	*1.02*	*0.52*
3	*0.62*	*0.62*	*0.52*	*0.72*	*1.02*	*0.52*	*0.62*	*0.82*
4	*0.82*	*0.72*	*0.82*	*0.62*	*0.82*	*0.72*	*0.52*	*0.72*
5	*0.42*	*0.52*	*0.72*	*0.92*	*0.32*	*0.92*	*0.82*	*0.92*
6	*0.72*	*0.92*	*0.62*	*0.82*	*0.92*	*0.62*	*0.92*	*0.62*
7	*0.52*	*0.82*	*0.42*	*0.72*	*0.72*	*0.42*	*0.72*	*1.02*

Continued …

	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}
1	*0.72*	*0.42*	*0.92*	*0.62*	*0.62*	*0.52*	*1.02*	*0.62*
2	*0.82*	*0.92*	*0.42*	*0.82*	*1.22*	*0.72*	*0.32*	*0.92*
3	*0.72*	*1.12*	*0.62*	*1.22*	*0.22*	*1.02*	*0.72*	*0.52*
4	*0.92*	*0.62*	*0.82*	*0.52*	*0.72*	*0.62*	*0.82*	*1.02*
5	*0.22*	*1.02*	*0.82*	*0.92*	*1.12*	*0.42*	*0.72*	*0.62*
6	*0.62*	*0.52*	*0.72*	*0.72*	*0.82*	*0.92*	*1.12*	*0.82*
7	*0.52*	*0.82*	*0.52*	*1.32*	*0.52*	*0.82*	*0.62*	*0.42*

Table 4:6 – Deduction wire test data

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
Avg.	0.62	0.648571	0.677143	0.762857	0.691429	0.677142	0.762857	0.748571
σ	0.2160247	0.269037	0.171825	0.171825	0.256348	0.171825	0.171825	0.179947
β_{RRX}	2.8736	1.7231	4.0272	4.7455	2.5744	4.0272	4.7455	4.613
η_{RRX}	0.6987	0.7521	0.7457	0.8305	0.7853	0.7457	0.8305	0.8154
β_{MLE}	3.359	2.954	5.423	4.974	3.442	5.127	4.983	4.897
η_{MLE}	0.6972	0.7135	0.7338	0.8439	0.753	0.7307	0.8172	0.8307
β_{Unbias}^{*}	2.6559819	2.335746	4.287999	3.932972	2.721611	4.053950	3.940087	3.872088
Upper	3.9289874	3.455263	6.343226	5.818036	4.026071	5.996998	5.828563	5.727970
Lower	1.4159595	1.245235	2.286022	2.096749	1.450947	2.161245	2.100543	2.064291

Continued …

	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}
Avg.	0.6485714	0.777142	0.691428	0.877142	0.748571	0.72	0.762857	0.705714
σ	0.2288688	0.263673	0.179947	0.299205	0.345032	0.216024	0.263673	0.219306
β_{RRX}	2.4112	2.9808	3.8768	3.2426	1.9252	3.4149	2.7236	3.4506
η_{RRX}	0.74	0.8727	0.7634	0.9764	0.8655	0.8017	0.8629	0.7836
β_{MLE}	3.779	3.559	4.865	3.467	2.364	4.34	3.728	3.497
η_{MLE}	0.7184	0.8571	0.7478	0.98	0.8484	0.7958	0.837	0.7833
β_{Unbias}^{*}	2.9880785	2.814123	3.846785	2.741378	1.869229	3.431664	2.947752	2.765099
Upper	4.4202571	4.162925	5.690540	4.055313	2.765146	5.076453	4.360602	4.090404
Lower	1.5930072	1.500267	2.050801	1.461486	0.996525	1.829492	1.571508	1.474132

* Unbiased factor = G_n = 0.79070614

Table 4:7 – Deduction wire shape parameter estimations

From the deduction method, many conclusions may be draw. The shape parameter in all of the estimations techniques is in the interval of theoretical expectation for normally distributed data, in other words, most of the readings are near the value of 3.44. Figure 4:23 shows the detection Weibull shape parameter values and the wire test shape parameter common values. For this figure it can be seen that the average of Weibull Shape parameter estimated by RRX, MLE and Unbiased methods are 3.3347, 4.047375 and 3.200284 respectively. Figure 4:24 illustrate a comparison of the common method and the deduction methods and its effect of the estimation of Weibull shape parameter in the wire test.

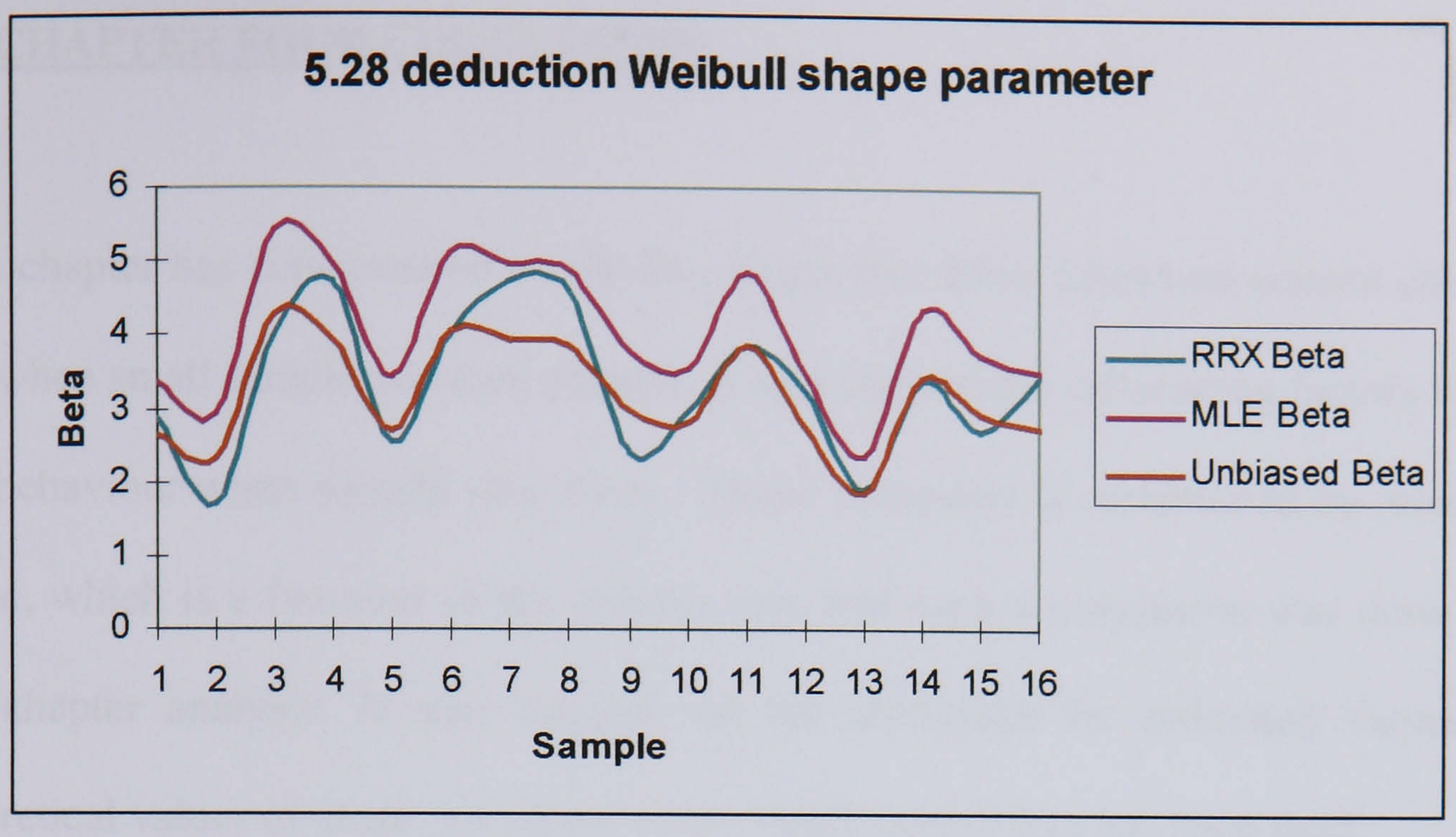


Figure 4:23 – Deduction Weibull Shape Parameter

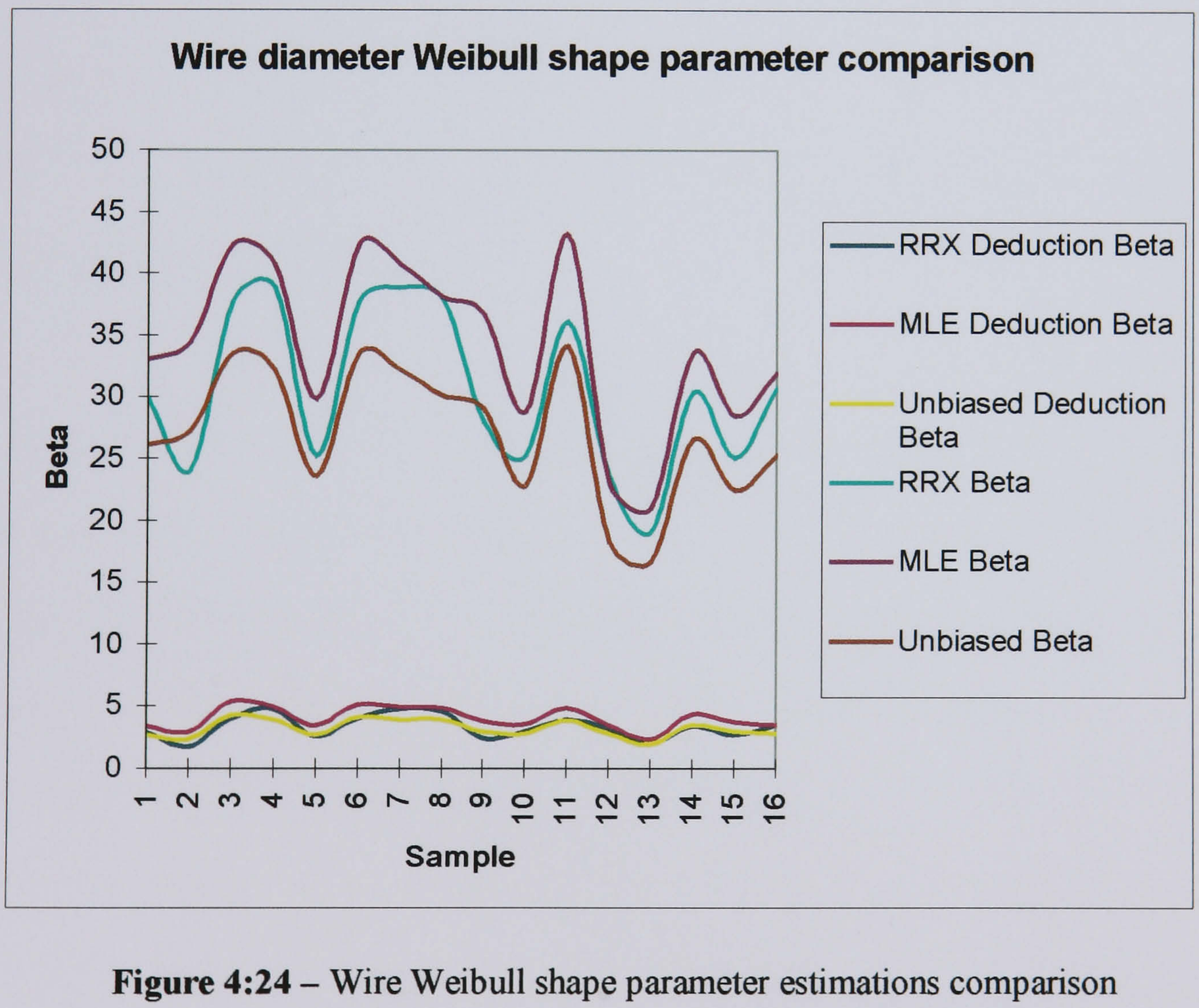


Figure 4:24 – Wire Weibull shape parameter estimations comparison

4.8 CHAPTER FOUR CONCLUSION

This chapter has concentrated on the key issues that allow Shewhart control charts to fail when small sample sizes are present. It also showed the influencing factors D_3 and D4 behaviour when sample size alters. Shape parameter is constrained by minimum value, which is a function of the sample size, and such a conclusion was draw from the chapter analysis. It also flagged out the difference be estimated values and theoretical values of shape parameter when small sample size has been used.

CHAPTER 5

NEW MODEL BASED ON WEIBULL ANALYSIS

5.0 CHAPTER FIVE REVIEW

This chapter summarises the main steps, which was used to develop a new Weibull based control chart model to compensate the existing Shewhart control charts when small sample sizes are used. In this chapter the modelling of the deduction percentage and formulate the result so a deduction model will be used to aid the Weibull analysis in achieving new control charts to achieve accurate limit ranges and detect variability and non-conformance. Also, data from a single lap shear test will be used to test the ability of the new Weibull deduction charts to overcome Shewhart control charts.

5.1 Shape Parameter Estimation Based on Deduction Technique

It has been detected that Weibull shape parameter estimation may be accurate if deduction method is implemented (section 4.7). The detection methods try to shift the p.d.f. of the data, which have Weibull shape parameter estimation away from the true theoretical method (refer to figure 5:1) . This shift showed a constructive influence on the result of Weibull shape parameter estimation.

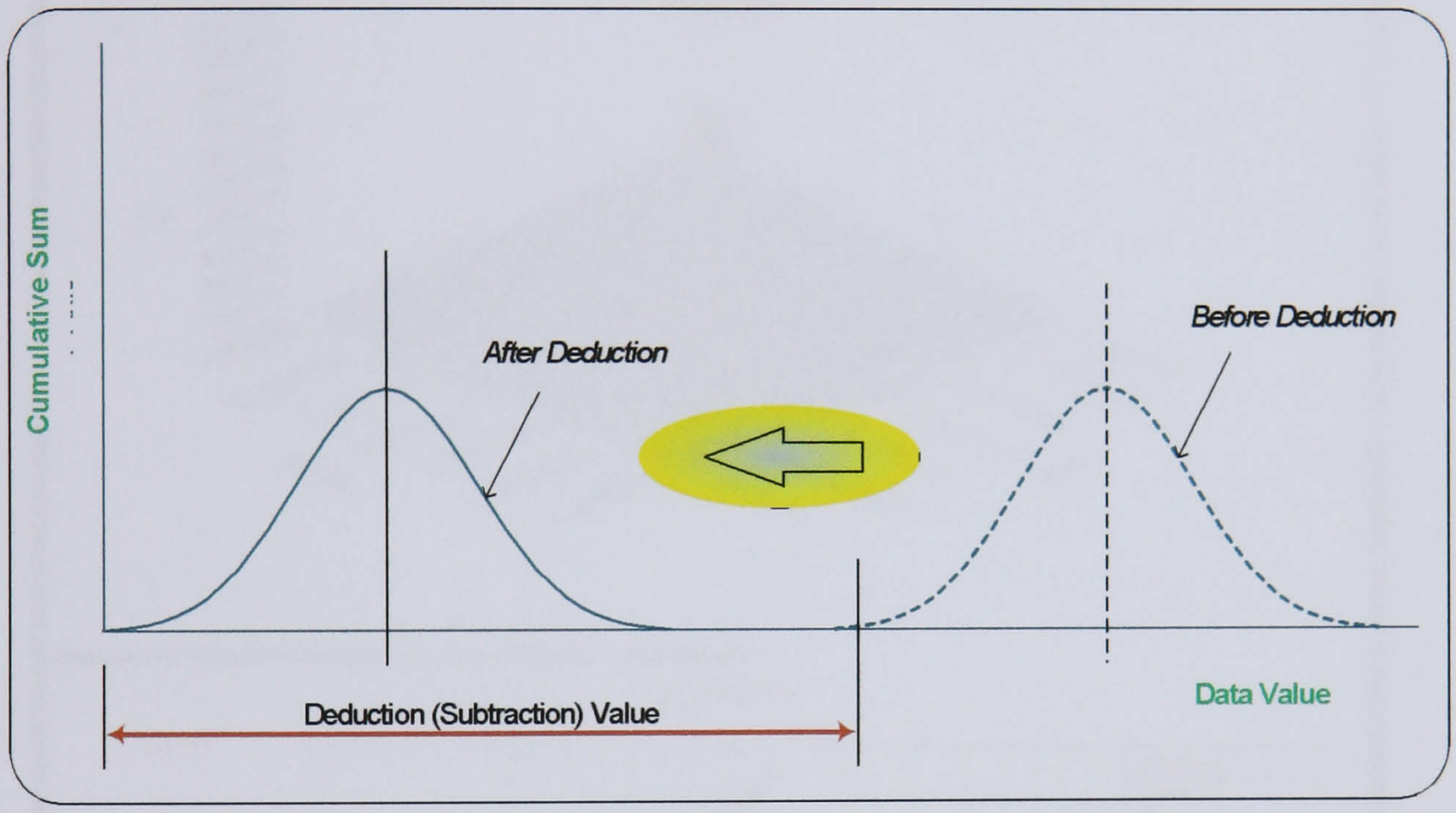


Figure 5:1 – Deduction Method

Weibull shape parameter estimation techniques showed different result of the value of the shape parameter, and using the deduction method should be accompanied with an estimation technique from one of the estimation techniques available (Probability plotting, RRX, MLE and unbiased shape parameter). From the wire test analysis, it has been noticed that the rank regression method showed an estimated value of Weibull shape parameter (after the use of deduction method) of 3.3347 (this value is an average value of shape parameters of 16 samples), which is the nearest value for the theoretical value 3.44. It also shows that MLE estimation was not suitable for

estimating population parameters with small samples[84]. Therefore, it can be noticed that deduction technique can be accompanied with rank regression method to predict an accurate shape parameter value when using small sample size. Figure 5:2, shows the different estimated shape parameter values (for all the 112 diameter values) when using deduction technique.

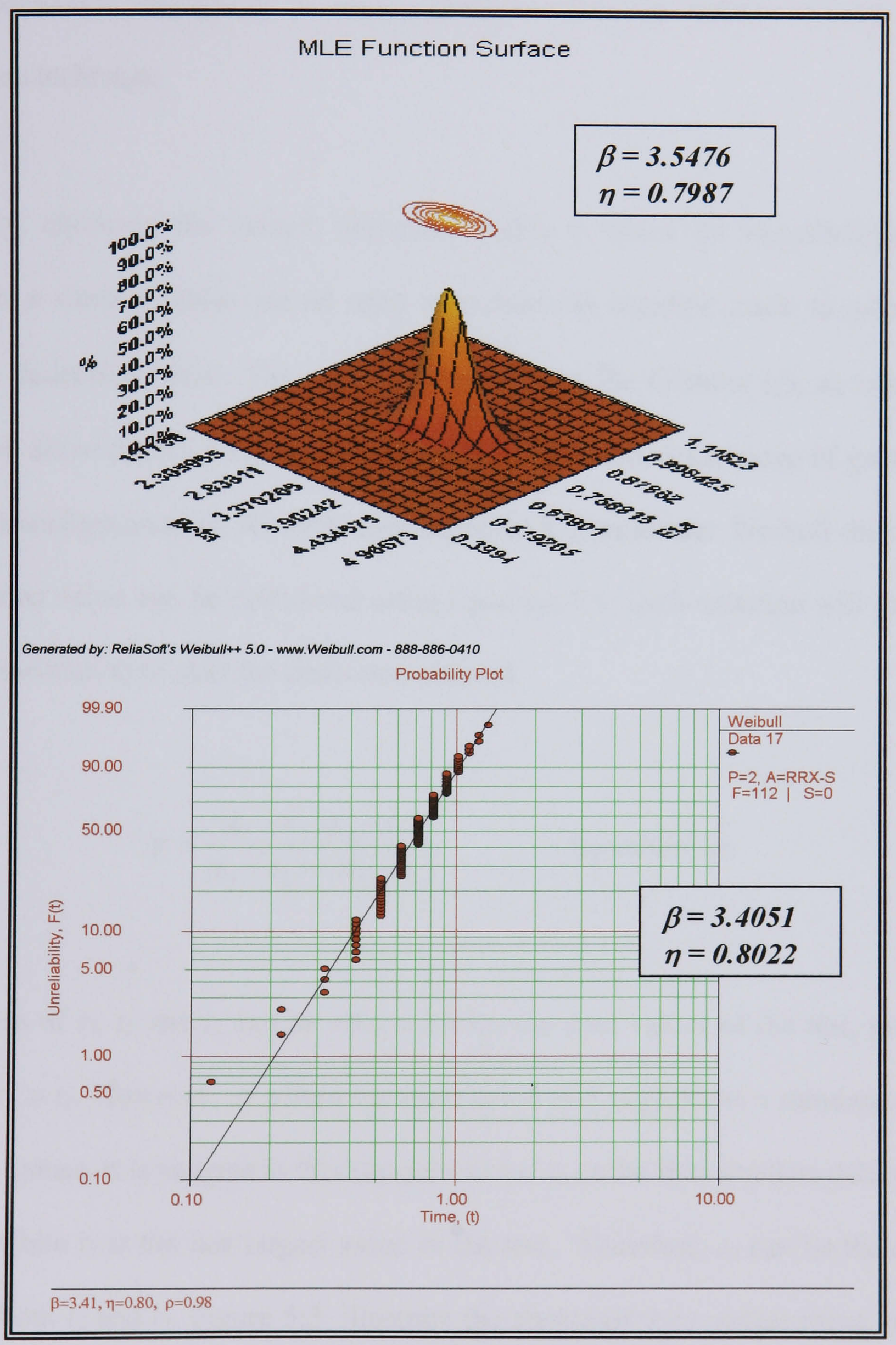


Figure 5:2 – Deduction shape parameter estimation in wire test

Achieving truthful deduction method should minimize the error of prediction of Weibull shape parameter. From figure 5.2, it is noticed that despite the change in the technique of estimation (RRX or MLE), there is no great impact on the estimation of scale parameter (η). For that reason, the deduction method will only concentrate on shape parameter estimation as this parameter value is visibly affected by the estimation technique.

Normally, choosing the correct deduction value is based on experimental trials, however, a starting value can be used to reduce the iteration made to achieve the accurate deduction value. The starting value can be the Gamma (γ), as using such value will allow the p.d.f. to be shifted and starts near zero. Such usage of gamma will allow a transformation of Weibull distribution to a 2 parameter Weibull distribution. The gamma value can be calculated using equation 5.1; such equation will provide a primary estimation to start the deduction method.

$$\gamma = \frac{(t_3 - t_2)(t_2 - t_1)}{(t_3 - t_2) - (t_2 - t_1)} \qquad \cdots \textbf{\textit{Equation}}\ 5.1$$

The values of t_3, t_2 and t_1 can be selected from the data values of the test, providing that $t_3 > t_2 > t_1$. However, to soften the calculation and implement a standard method for these values, it is suggest in this research to use t_1 as the first smallest data value in the test, while t_3 is the last largest value in the test. Therefore, t_2 can be the average value of both t_3 and t_1. Figure 5:3, illustrate the choice of the t values using any rank regression Weibull plot.

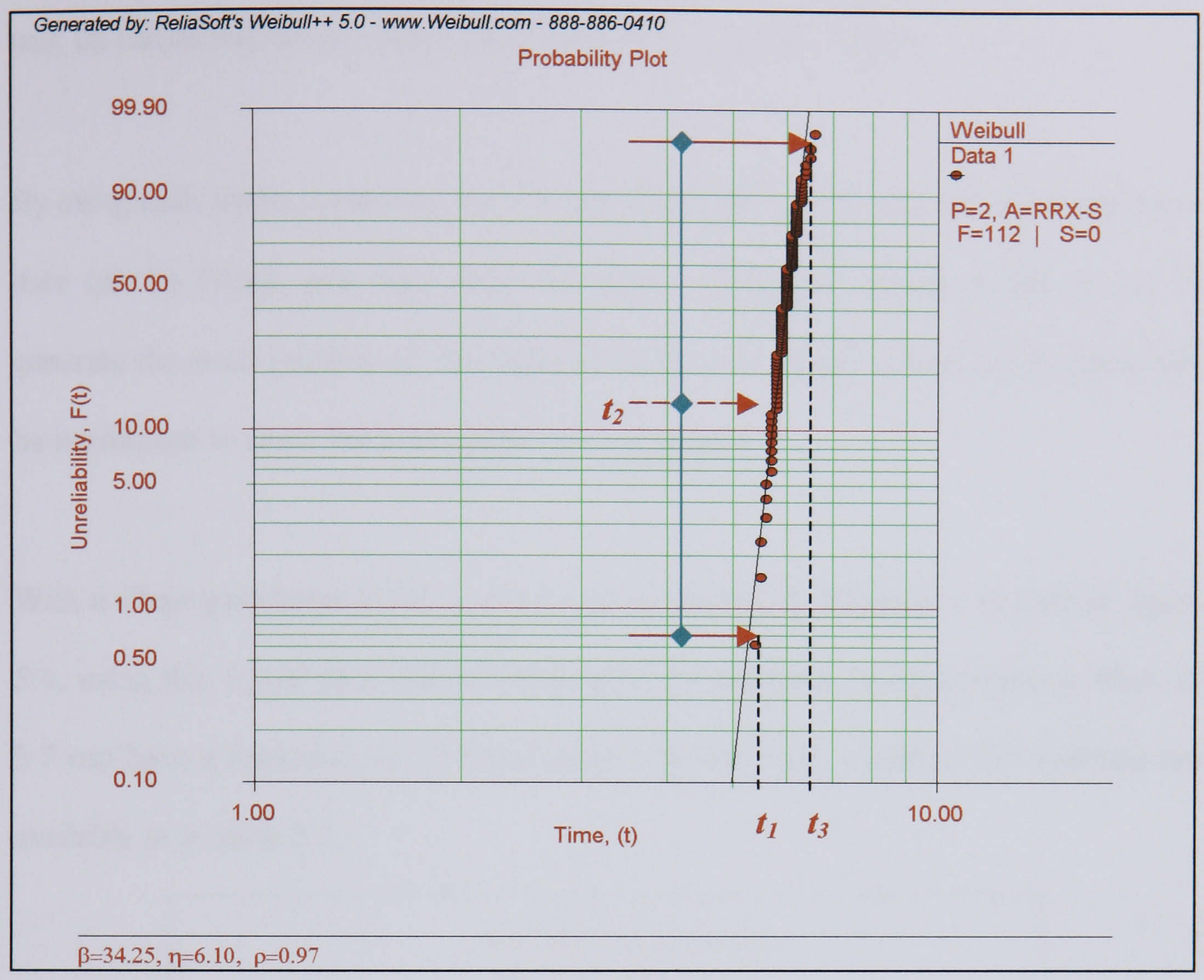


Figure 5:3 – t deduction values.

Using the iteration method to detect the deduction factor may be time consuming and need lots of effort and computer aid. To use the deduction method efficiently a development is required. The development can exist by exploring the relationship between the deduction factor and Weibull shape parameter.

To establish the relationship between the Weibull shape parameter and the deduction factor can be possible by analysing different data with different Weibull shape parameter, and use the deduction method to calculate the deduction factor by which the RRX estimation accompanied with deducted data will provide an accurate

estimation for the beta parameter. To accomplish such analysis, different beta will be use; its values will be, $\beta = 0.5, 1, 2.2, 3.44, 4.5, 7, 10, 12, 15, 20$.

By using each shape parameter, a p.d.f. can be plotted, and using such p.d.f. a primary data can be found, and each data will have a frequency, which enable its use to generate the exact number of data needed for the test. A fully explained example will be introduced to show the mechanism of such analysis.

With a shape parameter of 2.2, a p.d.f. can be plotted by Weibull++ as seen in figure 5:4, using this figure data will be obtained to be analysed, as an example a value of 5.7 can have a frequency of 17 observations. By this way, a total of 154 readings are available as in table 5:1.

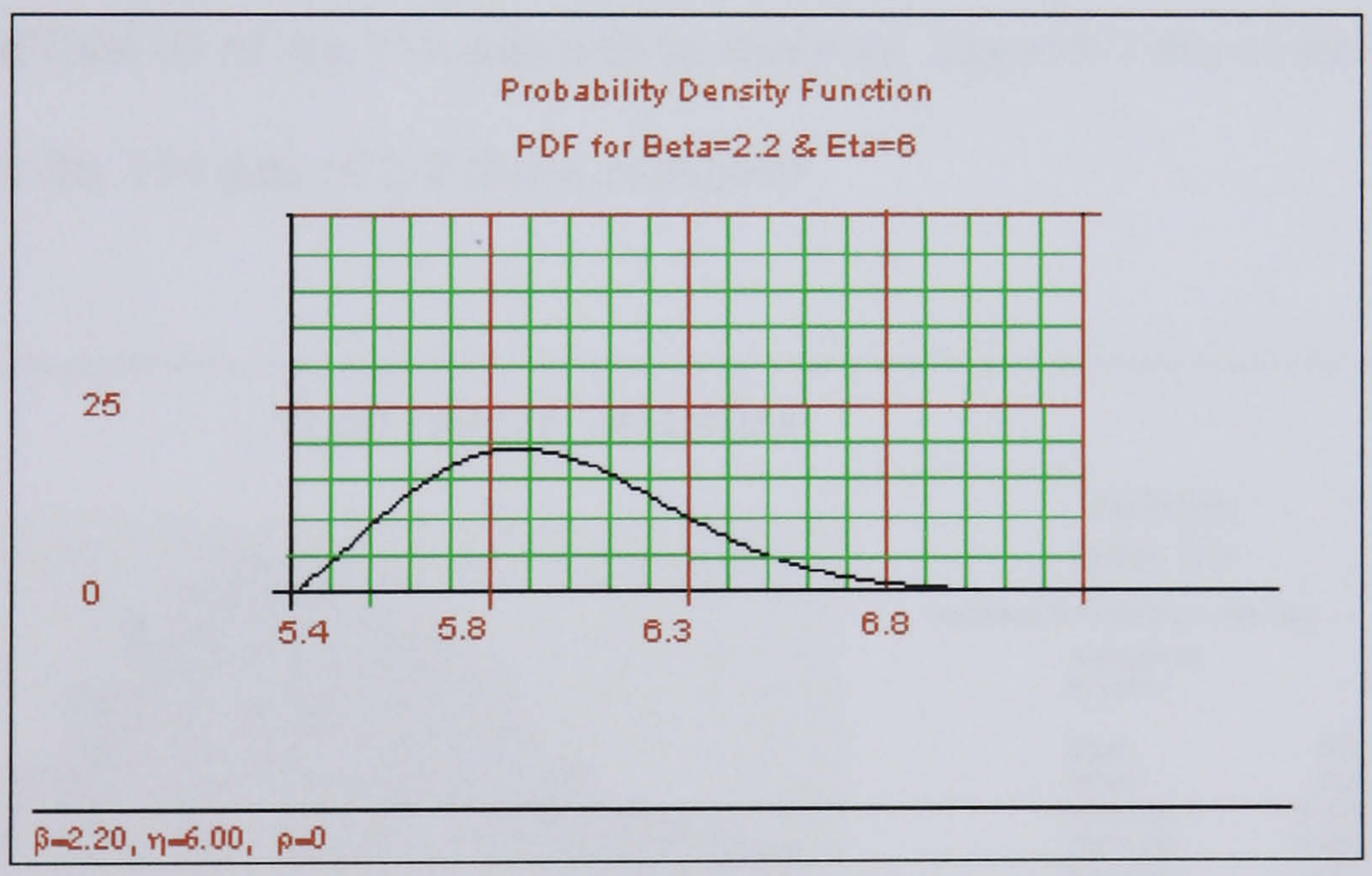


Figure 5:4 – Probability density function for a shape parameter of 2.2

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
1	*5.4*	*6*	*5.5*	*6.2*	*6*	*6.2*	*6.4*	*6.7*
2	*6.3*	*5.9*	*5.6*	*5.8*	*5.7*	*6.5*	*5.9*	*6*
3	*5.6*	*6.4*	*6.1*	*6.6*	*6.1*	*5.6*	*6.3*	*5.6*
4	*6*	*6.2*	*5.9*	*6.1*	*5.5*	*6*	*5.7*	*6.2*
5	*6.1*	*5.7*	*6.3*	*6*	*5.9*	*5.8*	*6.1*	*5.9*
6	*6.4*	*6.7*	*5.7*	*6.4*	*6.2*	*6.8*	*6.2*	*6.3*
7	*6.5*	*6.1*	*6*	*5.6*	*6.3*	*5.9*	*5.5*	*5.7*

Continued…

	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}
1	*6.1*	*6*	*6.5*	*6.1*	*5.9*	*5.8*	*6.4*	*6.1*
2	*5.9*	*6.3*	*5.9*	*6*	*6.3*	*6.8*	*5.6*	*5.8*
3	*5.8*	*5.7*	*6.4*	*5.9*	*5.6*	*5.7*	*5.8*	*6.3*
4	*6*	*5.9*	*5.7*	*5.8*	*5.5*	*5.8*	*6*	*5.8*
5	*5.8*	*6.9*	*6.1*	*6.2*	*5.8*	*6.1*	*5.9*	*6.6*
6	*6.5*	*5.7*	*5.9*	*5.6*	*5.7*	*6.2*	*5.8*	*5.8*
7	*5.6*	*6.2*	*6*	*6.6*	*6.1*	*5.9*	*6.5*	*5.7*

Continued…

	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}
1	*5.5*	*5.9*	*6.5*	*5.8*	*6.6*	*5.7*
2	*6.2*	*6.1*	*6*	*6.4*	*5.9*	*6.3*
3	*5.8*	*5.6*	*5.8*	*5.7*	*5.6*	*6*
4	*6.1*	*5.4*	*5.9*	*6*	*6.3*	*6.4*
5	*5.9*	*5.7*	*6.1*	*6.2*	*5.7*	*5.9*
6	*6*	*5.8*	*6.2*	*6.3*	*6*	*6.1*
7	*6.4*	*6.2*	*6.3*	*5.5*	*6.7*	*6.2*

Table 5:1 – Shape parameter 2.2 data.

Using MINITAB all of the 154 data will be analysed, figure 5:5 shows the descriptive statistics for the 154 data of 2.2 shape parameter.

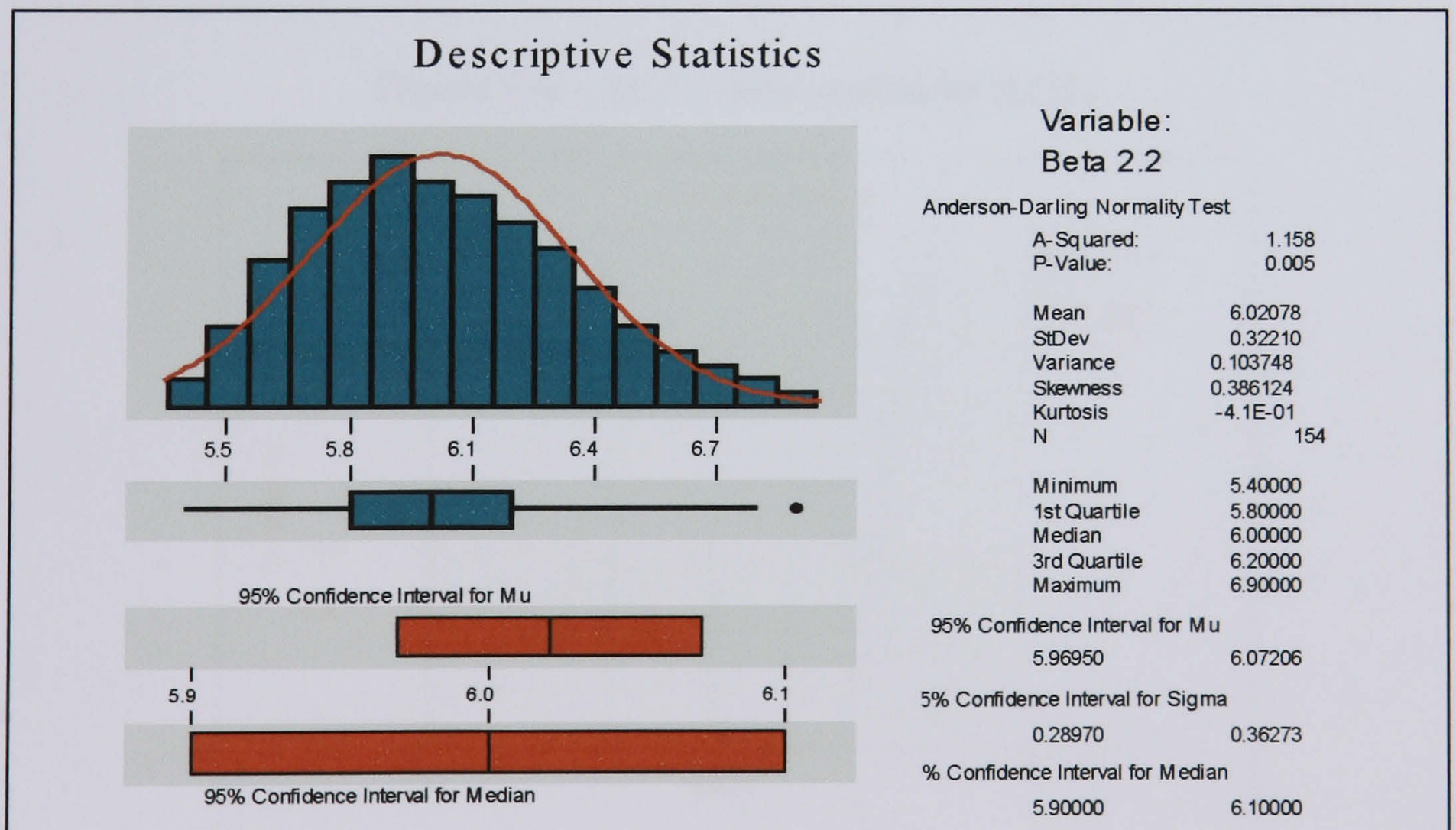


Figure 5.5 – 2.2 Shape parameter data descriptive

As S_{14} can be used to show the basic calculation regarding estimating the Weibull shape parameter, S_{14} consists of 7 data points with average of 6.042857143 and standard deviation of 0.377964473. S_{14} has a Weibull shape parameter of 20.7652 (RRX Weibull estimation) and 12.55 (MLE Weibull). Refer for figure 5:6 and 5:7.

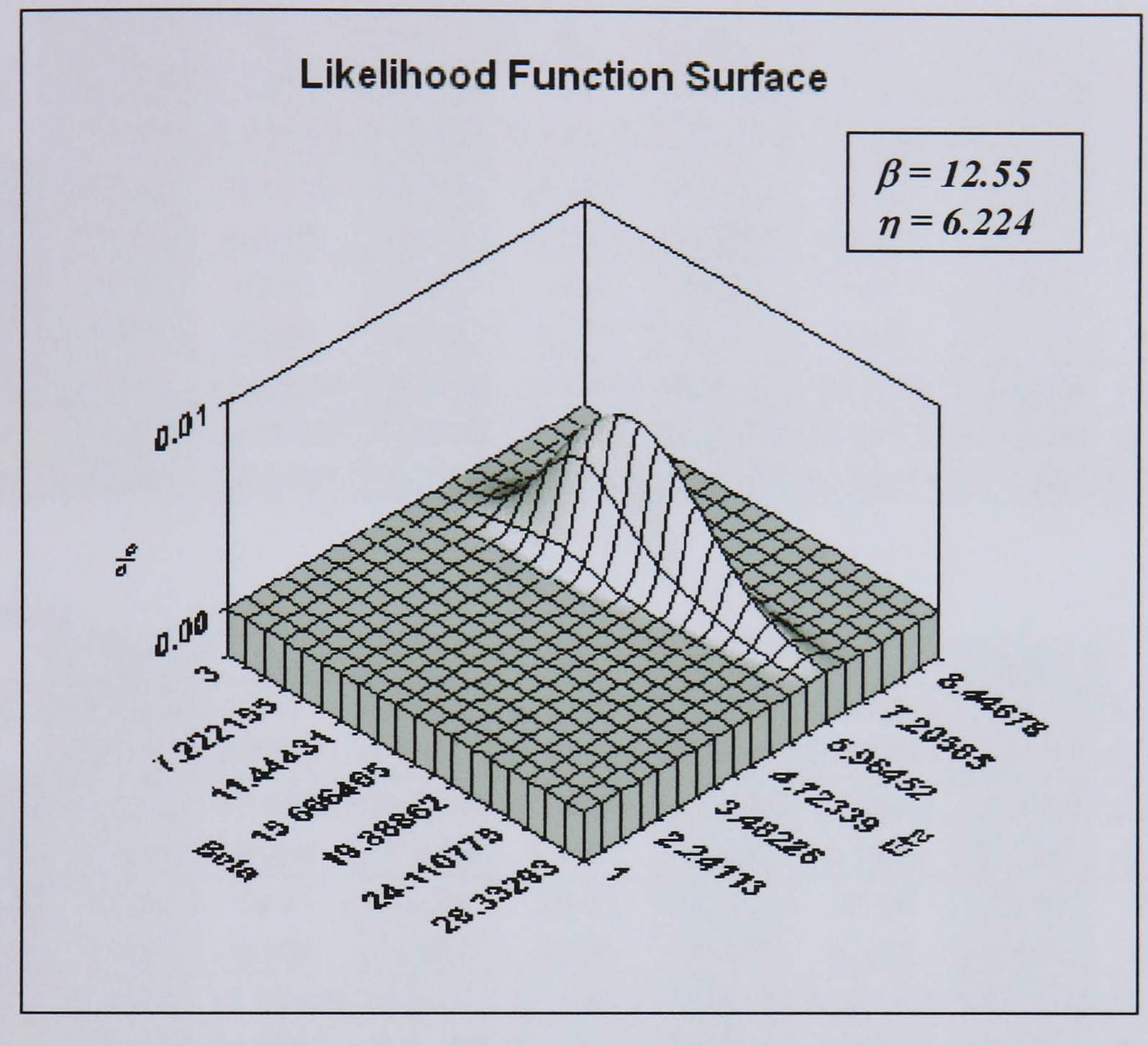


Figure 5:6 – MLE Shape parameter for S_{14}

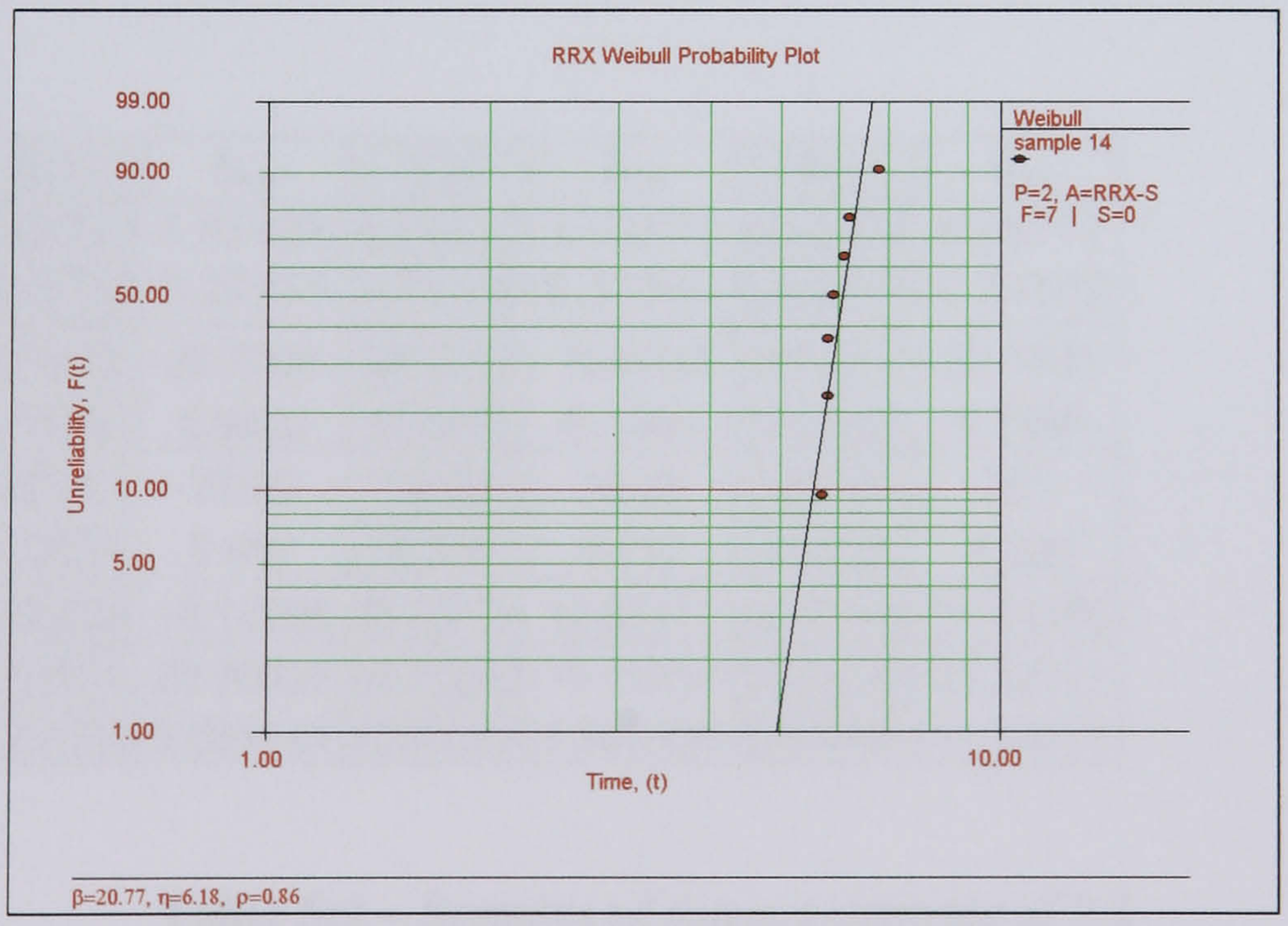


Figure 5:7 – RRX Weibull estimation for S_{14}

From figure 5:6 it can be noticed that the p.d.f. has a positive skewness which can be explained by the shape parameter of 12. Also the data of table 5:1, can be analysed by the conventional estimation techniques for each subgroup (sample), and table 5:2 summarises the results of the calculations. Figure 5:8 shows estimated beta.

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
Avg.	6.042857	6.14285	5.87142	6.1	5.957142	6.11428	6.014285	6.05714
σ	0.411732	0.33094	0.28702	0.34156	0.281999	0.41804	0.328778	0.37796
β_{RRX}	29.8885	24.0358	37.784	38.9227	25.2915	37.784	38.9227	37.9189
η_{RRX}	5.9978	6.0499	6.0357	6.1203	6.0879	6.0357	6.1203	6.1077
β_{MLE}	15.5	17.61	17.98	24.9	24.61	13.6	26.49	14.3
η_{MLE}	6.223	6.395	6.044	6.373	6.11	6.392	6.107	6.309
β_{Unbias}^{*}	12.25595	13.92434	14.2169	19.68858	19.45928	10.7536	20.94581	11.3071
Upper	18.13019	20.59823	21.03102	29.12527	28.78606	15.90778	30.98508	16.72656
Lower	6.533901	7.423355	7.579325	10.4964	10.37415	5.732971	11.16665	6.028051

Continued…

	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}
Avg.	5.957142	6.1	6.07142	6.02857	5.842857	6.04285	6	6.01428
σ	0.287849	0.42031	0.28702	0.31997	0.281999	0.37796	0.331662	0.33380
β_{RRX}	24.6329	17.8578	24.3794	21.4895	23.5555	20.7652	21.3904	22.7099
η_{RRX}	6.0755	6.264	6.1936	6.1657	5.9645	6.1828	6.1365	6.1426
β_{MLE}	15.25	14.41	14.22	20.42	22.72	12.55	13.85	21.86
η_{MLE}	6.158	6.326	6.151	6.206	5.88	6.224	6.187	6.072
β_{Unbias}^{*}	12.05827	11.39408	11.24384	16.14622	17.96484	9.923362	10.95128	17.28484
Upper	17.83777	16.85523	16.63299	23.88506	26.57535	14.6796	16.2002	25.56942
Lower	6.428515	6.07442	5.994327	8.607888	9.577434	5.290352	5.838357	9.214908

Continued…

	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}
Avg.	5.985714	5.81428	6.11428	5.98571	6.11428	6.08571
σ	0.291138	0.27945	0.24102	0.33380	0.429839	0.24102
β_{RRX}	22.1413	22.7006	28.2617	19.5323	16.764	27.1498
η_{RRX}	6.1186	5.9402	6.2212	6.1354	6.3047	6.1966
β_{MLE}	30.44	22.92	32.58	16.38	14.69	26.7
η_{MLE}	6.045	6.857	6.084	6.117	6.464	6.203
β_{Unbias}^{*}	24.06909	18.12298	25.76121	12.95177	11.61547	21.11185
Upper	35.60535	26.80929	38.10849	19.15952	17.18274	31.23071
Lower	12.83174	9.661743	13.73384	6.904858	6.192452	11.25517

Table 5:2 – Samples of shape parameter of 2.2

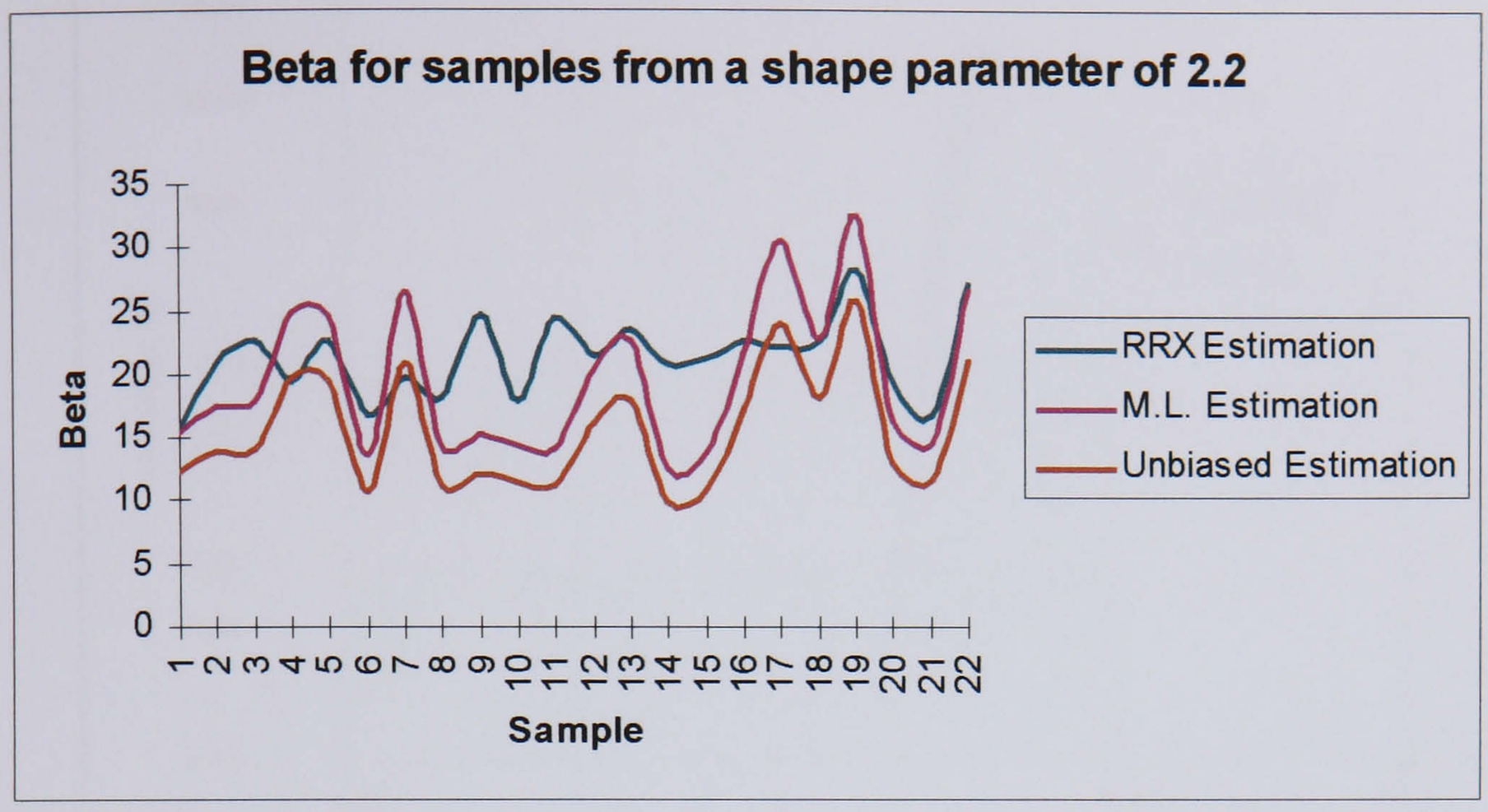


Figure 5:8 – Beta for samples from a shape parameter of 2.2

Using Weibull++, figure 5:9 is generated, which shows the overall data rank regression x estimation. It can be noticed that the data have a shape parameter of 25.02 (far away from 2.2). This graph will be used to calculate the primary deduction factor, and having little iterations afterward, it can be realised that the value of 5.3215 is the appropriate deduction value. This value will enable the rank regression x estimation of the data after the use of deduction method to have an estimated value of shape parameter near 2.2.

Using the deduction method will show the useful outcomes of such technique to calculate Weibull shape parameter accurately when using data with small sample size. Table 5:3 will show the data values after deduction, and table 5:4 will show the results of estimation techniques after the use of deduction method for each sample.

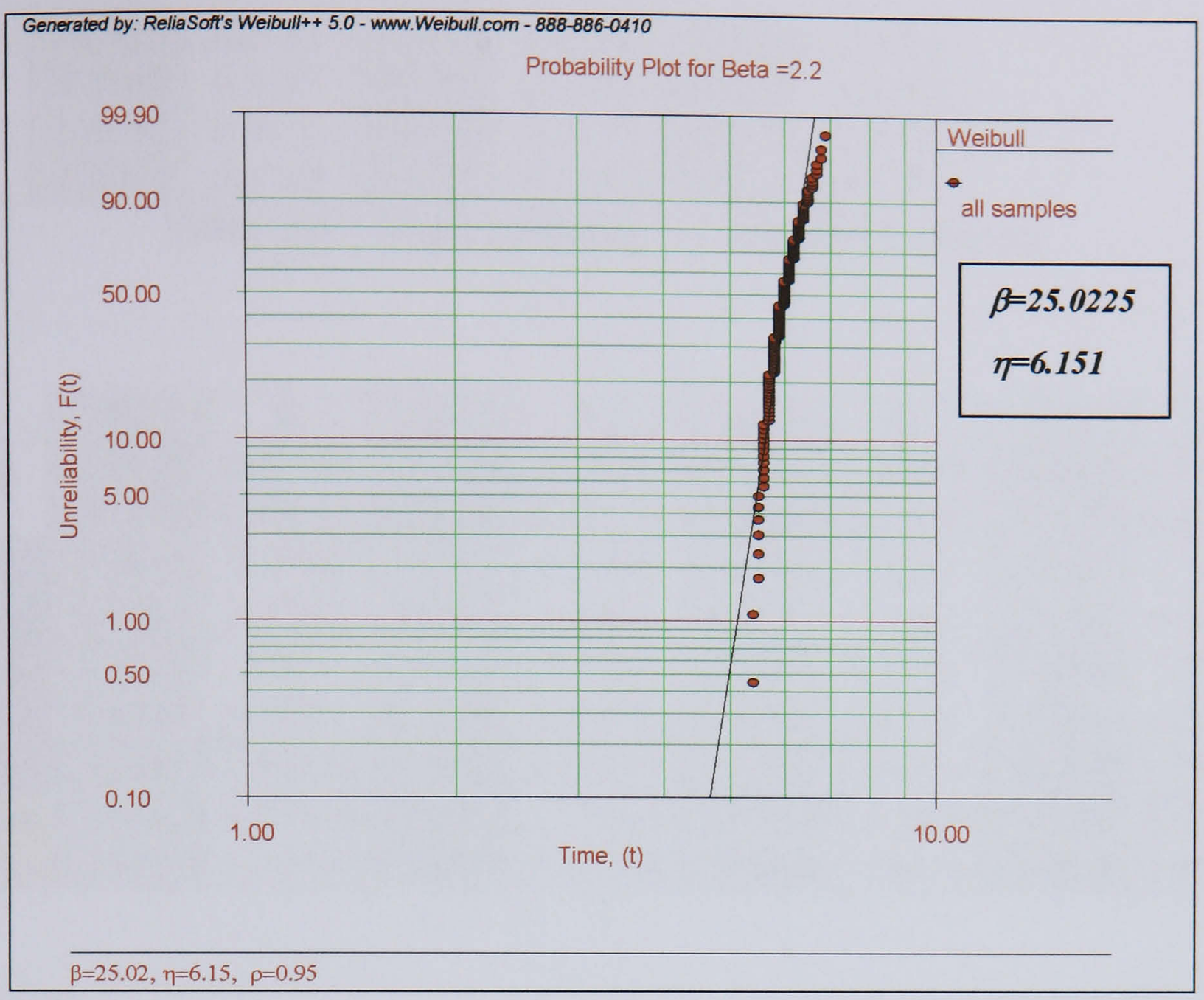


Figure 5:9 – RRX estimation for all data from shape parameter of 2.2

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
1	*0.0785*	*0.6785*	*0.1785*	*0.8785*	*0.6785*	*0.8785*	*1.0785*	*1.3785*
2	*0.9785*	*0.5785*	*0.2785*	*0.4785*	*0.3785*	*1.1785*	*0.5785*	*0.6785*
3	*0.2785*	*1.0785*	*0.7785*	*1.2785*	*0.7785*	*0.2785*	*0.9785*	*0.2785*
4	*0.6785*	*0.8785*	*0.5785*	*0.7785*	*0.1785*	*0.6785*	*0.3785*	*0.8785*
5	*0.7785*	*0.3785*	*0.9785*	*0.6785*	*0.5785*	*0.4785*	*0.7785*	*0.5785*
6	*1.0785*	*1.3785*	*0.3785*	*1.0785*	*0.8785*	*1.4785*	*0.8785*	*0.9785*
7	*1.1785*	*0.7785*	*0.6785*	*0.2785*	*0.9785*	*0.5785*	*0.1785*	*0.3785*

Continued…

	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}
1	*0.7785*	*0.6785*	*1.1785*	*0.7785*	*0.5785*	*0.4785*	*1.0785*	*0.7785*
2	*0.5785*	*0.9785*	*0.5785*	*0.6785*	*0.9785*	*1.4785*	*0.2785*	*0.4785*
3	*0.4785*	*0.3785*	*1.0785*	*0.5785*	*0.2785*	*0.3785*	*0.4785*	*0.9785*
4	*0.6785*	*0.5785*	*0.3785*	*0.4785*	*0.1785*	*0.4785*	*0.6785*	*0.4785*
5	*0.4785*	*1.5785*	*0.7785*	*0.8785*	*0.4785*	*0.7785*	*0.5785*	*1.2785*
6	*1.1785*	*0.3785*	*0.5785*	*0.2785*	*0.3785*	*0.8785*	*0.4785*	*0.4785*
7	*0.2785*	*0.8785*	*0.6785*	*1.2785*	*0.7785*	*0.5785*	*1.1785*	*0.3785*

Continued…

	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}
1	*0.1785*	*0.5785*	*1.1785*	*0.4785*	*1.2785*	*0.3785*
2	*0.8785*	*0.7785*	*0.6785*	*1.0785*	*0.5785*	*0.9785*
3	*0.4785*	*0.2785*	*0.4785*	*0.3785*	*0.2785*	*0.6785*

4	*0.7785*	*0.0785*	*0.5785*	*0.6785*	*0.9785*	*1.0785*
5	*0.5785*	*0.3785*	*0.7785*	*0.8785*	*0.3785*	*0.5785*
6	*0.6785*	*0.4785*	*0.8785*	*0.9785*	*0.6785*	*0.7785*
7	*1.0785*	*0.8785*	*0.9785*	*0.1785*	*1.3785*	*0.8785*

Table 5:3 – Shape parameter 2.2 data after deduction

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
Avg.	0.721357	0.82135	0.54992	0.7785	0.635642	0.79278	0.692785	0.73564
σ	0.411732	0.33094	0.28702	0.34156	0.281999	0.41804	0.328778	0.37796
β_{RRX}	1.1642	2.5704	1.786	2.1006	1.8696	1.9251	1.7259	1.9606
η_{RRX}	0.8444	0.9305	0.6343	0.8963	0.738	0.9081	0.8066	0.8426
β_{MLE}	1.7905	2.911	2.2313	2.7176	2.715	2.2046	2.4894	2.2656
η_{MLE}	0.8004	0.9231	0.6228	0.8771	0.7144	0.8991	0.7802	0.8338
β_{Unbias}^{*}	1.415759	2.30174	1.76430	2.14882	2.146767	1.74319	1.968383	1.79142
Upper	2.094329	3.40496	2.60992	3.17874	3.175707	2.57869	2.911825	2.65004
Lower	0.754770	1.22710	0.94058	1.14558	1.144486	0.92933	1.049386	0.95504

Continued…

	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}
Avg.	0.635642	0.7785	0.74992	0.70707	0.521357	0.72135	0.6785	0.69278
σ	0.287849	0.42031	0.28702	0.31997	0.281999	0.37796	0.331662	0.33380
β_{RRX}	2.4549	2.1855	2.8317	2.2499	1.8474	2.5105	2.2265	2.6321
η_{RRX}	0.7176	0.8739	0.8416	0.8069	0.5987	0.7999	0.7679	0.7971
β_{MLE}	2.5428	2.1512	3.862	2.5661	2.1449	2.2092	2.3939	2.4194
η_{MLE}	0.718	0.8841	0.8413	0.7982	0.5912	0.8193	0.7691	0.7857
β_{Unbias}^{*}	2.010607	1.70096	3.05370	2.02903	1.695985	1.74682	1.892871	1.91303
Upper	2.974286	2.51623	4.51734	3.00154	2.508867	2.58407	2.800120	2.82994
Lower	1.071896	0.90682	1.62799	1.08171	0.904165	0.9312	1.009129	1.01987

Continued…

	S_{17}	S_{18}	S_{19}	S_{20}	S_{21}	S_{22}
Avg.	0.664214	0.49278	0.79278	0.66421	0.792785	0.76421
σ	0.291138	0.27945	0.24102	0.33380	0.429839	0.24102
β_{RRX}	1.9021	1.3612	3.501	1.7114	1.8105	3.0427
η_{RRX}	0.7702	0.5776	0.8803	0.7717	0.9083	0.8595
β_{MLE}	2.7056	1.9269	3.9083	2.3204	2.1466	4.0106
η_{MLE}	0.7455	0.5527	0.8776	0.7502	0.8992	0.8457
β_{Unbias}^{*}	2.139334	1.52361	3.09031	1.83475	1.697329	3.17120
Upper	3.164712	2.25387	4.57149	2.71414	2.510855	4.69115
Lower	1.140524	0.81226	1.64751	0.97814	0.904882	1.69063

Table 5:4 – Samples of shape parameter of 2.2 calculations after deduction

Using the deduction technique showed the effectiveness and accuracy, which such method provide specially when using small sample (n=7). A sample size of seven was used as the D_3 factor starts to gain value after n=6. Figure 5:10 shows a comparison between the estimation of shape parameter of data from population of beta =2.2. It is clearly seen that using such method (deduction method) enables the estimation to be in the average of 2.2 rather in the average of 20.

Figure 5:11 shows the MLE for the overall data after deduction and it can be seen that the shape parameter equal to 2.3202. While, in figure 5:12, RRX has been used and it generated a shape parameter of 2.2945. This can be considered as another proof of the choice of RRX estimation associated with the deduction method.

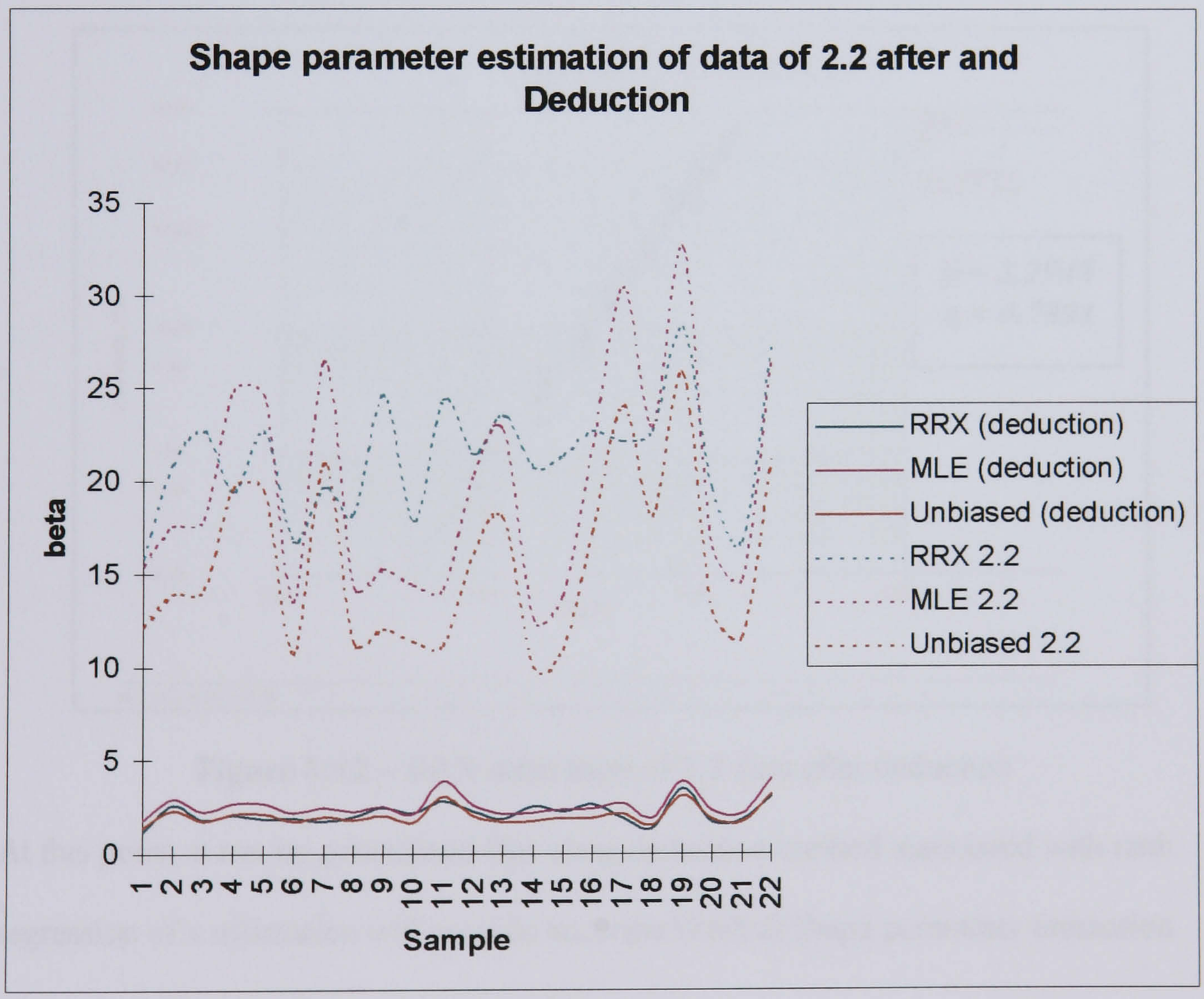


Figure 5:10 – Shape parameter estimation comparison

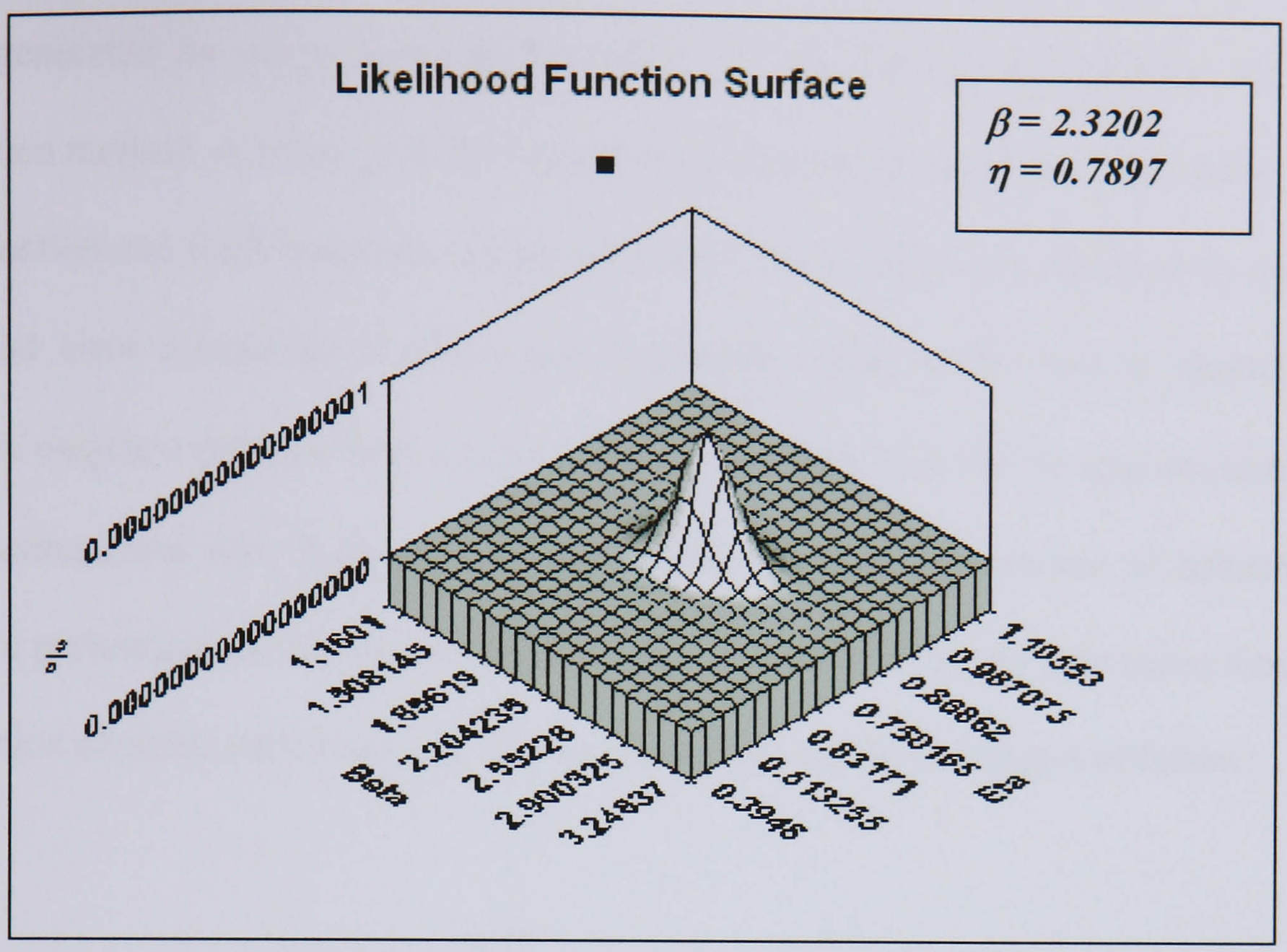


Figure 5:11 – MLE of 2.2 data after deduction

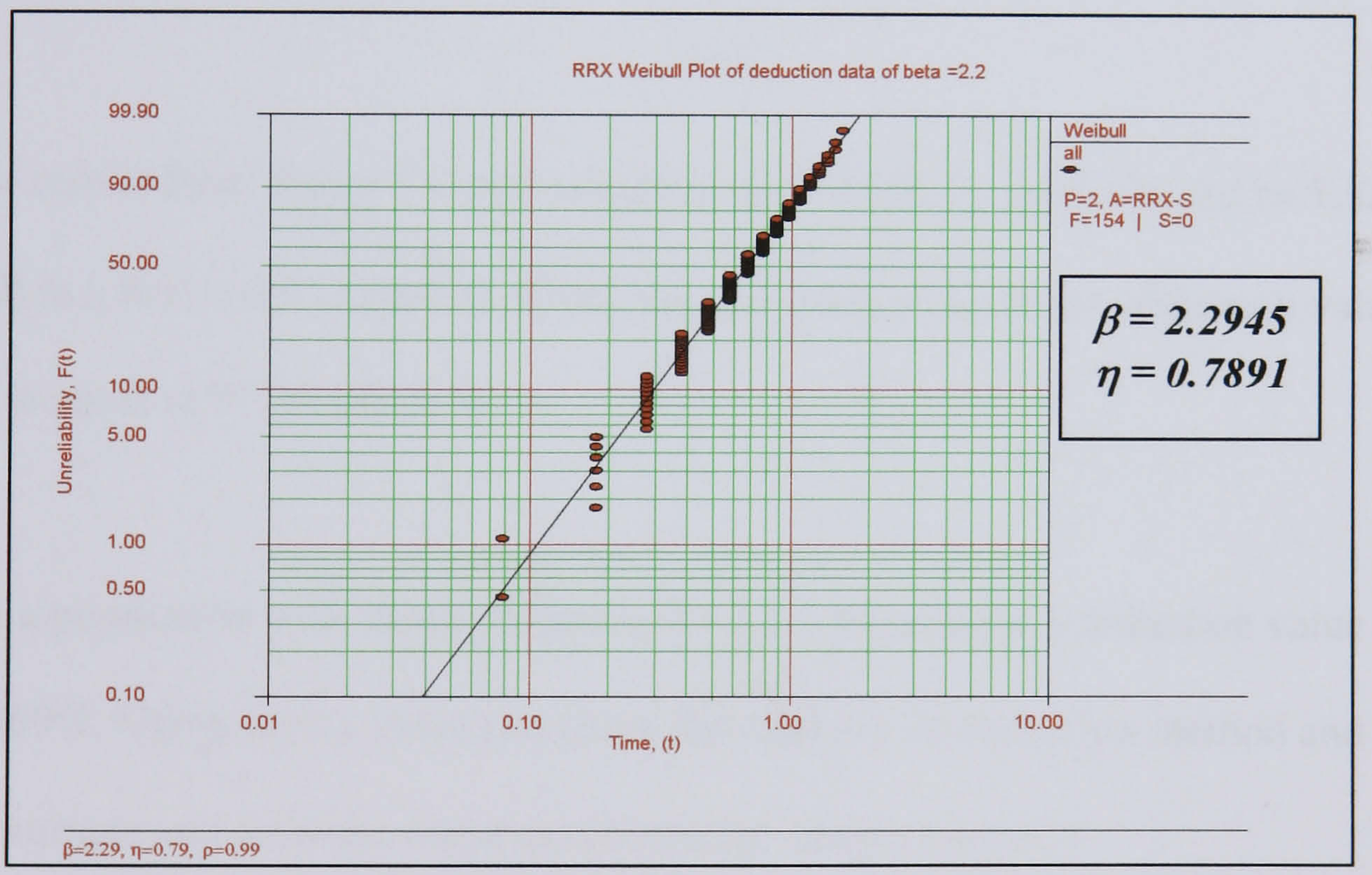


Figure 5:12 – RRX estimation of 2.2 data after deduction

At this point, it can be generalised that using deduction method associated with rank regression of x estimation will provide accurate Weibull shape parameter estimation when employing small sample size (n=7).

Data generated by using a shape parameter of 2.2, had to be modified by the deduction method. A value of 5.3215 had to be subtracted from all the data, so the use of deduction and RRX methods can be successful. Such value was obtained by many trial and error estimation to get a real applicable value to be used in deduction method, despite a primary value (γ) used, but such a procedure can be time consuming and accompanied with high level of uncertainty. To ease such choice of deduction value, a percentage can be developed, in order to be used with any data taken from a population of shape parameter of 2.2. The percentage can be calculated as follow:

$$R(t) = e^{-\left(\frac{t}{\eta}\right)^{\beta}} \qquad \cdots \textit{Equation } 4.15$$

Using the results from figure 5:9, and substituting β=25.0223, η=6.151 and t= 5.3215 will result in a R(t) = 0.973706059. Therefore, the percentage of the deduction value (5.3215) is equal to 97.3706059 %.

A data of a population with shape parameter of 2.2, should have a deduction value of 97.3706059%. Using such a value can allow the analysis of deduction method and RRX to estimate and accurate shape parameter for sample size of 7.

5.2 WEIBULL DEDUCTION CONTROL CHART MODEL

It is concluded that data taken from a population with shape parameter of 2.2 has a deduction factor of 97.3706059%. To make such conclusion wider, data of populations with different shape parameter (β = 0.5, 1, 3.44, 4.5, 7, 12, 15 and 20) were also used to calculate the deduction factor percentage. Calculations were carried out; table 5:5 shows the resulting deduction values and Weibull parameter before and after the deduction. (Refer to appendix for detailed calculations and results). Also, table 5:6, shows the deduction percentage of each population shape parameter.

Original population Beta	Parameters (Weibull 2P)	Deduction value	Modified parameters
β=0.5	β= 1.651 η= 4.981	-1.1925	β= 0.5012 η= 2.3236
β=1	β= 2.1936 η= 7.0552	-2.139	β= 1.0017 η= 4.135
β=2.2	β= 25.0223 η= 6.151	-5.3215	β= 2.2945 η= 0.7891
β=3.44	β= 34.253 η= 6.0963	-5.28	β= 3.4051 η= 0.7891
β=4.5	β= 19.5859 η= 7.1284	-5.147	β= 4.4832 η= 1.9623
β=7	β= 7.1753 η= 6.2674	-0.134	β= 7.0014 η= 6.132
β=10	β= 9.4392 η= 6.0102	-0.000001	β= 9.4392 η= 6.0102
β=12	β= 11.4539 η= 7.1458	+0.32	β= 12.027 η= 7.4664
β=15	β= 17.4513 η= 6.9368	-0.899	β= 15.0219 η= 6.0368
β=20	β= 20.6366 η= 6.9821	-0.2	β= 20.0141 η= 6.7819

Table 5:5 – Weibull parameters before and after deduction

Original population Beta	Deduction Percentage (%)
β=0.5	*90.99318508*
β=1	*92.96418432*
β=2.2	*97.3706059*
β=3.44	*99.275690*
β=4.5	*99.831362*
β=7	*99.999999*
β=10	*100*
β=12	*100*
β=15	*100*
β=20	*100*

Table 5:6 – Deduction Percentage

The percentage deduction values have a special trend, which is increasing in Beta interval of [0.5, 7], then it tends to stabilise between beta interval of [7, 20]. Figure 5:13 shows the trend of deduction percentages with respect to beta values. The deduction percentage can be modelled and formulated in a mathematical equation. Such equation (model) can be used to predict any percentage when knowing the parent population shape parameter with minimum error; also it must fit the percentage data in table 5.6 with high goodness-of-fit.

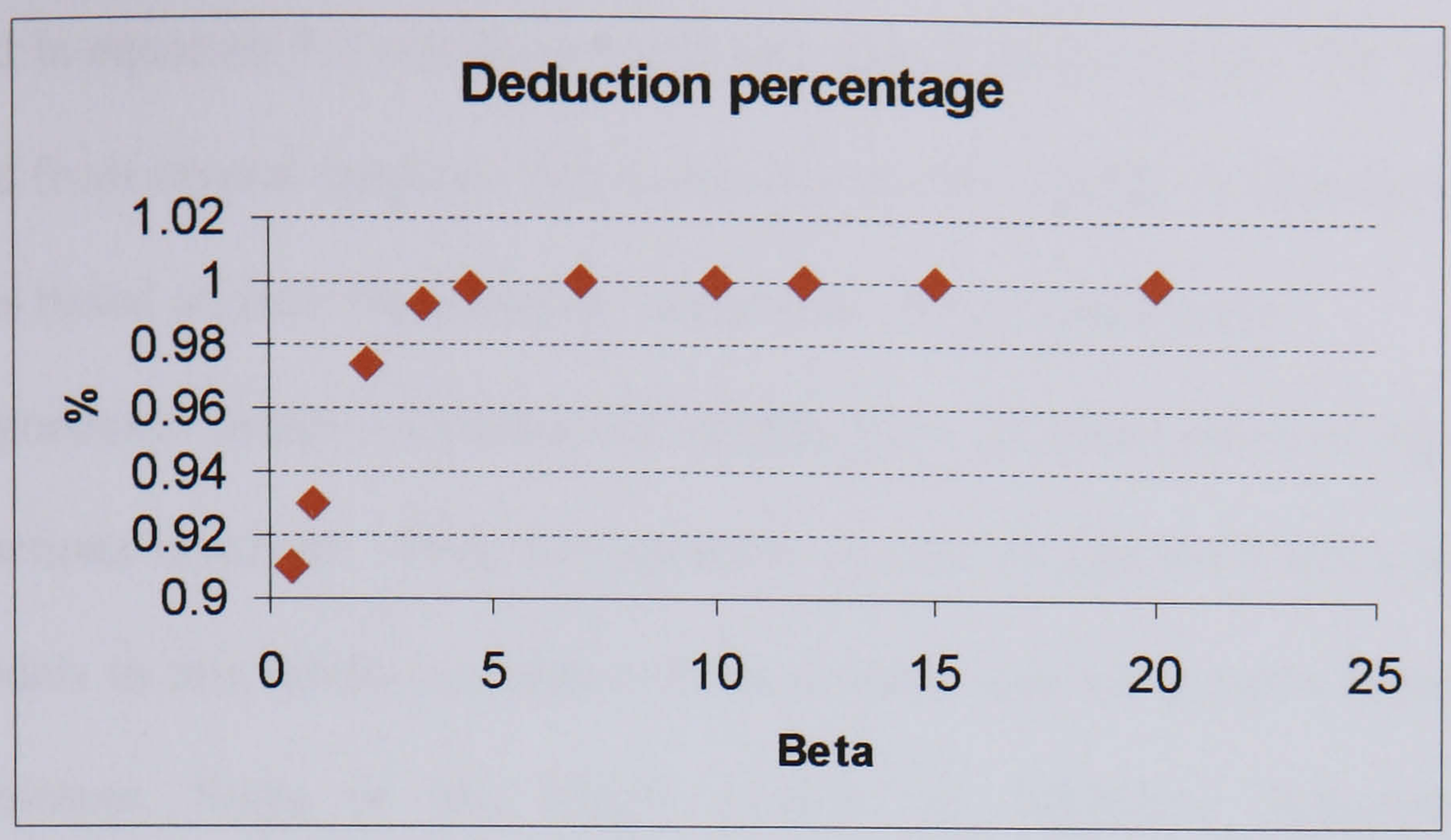


Figure 5:13 – Deduction percentages trend

After many modelling tests with the use of CurveExpert 1.3, a model has been found which satisfy the prediction of deduction percentages near the true experimental results (available from table 5:6) with minimum error. The model formula of deduction percentage is:

$$\textit{Deduction percentage} = a - b\,e^{-c\,n^{d}} \qquad \cdots \textit{Equation}\,5.2$$

Where:

$a = 0.99994$

$b = 0.10225931$

$c = 0.38041323$

$d = 1.5959549$

n= *Sample size.*

Equation 5.2 has been conducted as a result of different trials to fit percentage data, many models have been tested. Out of the 32 different models, the model that is

symbolised in equation 5.2 was the nearest best model to fit the data. The 32 models where used from several families. The non-linear models used have been divided into six families based on their characteristic behaviour. These families are:

1. Exponential family - Exponential models have the exponential or logarithmic functions involved. They are generally convex or concave curves, but some models in this family are able to have an inflection point and a maximum or minimum. Some of this family models are: Modified Exponential and Reciprocal Logarithm.
2. Power family - The Power Family involves raising one or more parameters to the power of the independent variable, or raising the dependent variable to the power of a given parameter. This family is generally a set of convex or concave curves with no inflection points or maximum or minimum. Some of this family models are: Root Fit Model and Hoerl Model.
3. Yield density family- two types of response are observed in practice: the "asymptotic" and "parabolic" yield-density relations. If the response is such that as density (x) increases, but the yield (y) approaches a fixed value, the relationship is asymptotic. If the response is such that there is a distinct optimum as the density increases, the relationship is parabolic. Of course, these types of relationships occur commonly in other scientific areas; therefore, this family of models is very useful. Some of this family models are: Harris Model and Bleasdale Model.
4. Growth family - Growth models are characterized by a monotonic growth from some fixed value to an asymptote. These models are most common the engineering sciences. Some of this family model is saturation growth.

5. Sigmoidal family - Processes producing sigmoidal or "S-shaped" growth curves are common in a wide variety of applications such as biology, engineering, agriculture, and economics. These curves start at a fixed point and increase their growth rate monotonically to reach an inflection point. After this, the growth rate approaches a final value asymptotically. Occasionally, some scientists consider this family is a subset of the Growth Family. Due to the behaviour of equation 5.2, the Deduction Percentage Model is considered to be from this family, other models in this family are: Richard Model and Gompertz Model.
6. Miscellaneous family - Some models just don't fit into previous families. The miscellaneous family is the one in which these "different" nonlinear regression models exist. Some of this family models are: Sinusoidal Fit Model, Gaussian Model and Hyperbolic Fit Model.

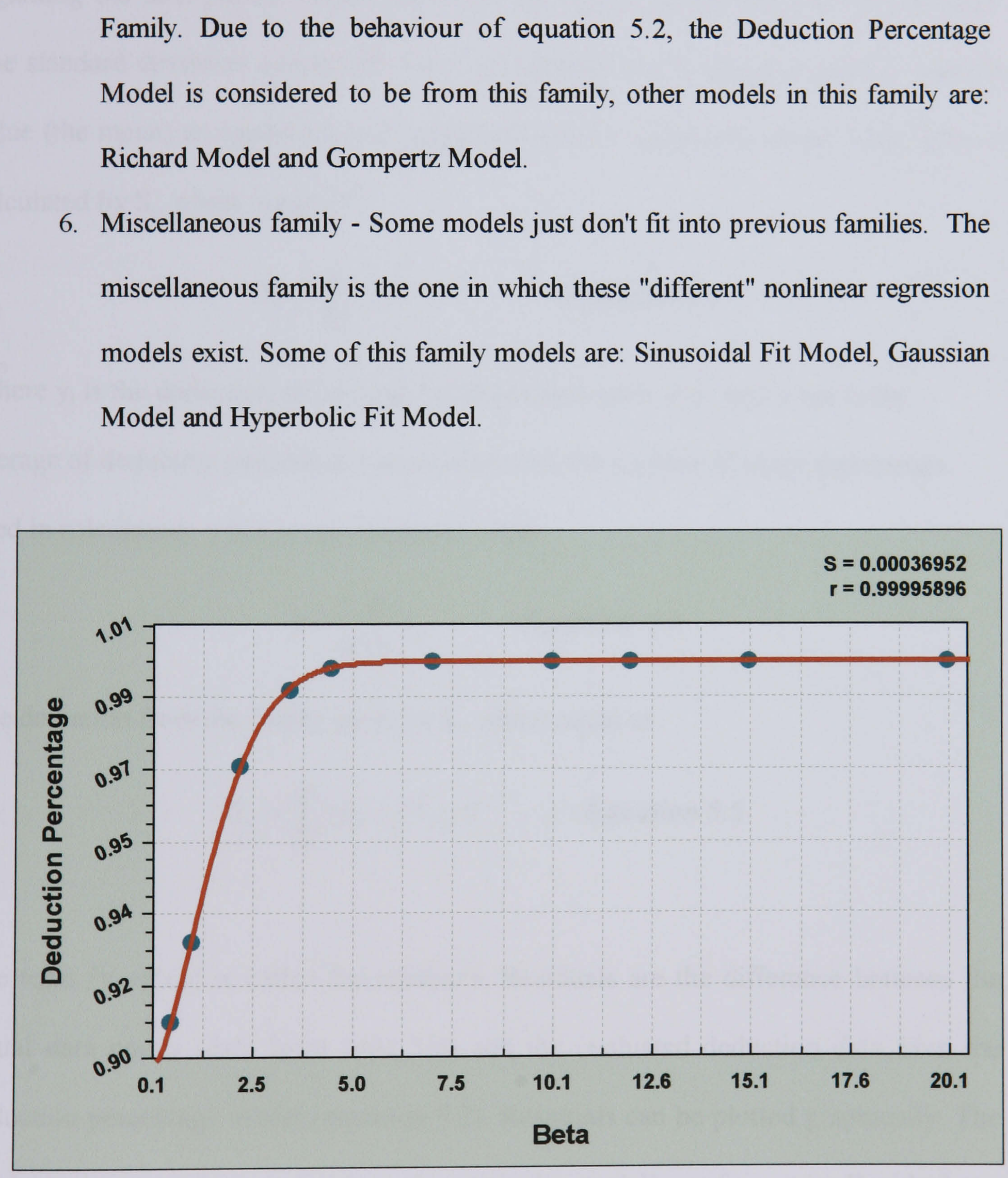


Figure 5:14 – Best Curve fitting for the deduction percentage model

The deduction percentage model has the best goodness-of-fit, as it is seen in figure 5:14 the curve of the model cover most data with the minimum error and it has a correlation coefficient (r) of 0.99995896, which can express a high level of goodness-of-fit. The correlation coefficient is considered to be a measure of the goodness of fit. To explain the meaning of this measure, the standard deviation should be defined regarding the data points, which quantifies the spread of the data around the mean. The standard deviation around the mean is regarded as the spread around a constant value (the mean) as opposed to the spread around the regression model. This value is calculated by S_t, which equal to[84]:

$$S_t = \sum_{i=1}^{n} (\bar{y} - y_i)^2 \qquad \cdots \textbf{\textit{Equation}}\ 5.3$$

Where y_i is the deduction percentage for shape parameter of i. And y bar is the average of deduction percentages associated with the number of shape parameters used in calculation. y bar is calculated as follow:

$$\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i \qquad \cdots \textbf{\textit{Equation}}\ 5.4$$

The deviation from the fitting curve as S_r, which equal to

$$S_r = \sum_{i=1}^{n} (y_i - f(x_i))^2 \qquad \cdots \textbf{\textit{Equation}}\ 5.5$$

The term $[y_i - f(x_i)]$ is called the residuals. Residuals are the difference between the actual data points (data from table 5:6) and the evaluated deduction data from the deduction percentage model (equation 5.2). Residuals can be plotted graphically. The residuals can provide an indication of a particular model's performance. Residual can be positive and negative residuals. Positive residuals mean that the predicted deduction percentage is over the curve of the fit; while negative residuals indicate that

the prediction values of deduction percentages in under the model curve. Optimally, the residuals should exhibit a random scatter around zero, which indicates that the data points are randomly distributed around the curve.

The correlation coefficient (r) can be calculated by the use of equation 5.3 and 5.5, the value of r is calculated by equation 5.6[85]

$$r \equiv \sqrt{\frac{S_t - S_r}{S_t}} \qquad \cdots \textit{Equation } 5.6$$

As the regression model better describes the data, the correlation coefficient will approach unity[86]. In other words, for a perfect fit, the standard error of the estimate will approach Sr =0 and the correlation coefficient will approach r =1.

Noting that the this method to calculate the correlation coefficient is based on a linear regression modelling, as it consists of a linear combination of a particular set of functions. It should be clear that the word "linear" refers only to dependence of the regression model on the parameters, not to the function of deduction percentage.

Table 5:7 shows calculated mathematical terms used in the calculations of the correlation coefficient (r).

	y_i	$f(x_i)$	$y_i - f(x_i)$	ybar - y_i	(ybar - y_i)^2
i_1 (β = 0.5)	*0.909932*	*0.909772*	*0.00016*	*-1.8902*	*3.572867*
i_2 (β = 1)	*0.929642*	*0.930038*	*-0.0004*	*-1.91047*	*3.649892*
i_3 (β = 2.2)	*0.973706*	*0.973134*	*0.000572*	*-1.95357*	*3.816416*

i_4 (β = 3.44)	*0.992757*	*0.993288*	*-0.00053*	*-1.97372*	*3.895567*
i_5 (β = 4.5)	*0.998314*	*0.998399*	*-8.5E-05*	*-1.97883*	*3.915768*
i_6 (β = 7)	*1*	*0.999919*	*8.1E-05*	*-1.98035*	*3.921786*
i_7 (β = 10)	*1*	*0.999940*	*6E-05*	*-1.98037*	*3.921869*
i_8 (β = 12)	*1*	*0.999940*	*6E-05*	*-1.98037*	*3.921869*
i_9 (β = 15)	*1*	*0.999940*	*6E-05*	*-1.98037*	*3.921869*
i_{10} (β = 20)	*1*	*0.999940*	*6E-05*	*-1.98037*	*3.921869*

Table 5:7 – Correlation coefficient mathematical calculation terms

Using calculated data in table 5:7, and employing them in equation 5.3, 5.4, 5.5, and 5.6 the following result can be found:

ybar = 0.980431,

S_r = 8.19747E-07,

S_t = 38.45977356

Therefore, deduction percentages curve correlation coefficient (*r*) equal to 0.99995795.

Furthermore, using the results of $y_i - f(x_i)$ from the table 5:7, a residual graph can be plotting as seen in figure 5:15.

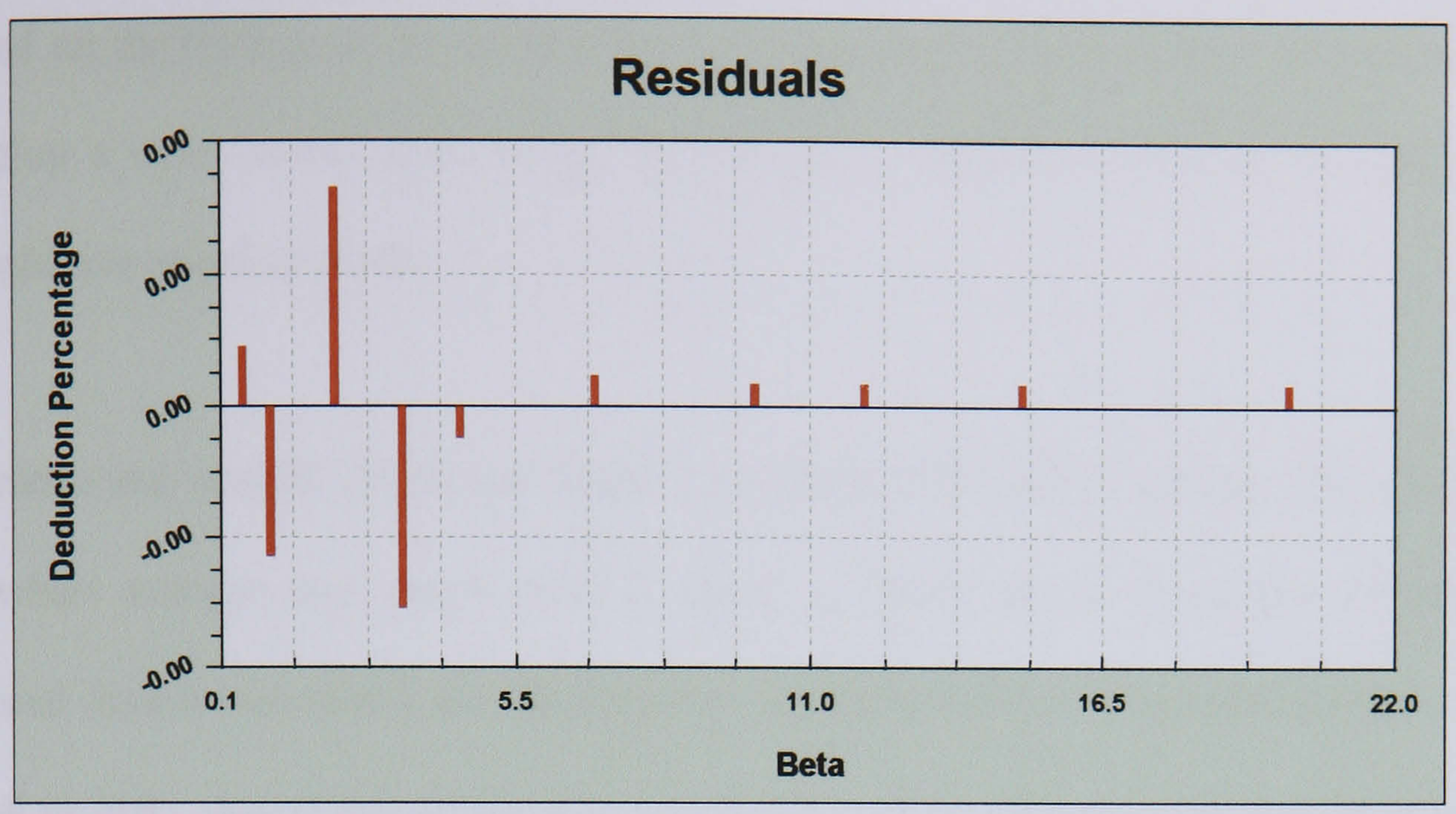


Figure 5:15 – residual graph for deduction percentage model

In addition, the deduction percentage model can be plotting by using a logarithmic scale. (refer to figure 5:16)

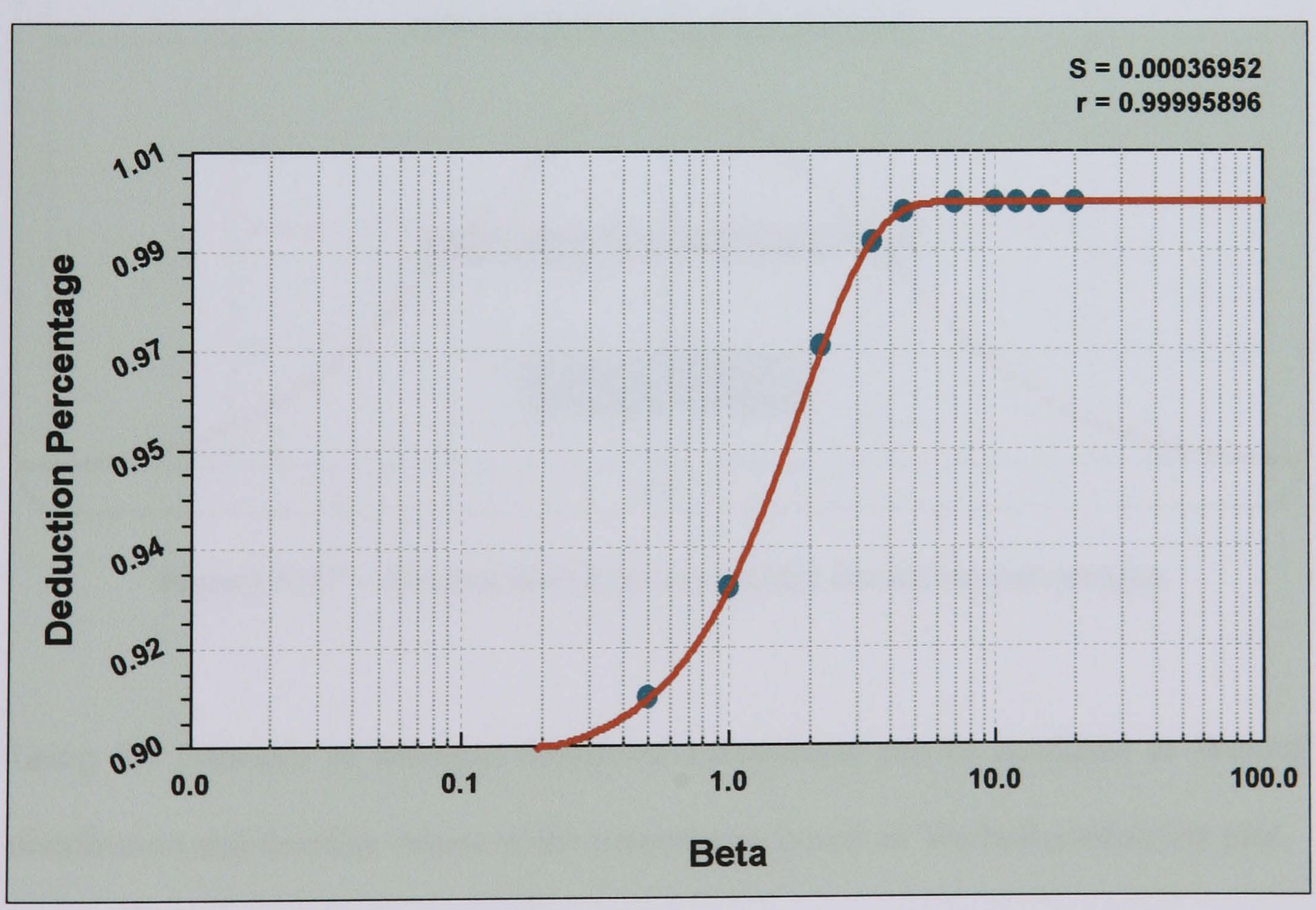


Figure 5:16 – logarithmic residual graph for deduction percentage model

Based on the finding of the deduction percentage model, such model can be used to develop a new control chart based on deduction Weibull analysis to be used with sample size equal to seven.

Conventional control charts are based on normal distribution analysis, the common Shewhart average and range control charts is based on 3σ (standard deviation). Normal distribution has a unique property, which is the area under the normal curve equal to one. And it has predictable proportions of its total area within one, two and three standard deviations of the mean, regardless the magnitude of the standard deviation. Figure 5:17 shows the percentages of areas covered by 1σ, 2σ and 3σ.

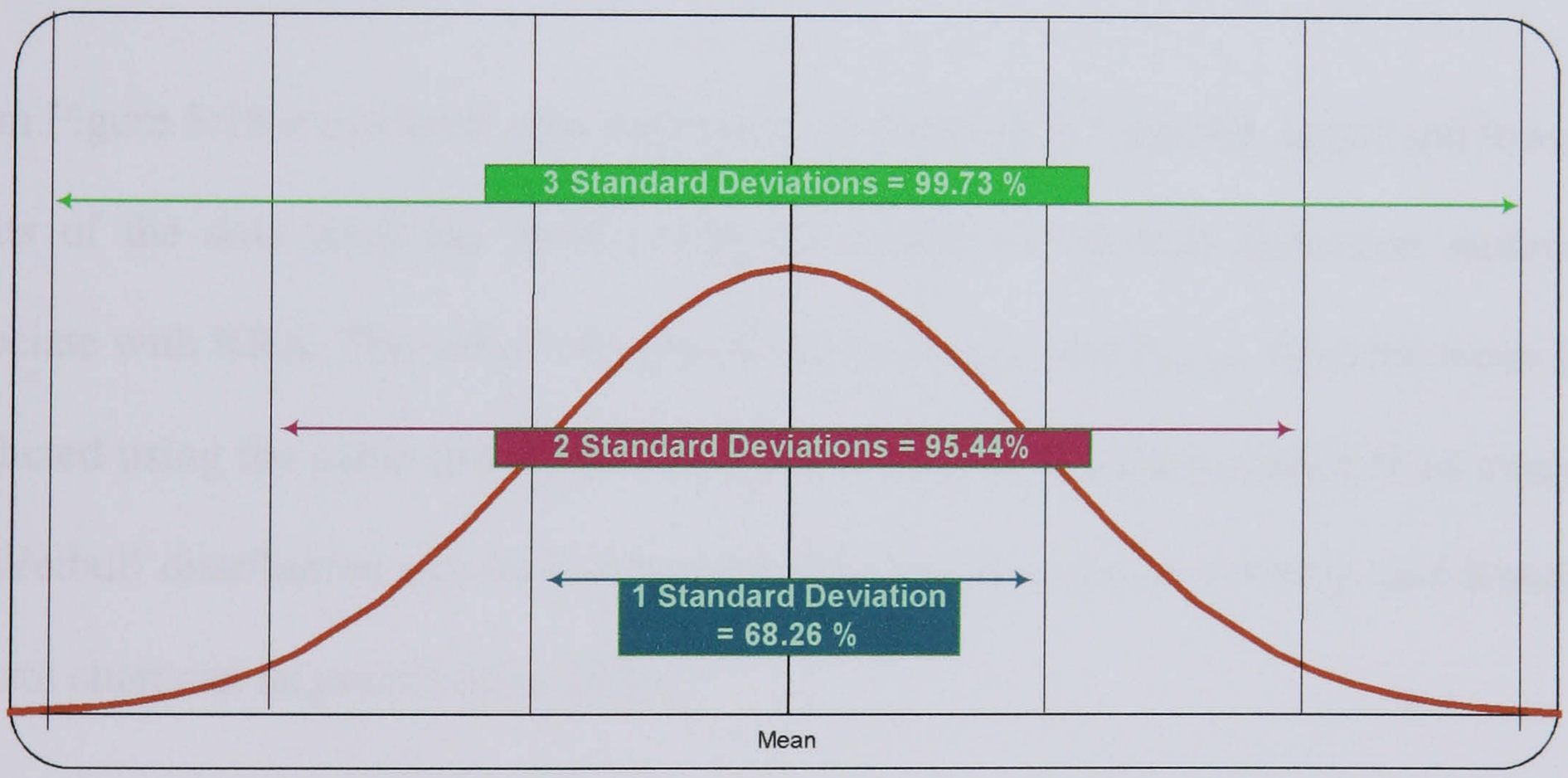


Figure 5:17 – Normal distribution standard deviations percentages

Using the principle of standard deviations percentages can be modified to Weibull distribution and develop values of the percentages based on Weibull probability plot.

Using 3 standard deviations in this analysis, it can be found that Weibull have an upper and lower range limits as follow

$$\textit{Upper Range Limit} = 50\ \% + 49.865\ \% = 99.865\ \% \qquad \textit{...Equation } 5.7$$

$$\textit{Lower Range Limit} = 50\ \% - 49.865\ \% = 0.135\ \% \qquad \textit{...Equation } 5.8$$

Using such limits, a correspondent data values can be obtained by Weibull probability plot, which was originated using any test data after the use of deduction method to achieve accurate representative Weibull probability plot based on rank regression on x. Figure 5:18 represents the previous idea used to achieve the range limits based on Weibull analysis and deduction method.

From Figure 5:18 it can been seen that based on equation 5.7 and 5.8, upper and lower limits of the data used has been configured based on Weibull deduction method associate with RRX. The value are symbolised by P_{Upper} and P_{Lower}. Also the mean is predicted using the same technique. Using these predicted values control chart based on Weibull distribution can be constructed. The construction of Average and Range control chart can be described as follow:

From the Values of Figure 5:18, it can be easily noticed that the average Weibull control chart limits are calculated as follow, (also refer to Figure 5:19)

$$\textit{Average Weibull Centre Line} = \textit{Mean} = \mu \qquad \textit{... Equation } 5.9$$

$$\textit{Range} = P_{Upper} - P_{Lower} \qquad \textit{... Equation } 5.10$$

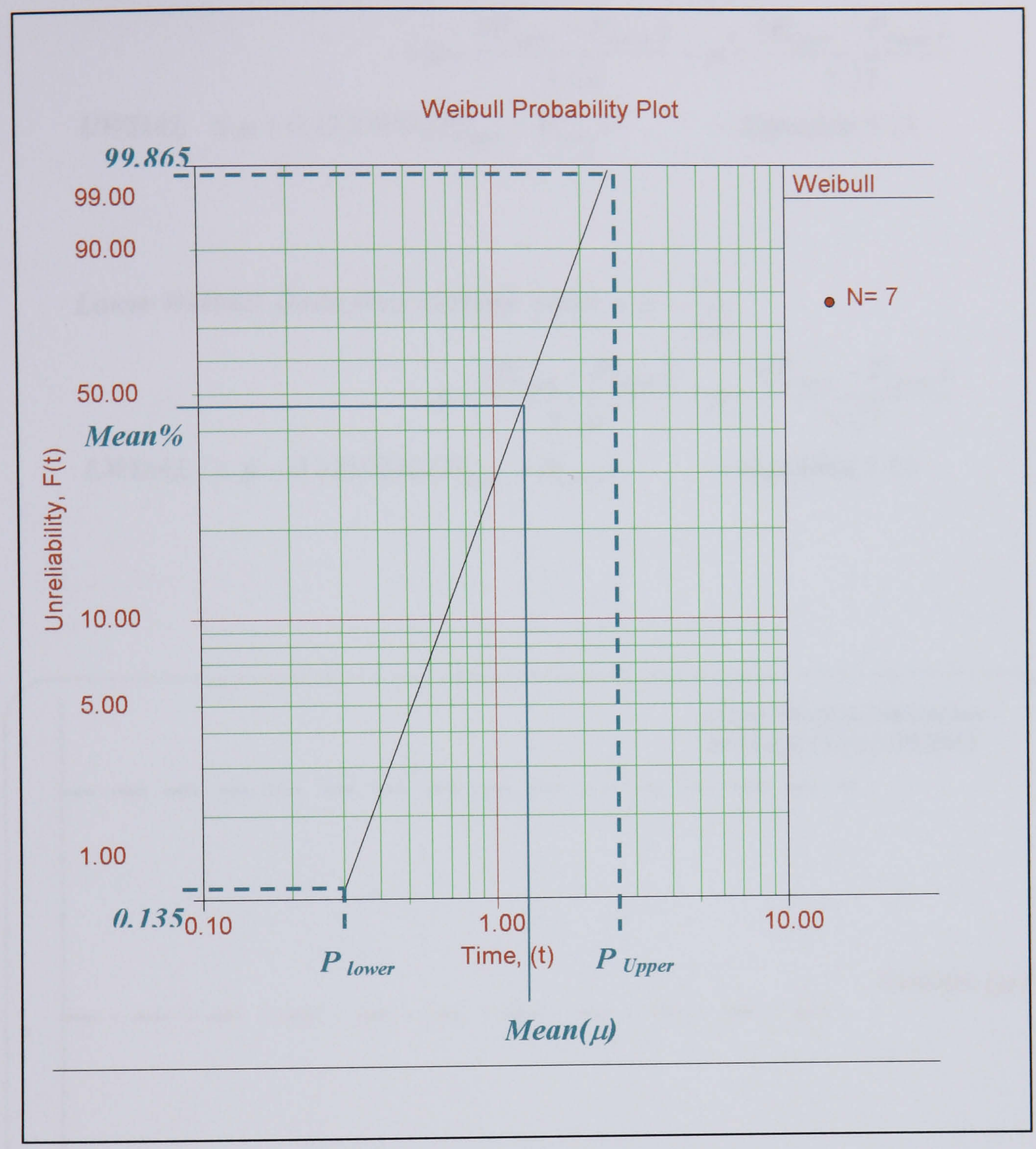


Figure 5:18 – Weibull Range Percentages (based on deduction and RRX)

$$\textbf{\textit{Range}} = R = P_{Upper} - P_{Lower} = 3\,\sigma \qquad \cdots \textbf{\textit{Equation}}\ 5.11$$

$$\textbf{\textit{Therefore}},\ \sigma = \frac{R}{3} = \frac{(P_{Upper} - P_{Lower})}{3} \qquad \cdots \textbf{\textit{Equation}}\ 5.12$$

$$\textbf{\textit{Uppper Weibull Deduction Average Limit}} = \mu + \frac{\sigma}{\sqrt{n}}$$

$$= \mu + \frac{(P_{Upper} - P_{Lower})}{3\sqrt{n}} = \mu + \frac{(P_{Upper} - P_{Lower})}{3\sqrt{7}}$$

$$\textbf{\textit{UWDAL}} \cong \mu + 0.1259882\,(P_{Upper} - P_{Lower}) \qquad \cdots \textbf{\textit{Equation}}\ 5.13$$

$$\textbf{\textit{Lower Weibull Deduction Average Limit}} = \mu - \frac{\sigma}{\sqrt{n}}$$

$$= \mu - \frac{(P_{Upper} - P_{Lower})}{3\sqrt{n}} = \mu - \frac{(P_{Upper} - P_{Lower})}{3\sqrt{7}}$$

$$\textbf{\textit{LWDAL}} \cong \mu - 0.1259882\,(P_{Upper} - P_{Lower}) \qquad \cdots \textbf{\textit{Equation}}\ 5.14$$

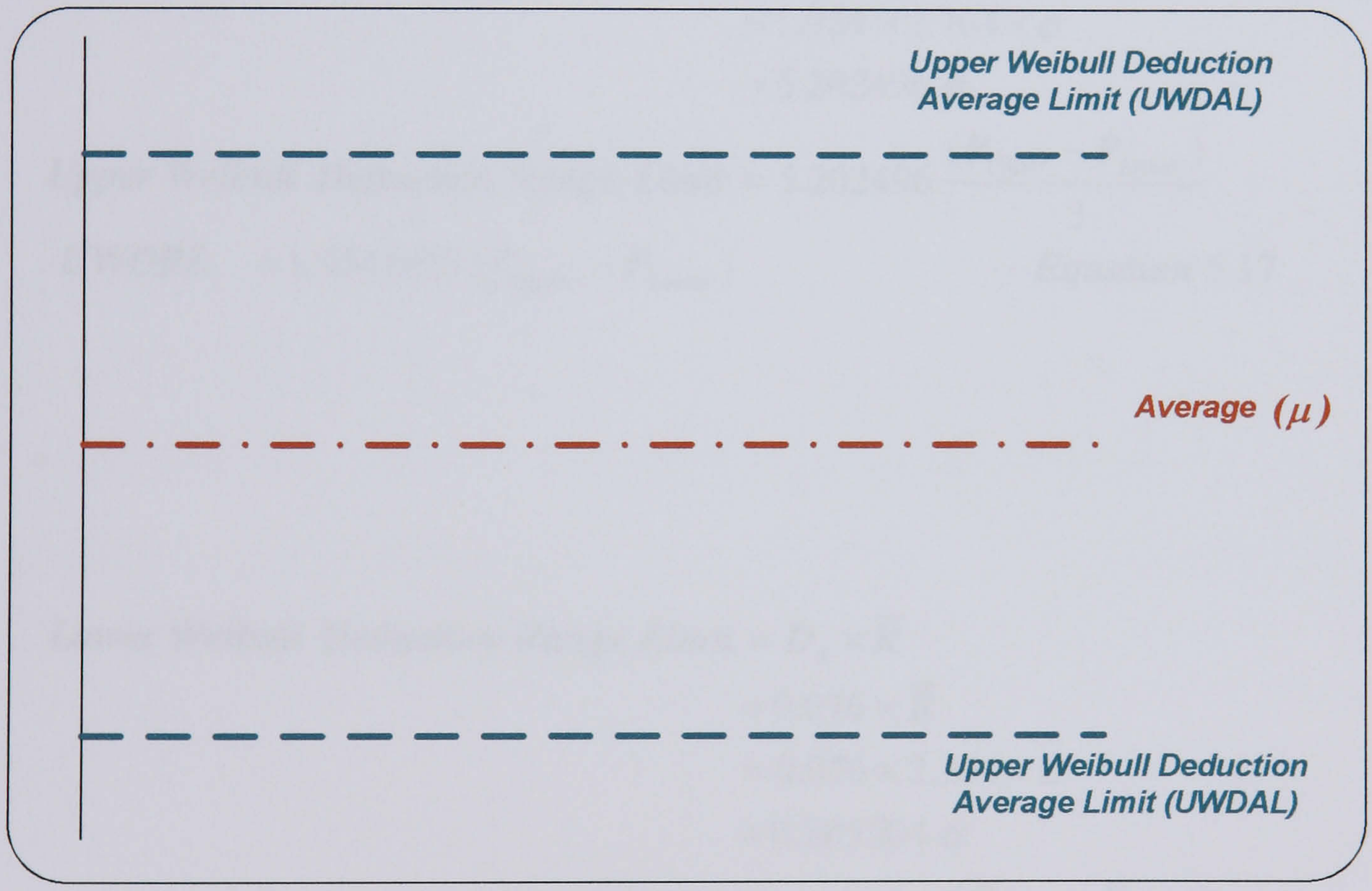


Figure 5:19 – Weibull Deduction Average Control Chart

Also the Weibull Deduction Range Control Chart Limits are calculated as follow; (refer to figure 5:20)

$$\overline{R} = d_2 \times \sigma \qquad \cdots \textit{Equation } 5.15$$

When n=7 , $d_2 = 2.704$ [68]. Therefore,

$$\overline{R} = 2.704\ \sigma \qquad \cdots \textit{Equation } 5.16$$

$$\begin{aligned} \textit{Upper Weibull Deduction Range Limit} &= D_4 \times \overline{R} \\ &= 1.924 \times \overline{R} \\ &= 1.924 \times 2.704 \times \sigma \\ &= 5.202496\ \sigma \end{aligned}$$

$$\textit{Upper Weibull Deduction Range Limit} = 5.202496\ \frac{(P_{Upper} - P_{Lower})}{3}$$

$$\textit{UWDRL} \quad = 1.7341653\ (P_{Upper} - P_{Lower}) \qquad \cdots \textit{Equation } 5.17$$

$$\begin{aligned} \textit{Lower Weibull Deduction Range Limit} &= D_3 \times \overline{R} \\ &= 0.076 \times \overline{R} \\ &= 0.076 \times 2.704 \times \sigma \\ &= 0.205504\ \sigma \end{aligned}$$

$$\textit{Lower Weibull Deduction Range Limit} = 0.205504\ \frac{(P_{Upper} - P_{Lower})}{3}$$

$$\textit{LWDRL} \quad = 0.0685013\ (P_{Upper} - P_{Lower}) \qquad \cdots \textit{Equation } 5.18$$

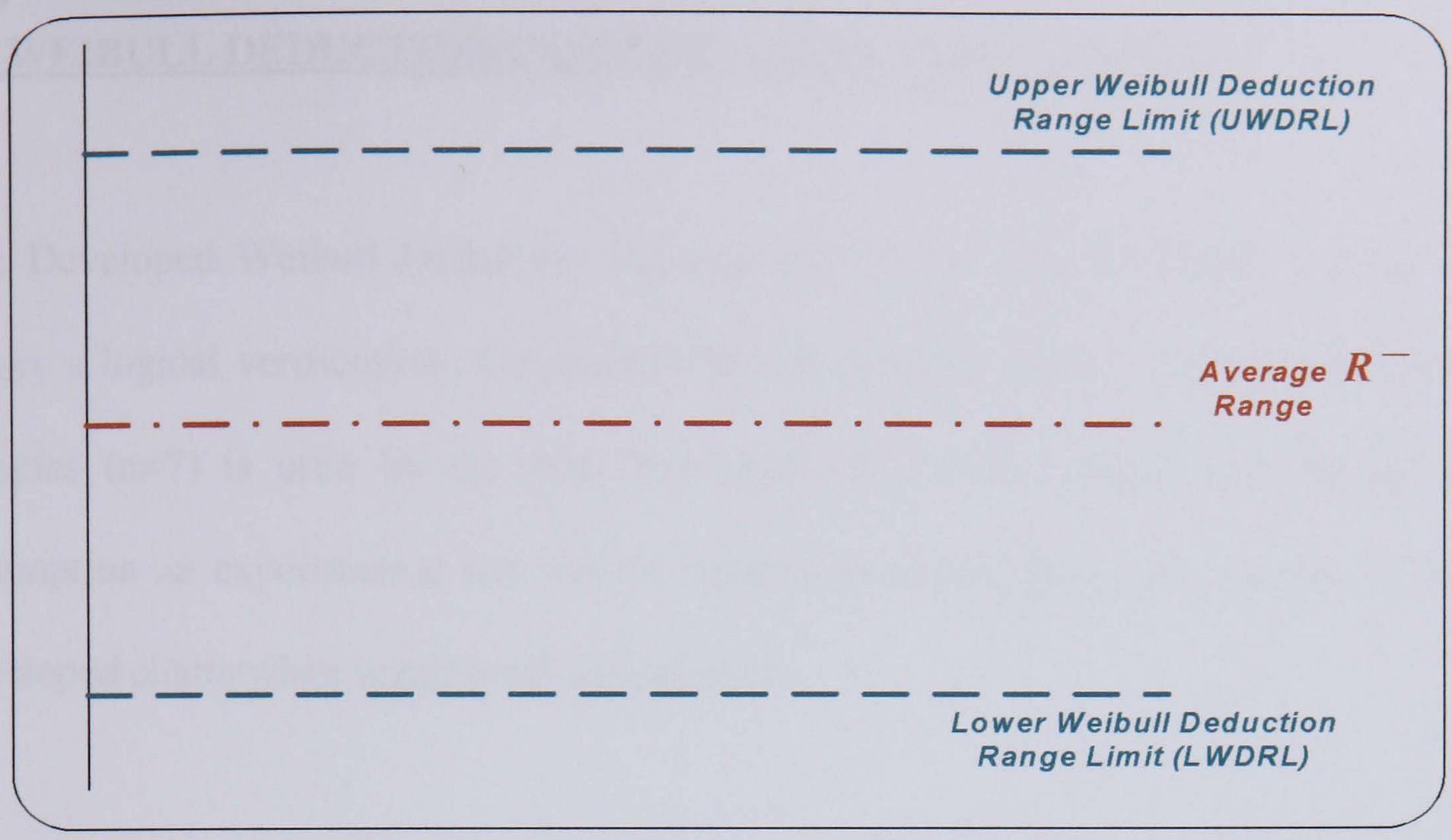


Figure 5:20 - Weibull Deduction Range Control Chart

It is a fact now, that Weibull distribution can be used with the association of deduction model and rank regression x; to establish control charts (Average and Range Control Charts), which are based on small sample size (n=7). These Weibull Deduction control charts will provide accurate results more than the use of conventional Shewhart control charts.

5.3 WEIBULL DEDUCTION CONTROL CHART APPLICATION

The Developed Weibull Deduction Average and Range Control Charts showed in theory a logical verification of replacing the conventional control charts when small samples (n=7) is used by the new Developed charts. To validate the theoretical assumption an experimental test will be used to positively insure the success of the developed charts when using small sample size.

The experimental test, which will be used in this research, is the Lap Shear Strength Test (LSST). In this test strength of a lap sheared adhesive bond will be stressed in shear to determine the strength of the joint, which have specific type of adhesive. The joint will be exposed to a concentric parallel force. The maximum shear force or stress rapture will be calculated. Figure 5:21 shows the behaviour of shear force.

When Force F is applied then the rigid adhesive will deform only in shear, then the average adhesive shear stress t can be calculated by equation 5:19[87]. Where F is the applied load, L is the length of the joint, and b is the width of the joint.

$$\textbf{\textit{Shear Stress}} = t = \frac{F}{Lb} \qquad \cdots \textbf{\textit{Equation}}\ 5:19$$

Lap shear specimens were prepared using 100mm x 3mm aluminium sheet[88]. In order to hold the specimens, two holes were drilled in the size of the clamp pin. Joints were assembled with a 12.5mm overlap, and then stacked on a special clamping jig using guide pins on a metal base to control dimensional changes. The adhesive was placed in between the substrates leaving a 19mm gap for wedge insertion. The crack length

was measured which gave the strength retention data of the joint as a function of time. Maximum lap-shear strengths of the joints were measured with a constant cross-head speed of 0.42 mm/s (1 inch per minute)[89].

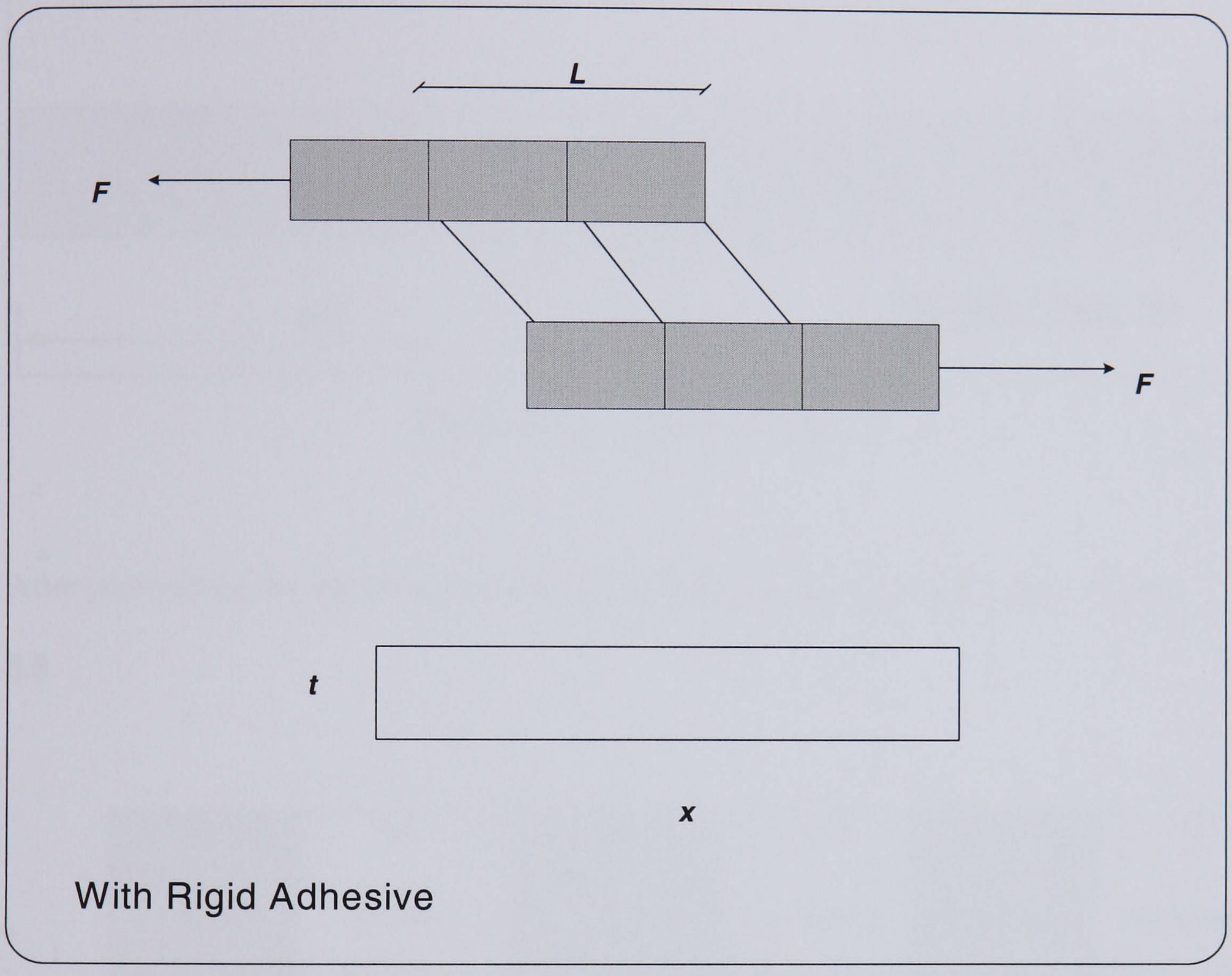


Figure 5:21 – Deformation in loaded Single Lap joints

Figure 5:22 shows the specimen specification used in lap shear test, a set of 7 specimens were used in each test, and the shear stress was calculated for each test. Tests were performed under the room temperature and relative humidity. Test where done by using different adhesives (O-XD4600, Q-Citec FM73 and P-Araldite 2012), also the tests were done twice, first with untreated surface and the second with B-Si treated surface.

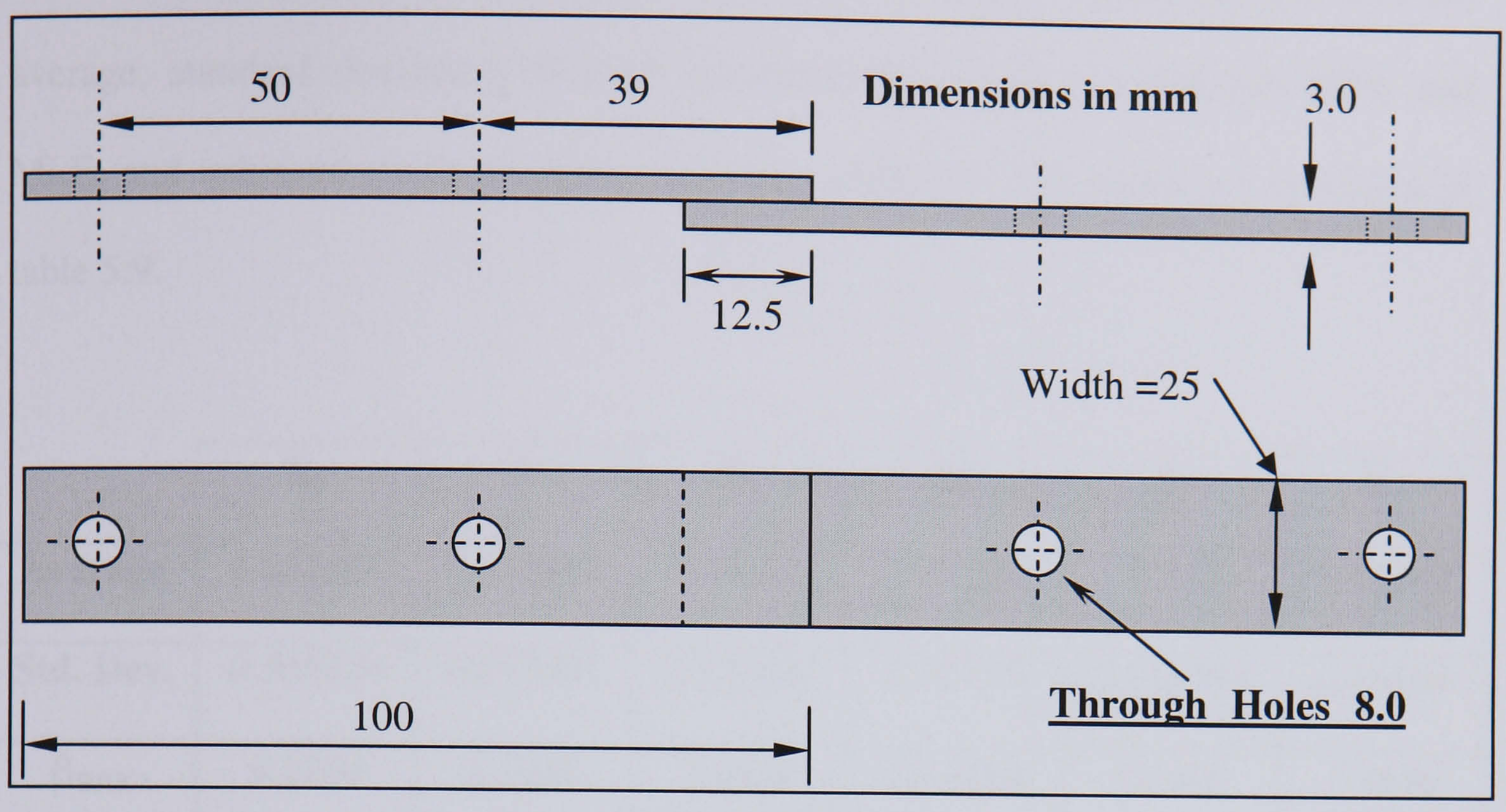


Figure 5:22 - Lap Shear Test

After performing the lap shear tests the following data were obtained, refer to table 5.8.

	S_1 O-XD4600	S_2 O-XD4600	S_3 Q-CitecFM73	S_4 Q-CitecFM73	S_5 P-Araldite 2012	S_6 P-Araldite 2012
	Un treated	**B-Si Only**	**Un treated**	**B-Si Only**	**Un treated**	**B-Si Only**
i_1	*4.62*	*5.37*	*3.165*	*4.72*	*0.834*	*0.15*
i_2	*4.06*	*5.41*	*3.045*	*5.55*	*0.914*	*0.313*
i_3	*5.03*	*5.89*	*4.18*	*5.095*	*0.99*	*0.592*
i_4	*5.4*	*5.6*	*4.035*	*5.12*	*0.85*	*0.0755*
i_5	*5.69*	*4.79*	*4.285*	*4.65*	*0.97*	*0.191*
i_6	*4.86*	*5.17*	*3.65*	*4.99*	*0.87*	*0.2203*
i_7	*4.38*	*3.98*	*3.19*	*4.86*	*0.676*	*0.22*

Table 5:8 – Lap Shear Test Results

Using the data in table 5:8, few calculations will take place on each sample, such as, average, standard deviation, Weibull parameter based on conventional RRX and MLE, and unbiased shape parameter will be calculated. The results are tabulated in table 5:9.

	S_1	S_2	S_3	S_4	S_5	S_6
Average	*4.862857*	*5.172857*	*3.65*	*4.997857*	*0.872*	*0.25168*
Std. Dev.	*0.568234*	*0.627341*	*0.523641*	*0.30179*	*0.104594*	*0.16664*
β_{RRX}	*9.2904*	*8.5984*	*7.9804*	*19.4164*	*8.7902*	*1.7686*
η_{RRx}	*5.1065*	*5.4505*	*3.8559*	*5.1227*	*0.9178*	*0.2830*
β_{MLE}	*10.2638*	*12.0521*	*8.1447*	*17.1691*	*11.6478*	*1.7635*
η_{MLE}	*5.1014*	*5.4132*	*3.8648*	*5.1367*	*0.9129*	*0.2848*
$\beta_{Unbiased}$	*8.1156496*	*9.529669*	*6.9144879*	*13.575713*	*9.2099870*	*1.39441*

Table 5:9 – Weibull parameter estimations

Observing the results of Weibull parameters estimations in table 5:9 shows a lack of prediction and high level of variation. As these are sample sizes of 7 and the sample mean is normally distributed (Based on Central Limit Theorem), therefore, it is known that the parent population is normally distributed, with a shape parameter of 3.44. Using this fact, it is recommended to run Weibull Deduction Method to achieve high level of confidence in shape parameters estimations.

Having a parent population with a shape parameter of 3.77, and using the Deduction Model, then the deduction percentage can be calculated. The deduction percentage resulting from equation 5.2 is equal to 99.3288 %, table 5:10 shows the correspondent

deduction values for deduction percentage of 0.993288. Also, table 5:11, shows the lap shear test data after the deduction.

Sample	Deduction Value
S_1	*2.9810*
S_2	*3.0470*
S_3	*2.0606*
S_4	*3.9596*
S_5	*0.5196*
S_6	*0.0167*

Table 5:10 – Deduction Values

	S_1 O-XD4600 Un treated	S_2 O-XD4600 B-Si Only	S_3 Q-CitecFM73 Un treated	S_4 Q-CitecFM73 B-Si Only	S_5 P-Araldite 2012 Un treated	S_6 P-Araldite 2012 B-Si Only
i_1	*1.639*	*2.323*	*1.1044*	*0.7604*	*0.3144*	*0.1333*
i_2	*1.079*	*2.363*	*0.9844*	*1.5904*	*0.3944*	*0.2963*
i_3	*2.049*	*2.843*	*2.1194*	*1.1354*	*0.4704*	*0.5753*
i_4	*2.419*	*2.553*	*1.9744*	*1.1604*	*0.3304*	*0.0588*
i_5	*2.709*	*1.743*	*2.2244*	*0.6904*	*0.4504*	*0.1743*
i_6	*1.879*	*2.123*	*1.5894*	*1.0304*	*0.3504*	*0.2036*
i_7	*1.399*	*0.933*	*1.1294*	*0.9004*	*0.1564*	*0.2033*

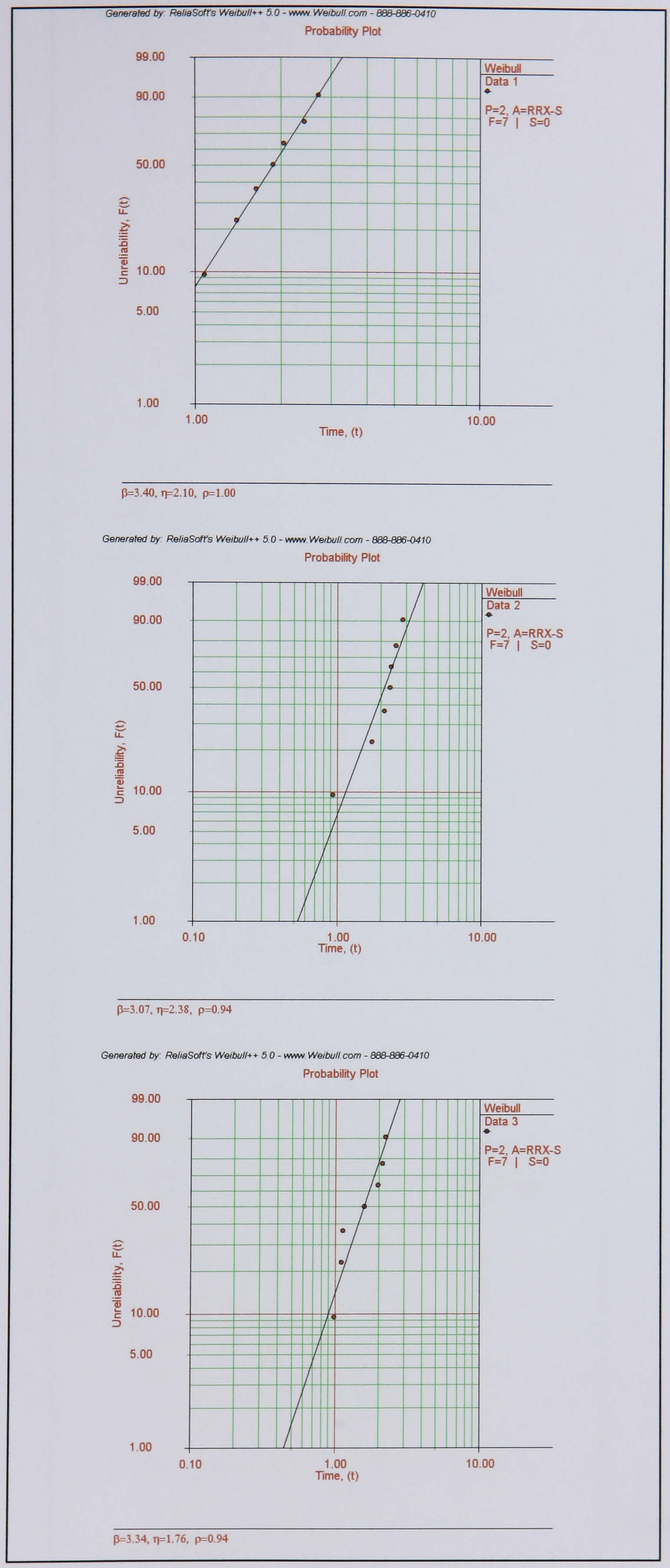
Table 5:11 – Deducted data.

Using the deduction data in table 5:11, the Weibull shape and scale parameters will be estimated based on deduction method associated with RRX, The results of deduction method estimation can be found in table 5:12.

	S_1	S_2	S_3	S_4	S_5	S_6
β_{RRX} **(Deduction)**	*3.4028*	*3.0708*	*3.3406*	*3.9667*	*3.1113*	*1.5935*
η_{RRX} **(Deduction)**	*2.0961*	*2.3848*	*1.7621*	*1.1402*	*0.3948*	*0.2643*

Table 5:12 – Weibull Deduction Parameters Estimations

The following graphs (see Figure 5:23) are the probability plots for each sample based on Weibull deduction method associated with rank regression x estimation. These graphs will be used to calculate the *mean* and P_{Upper} and P_{Lower} for each sample, and these values will be used to generate average and range Weibull deduction graphs.



β=3.40, η=2.10, ρ=1.00

β=3.07, η=2.38, ρ=0.94

β=3.34, η=1.76, ρ=0.94

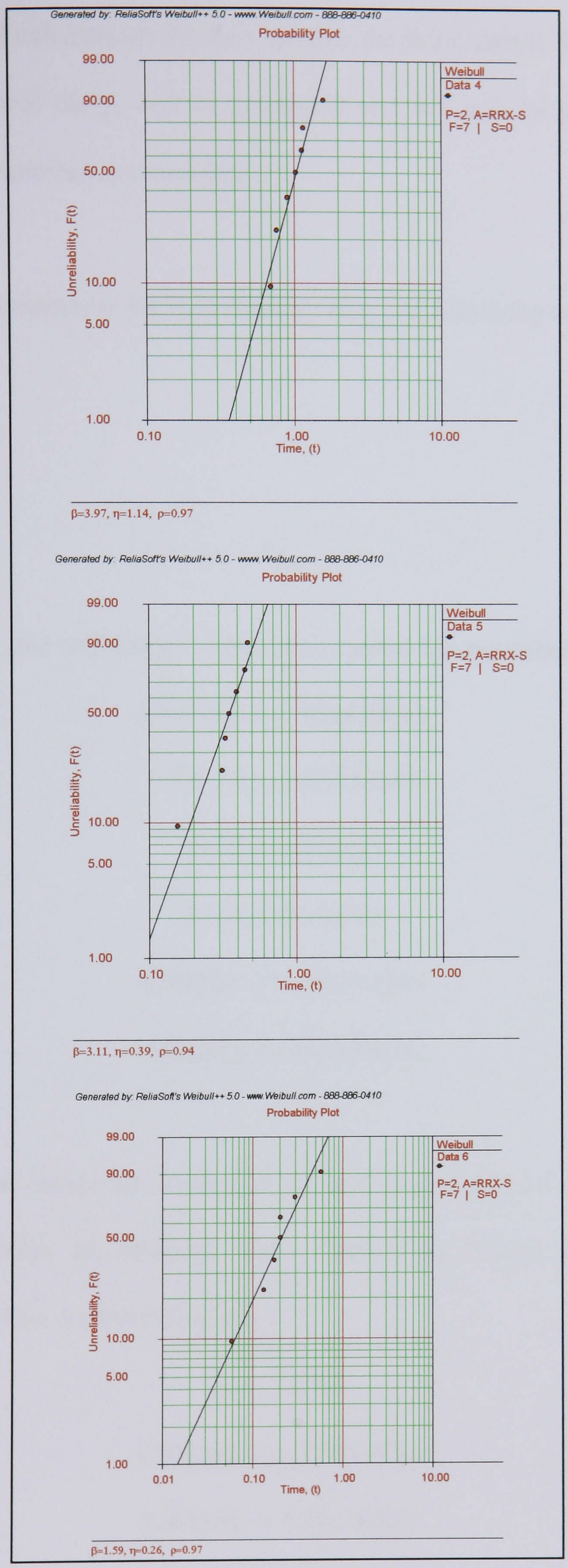

Generated by: ReliaSoft's Weibull++ 5.0 - www.Weibull.com - 888-886-0410
Probability Plot
99.00
90.00
50.00
10.00
5.00
1.00
Unreliability, F(t)
0.10
1.00
10.00
Time, (t)
Weibull
Data 4
P=2, A=RRX-S
F=7 | S=0
β=3.97, η=1.14, ρ=0.97
Generated by: ReliaSoft's Weibull++ 5.0 - www.Weibull.com - 888-886-0410
Probability Plot
99.00
90.00
50.00
10.00
5.00
1.00
Unreliability, F(t)
0.10
1.00
10.00
Time, (t)
Weibull
Data 5
P=2, A=RRX-S
F=7 | S=0
β=3.11, η=0.39, ρ=0.94
Generated by: ReliaSoft's Weibull++ 5.0 - www.Weibull.com - 888-886-0410
Probability Plot
99.00
90.00
50.00
10.00
5.00
1.00
Unreliability, F(t)
0.01
0.10
1.00
10.00
Time, (t)
Weibull
Data 6
P=2, A=RRX-S
F=7 | S=0
β=1.59, η=0.26, ρ=0.97

Figure 5:23

Due to the retrieval calculations for each sample, the third sample S_3 will be taken and analysed. The control charts will be generated and this will be a prototype off the other Samples calculations and analysis.

From the Weibull Deduction RRX Probability Plot, the following can be calculated:

$\mu = 1.5790$

$P_{Upper} = 3.00135$

$P_{Lower} = 0.2438$

Using these values, the following Control charts terms can be calculated,

$$UWDAL = 1.926418761$$

$$LWDAL = 1.23158124$$

$$\sigma = 0.115806253$$

$$\overline{R} = 0.313140109$$

$$UWDRL = 0.602481558$$

$$LWDRL = 0.02798636$$

The Above control limits are deduction control limits, and in order to have the absolute control limit an addition value (Deduction Value) should be added. Therefore, the absolute control limits are:

$\mu = 3.6396$

$$UWDAL = 3.987018761$$

$$LWDAL = 3.29218124$$

$$UWDRL = 2.663081558$$

$$LWDRL = 2.08858636$$

The results obtained from the use of Weibull deduction technique can be employed graphically to generate a graphical representation to control charts based on Weibull distribution and rank regression on x for small sample size (n=7). Two main graphs can be generated, a Weibull Deduction Average Control Chart and a Weibull Deduction Range Control Chart. Figure 5:24 is a Weibull Deduction Average Control Chart, which show the behaviour of the average strength for a joint with Q-CitecFM73 adhesive and no surface treatment resulting form a single lap shear test.

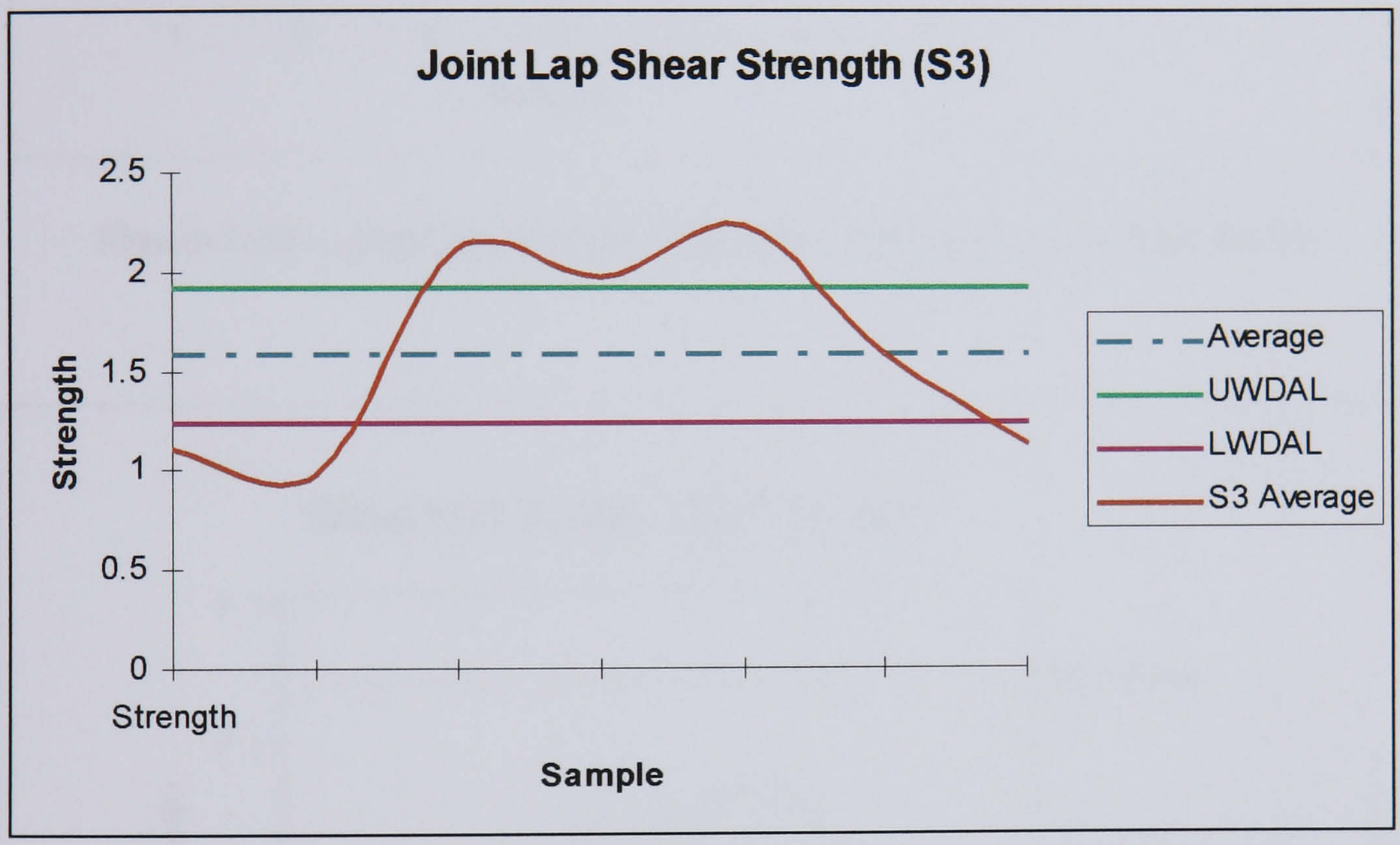


Figure 5:24 – Weibull Deduction RRX Average Control Chart

Also to make such analysis softer and easy to digest the Weibull Deduction Average Control Charts can be re-plotted by using absolute values, which is achieved by adding the deduction value to the limits and averages. Figure 5:25 shows Weibull deduction average charts based on absolute values. Figure 5:26 present a conventional average Shewhart chat for S3, and the difference between Shewhart and Weibull deduction chart is so apparent.

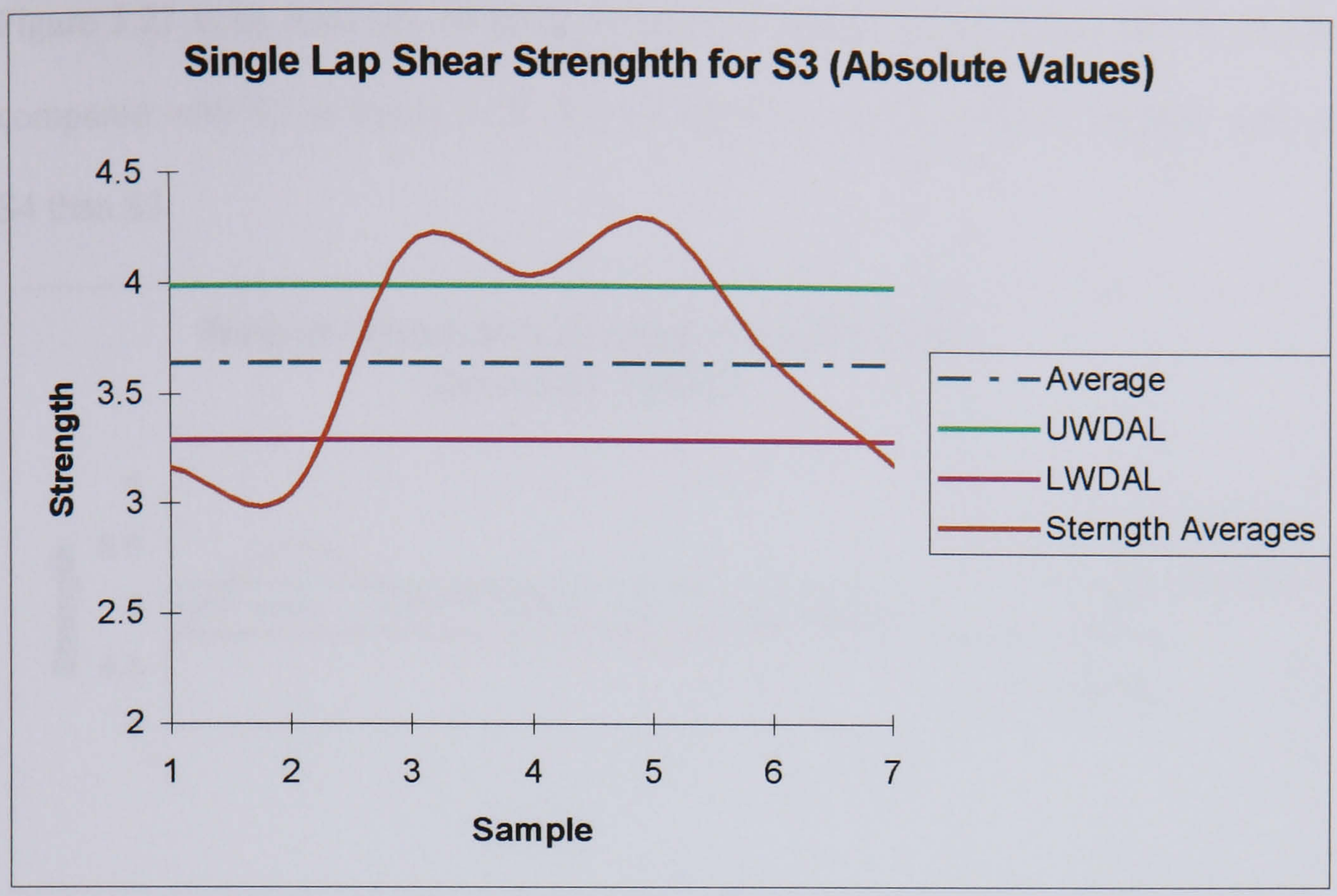


Figure 5:25 – Absolute Weibull Deduction Average Control Chart for S3

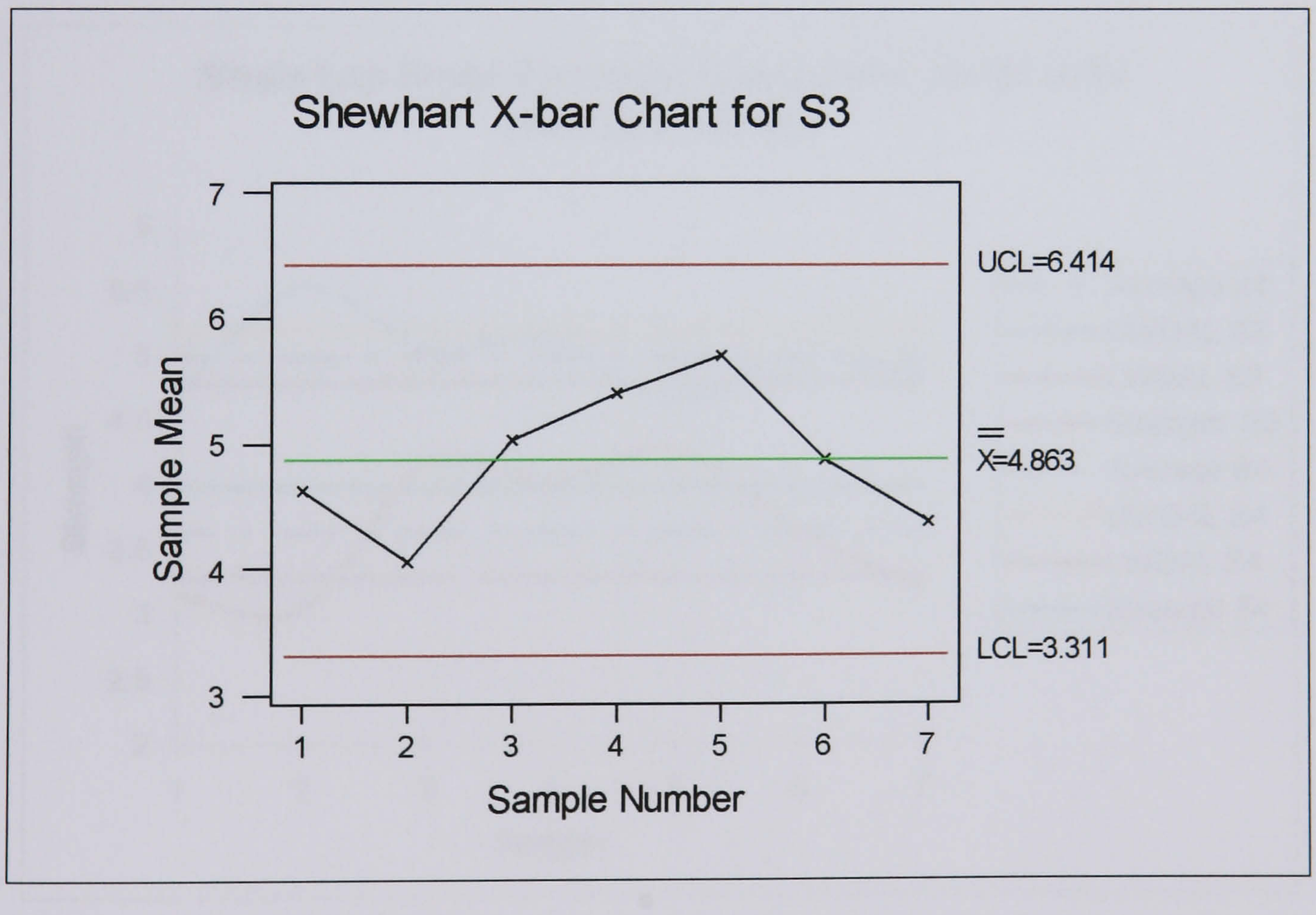


Figure 5:26 – Shewhart Control Chart for S3

Figure 5:27 is an Absolute Weibull deduction Average control chart for S4, and it is compared with S3 in figure 5:28. The comparison shows a higher strength value for S4 than S3.

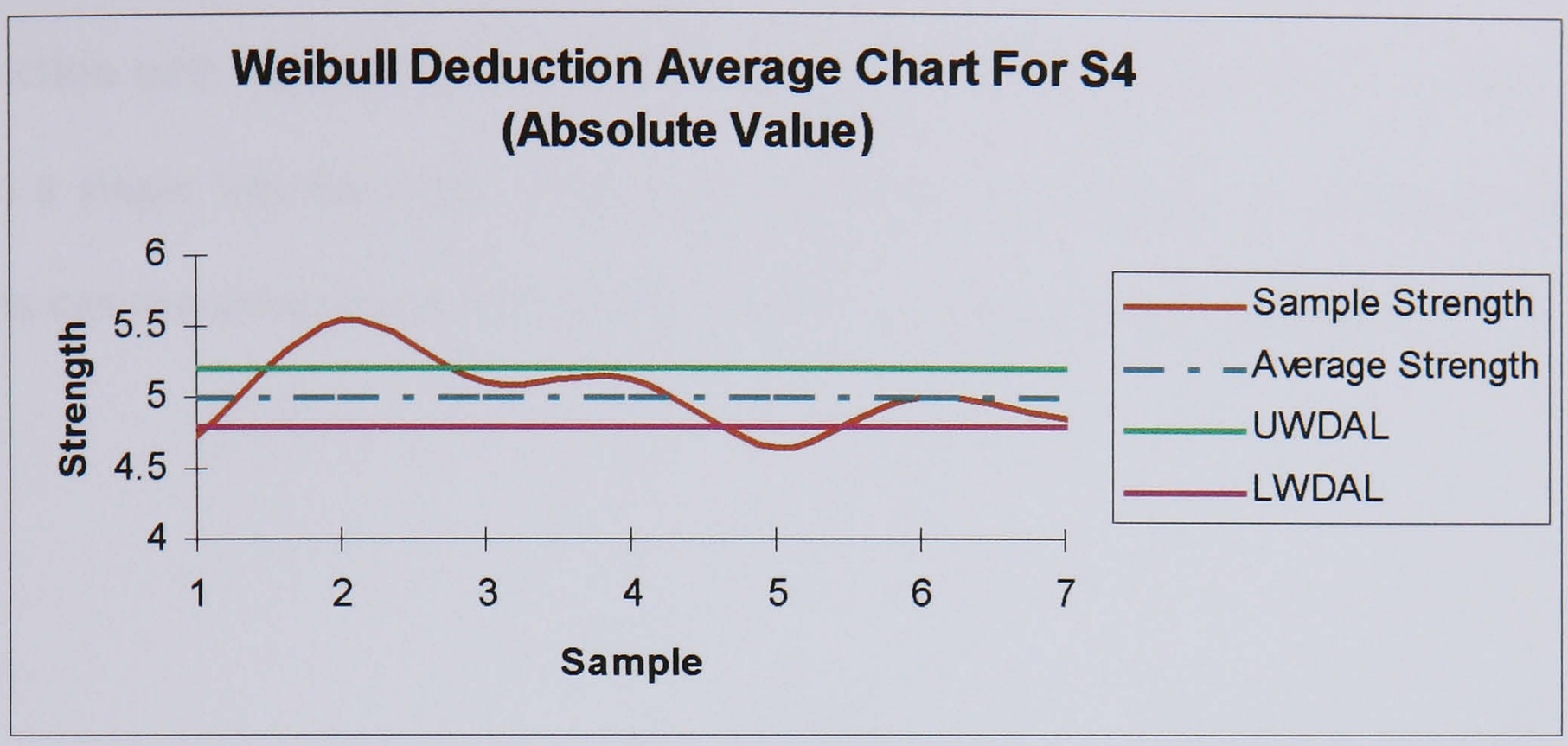


Figure 5:27 – Absolute Value Weibull Deduction Average Control Chart for S4

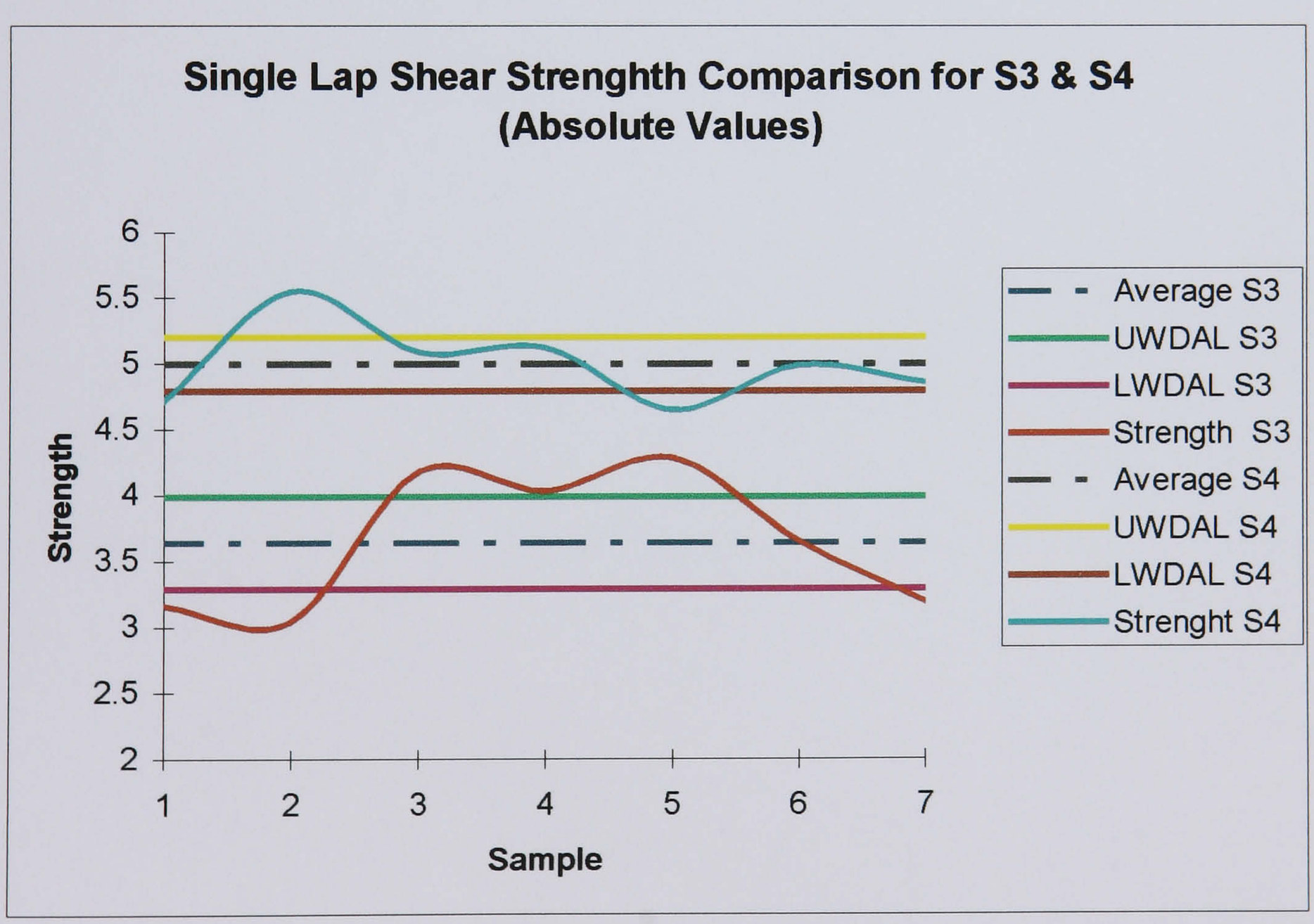


Figure 5:28 – Comparison between S3 versus S4 absolute Weibull Deduction Average Chart

5.4 CHAPTER FIVE CONCLUSION

This chapter has presented remarkable conclusion, as the success of the new Weibull deduction rank regression on x control charts was proven by the use of strength data from a single lap shear test. Therefore, it is believed that the new Weibull control charts can accommodate small sample analysis with high level of accuracy.

CHAPTER 6

DISCUSSION

6.0 CHAPTER SIX REVIEW

In this chapter a discussion of all primary finding and model analysis will be handled. A brief argument regarding the concept of quality and its philosophies will be addressed, as many researchers have different points of view regarding the definition of quality. Moreover, A comparison of estimation techniques will be held to show the technique, which will be adopted to be used with Weibull deduction method. Finally, results of single lap shear test will be analysed to show the effectiveness of the new Weibull deduction rank regression on x control charts with the comparison of Shewhart control charts.

6.1 THE LONG AND THE SHORT OF QUALITY

Quality is may be considered as the religion of engineers, it guides them to the successes and goal achievement. It is based on the idea that they all agree on, which is worshiping a god. However, religions have many paths to fulfil their objective, but they all lead to one ending. Similarly, in quality there were many philosophies concerning the definition, objectives and strategy. Nevertheless these philosophies revolve around basic pillars of quality. Quality Philosophers tried to explain their own prospective view about quality as a separate science overlapping with many other life ventures.

Quality had been defined in many different ways (Section 1.1), it was defined as fitness for use[2], complying with specification and achieving customer needs[4]. Due to the nature of the new technology in these days, such definitions can be general and open-ended definitions; therefore, it is the author opinion to constrain quality is a simple professional view. For the present time, Quality can be defined as exceeding customer satisfaction by minimising the variation between the process and a service and the requirement by market. Such a definition will lead to unmitigated quality and allow quality providers to compete successfully in the global market.

Many quality researchers showed impressive contributions in managing quality through fixing a comprehensive quality management system to provide a world-class quality level. Some of the developed quality management viewpoints have been conducted through a personal experience by the researcher himself. Shewhart concentrated on developing statistical control charts to monitor and develop quality

(Section 2.2.1), while Juran and Deming focused on the managerial side to develop quality through implementing organisational strategy to be committed to quality improvement (Section 2.2.2, 2.2.3 and figure 2:1). Dr Ishikawa established many statistical tools to be integrated with quality improvement (Section 2.6).

The current research involves the study of Shewhart control charts and the statistical aspect when small sample size is associated. Therefore, improving the quality in this research has been through developing new statistical tool to increase the accuracy of monitoring and to allow transparency in process control, which will enhance the confidence in decision making by managers (Section 2.6, 3.1 and figure 3:1)

6.2 LACKS IN CONVENTIONAL STATISTICAL CONTROL CHARTS

Statistical control charts play a vital role in the quality development and enhancement to manufacturing processes. After many years of concerns and debates, many dispute and lack of agreement may be pointed out[90]. SPC is considered a sub-division of Statistical Quality Control (SQC), however, many companies these days invest heavily in SPC, and this is the reason that this current research in valuable to address the lack in the current SPC charts and provide an alternative solution for the present obstacles in adopting SPC charts in some areas. In high cost and low volume manufacturing environment, conventional SPC charts does not fully satisfy the needs of the manufacturers to monitor quality, as the problem issued in this case is considered as a financial problem. In such an environment, it cannot be wise and efficient to use large sample sizes, as this maybe expensive (specially in case of destructive testing). Also, if the large samples use expensive items, it needs to be replaced by small sample sizes; the current SPC charts are not capable to ensure the required level of accuracy and detection of variability and omitting non-conformance[91] (section 1.2).

High levels of acceptable accuracy obtained by conventional (Shewhart) Control charts may be only established when using large sample size (n>50), which is highly not recommended in low volume high cost process environment. Nevertheless, the conventional control charts establish its control by developing limits to control the process; these limits are based on practical calculations. Nevertheless, a control chart with a large number of sample sizes may not predict variability, as the specification is met in control chart but the process may vary while it is within the control. In such a

case the conventional control charts fail to achieve acceptable accuracy and early warning status even with large sample size[92,93].

Shewhart Control Charts are based on normal distribution, and such distribution showed a lack in performance when small samples sizes are use[94,95]. Such charts were unable to detect variability and variations when small size samples are used. Consequently, the effectiveness of these charts was so low and they did not reflect the true behaviour of the process, which made many processes produce non-conforming items. Moreover, normality of small sample size is a debateable matter of concern, as normality tests did not reflect accurate nature of small samples size. (Section 2.6.2, 2.9, 4.1, and 4.3).

When Shewhart control charts are used with small sample size, they may establish a trend figure, which will deceive the interpreter of such graphs, as they will not detect the variability in a correct manner and draw different trend than the reality. Conventional Shewhart charts limit calculations depend on the many empirical factors, such as D_3, D_4, A_2 and d_2. Such factors have been tabulated and the accuracy of the average and range control charts limits depend on the values of these factors. Each factor changes its value depending on the sample size (n) used in construction average and range control charts. In this current research, values of D_3 and D_4 have been analysed, it has been clear that D_3 and D_4 shows a contrary behaviour with sample size change. D4 Start with a value of 3.268 at n=2 and decreases till value of 1.777 at n=10, then it tries to stabilises around 1.6 when n is greater than 11. On the other hand, D3 start with value of zero a it keeps this value till n=6, afterward, when n=7 the D3 value increases to 0.076, then it keeps increasing till the value of 0.223 at

n=10 then it tried to stabilise around the value of 0.23 when n goes greater than 11 (Figure 4:14). Therefore, this research has shown that when the sample size increases (n=20 items or more), Shewhart control charts tends to achieve acceptable reflection to process behaviour in reality as D_4 and D_3 tend to converge and stabilise near a specific value. On the contrary, it is clear that when small sample sizes are implemented with Shewhart control charts the values of D_3 and D_4 fluctuate significantly, and it will affect the accuracy of control charts to trace variability (Equation 2.5 and 2.6, Section 4.4).

The control charts are usually based on three standard deviation range, as this range is wide, and also the small sample effect associated with control chart, a poor prediction to non-conformance will occur with the conventional control charts despite the different control charting[96]. Also, for the pre-control charts, when the specification limits are set (assume it to be) as an upper and lower control limits, this may also increase the range and widen the interval of detection and small variation will be hard to notice. Also, such an assumption may cause confusion as if any point exceeding the limits means it exceeded the specification, and this will cause a problem to set up process parameters and detect problems affecting of limits points (Figure 2.7, Section 2.6.2, 2.8 and 2.12).

6.3 CONTROL CHARTS BASED ON WEIBULL ANALYSIS

Weibull distribution has showed a great success with small sample size analysis. It has an acceptable level of accuracy with small sample sizes. As the control charts are based on normal distribution and it fails the accuracy in small sample size analysis[97], it has been suggested in this research to use Weibull distribution to overcome the problems with normal distribution based control charts.

Weibull distribution is considered a reliability tool, but in this research it has been used in quality to achieve success in small sample size control charts. Weibull distribution analysis accuracy depends on the accuracy of estimating Weibull parameters. Many methods have been developed to overcome some errors in predicting Weibull parameters. These methods are Weibull probability plot, rank regression estimation and Maximum likelihood estimation[98, 99,100] (Section 4.6.1, 4.6.2. and 4.6.3)

Weibull distribution when used with small samples showed estimated values for Weibull parameters, which differ from the theoretical expected Weibull parameters values. For example, when a small sizes sample data adopted from a normally distributed population the Weibull parameters estimation techniques showed results far away from the expected value for Weibull parameter. The expected values for shape parameter is 3.44, but with the use of small sample with Weibull probability plot, the estimated shape parameter was 82.3, 77.19 Weibull rank regression on x, and 42.28 for Weibull maximum likelihood estimation (Section 4.3 and figure 4:12). This occurred as Weibull 2- parameters estimation techniques are based on beta with

location parameter of zero. However, in the case of practical data, manufacturing data have positive location parameter. For such reason the Weibull estimation techniques failed to ensure estimated values near 3.44.

It has been proven that Weibull shape parameter based on the conventional techniques of estimation has a specific lower value dependent on the sample size, where it has a starting point for estimation and increase to positive infinity depending on the nature of the original data. It was shown that with sample size of 5 the value of the shape parameter would lay in the interval of $[9.06, \propto]$. (Section 4.5 and figure 4.15)

To have an accurate method to predict Weibull parameters and use its results in constructing control charts based on Weibull analysis, it was found in this research that deduction a specific value from the absolute original data enhances the accuracy in calculating Weibull parameter. This deduction will solve the problem of data offset, as their gamma value will be more than zero (Figure 4.8, 5.1).

Data samples from many different populations with known shape parameter were used and deduction values were calculated. Based on calculated deduction values, a mathematical model was established to formulate the relation between the deduction percentage and parent population shape parameter (Section 5.1, Equation 5.2).

A Weibull parameters estimation technique was needed in Weibull deduction method. Therefore, the three previously mentioned techniques were tested with real experimental data obtained from single lap shear strength test for aluminium joint with different adhesive. Rank regression on X was selected after running the test as

the RRX showed accurate results when adopting deduction data in Weibull deduction analysis when n=7. Reaching this selection, a Weibull deduction average and range limits were derived and formulated. Such limits can be drawn graphically to construct a new control chart based on Weibull deduction rank regression on x method (Section 5.2, table 5:12).

Data were applied in Weibull Deduction Average Control chart, and clear observation of the success of such method and ensuring an accurate limiting charts to monitor the strength of the lap shear joints with different adhesives. The Weibull deduction average control limits were tighter than Shewhart average control charts (Section 5:2. Equation 5.13, 5.14, ,5.16 and 5.17).

A compression of two single lap shear strength test samples was used to show the success of the Weibull deduction average control charts. The charts showed that using a Q-CitecFM73 adhesive with a B-Si treated joint will have a higher strength (Strength of 4.9991 KN) than the untreated joint (Strength of 3.6396 KN) (Section 5.3, Figure 5:25, 5:27 and 5:28).

Finally, it is the author's believe that the Weibull Deduction Rank Regression on X Control charts demonstrate a successful monitoring and provide an accurate results of manufacturing data when small sample of seven items are used, and such charts compensate the weakness of Shewhart conventional control charts when small sample size is employed (Figure 5:25 and 5:26).

6.4 CHAPTER SIX CONCLUSION

It was concluded from the previous discussion, that Shewhart control charts failed to show capability in detecting variation with small sample. The deduction percentage model was an accurate model to be adopted in order to overcome the offset of the manufacturing data. The new Weibull Deduction control charts show a distinguish potential to detect variability and establish new tight control limit, and these charts compensated the disadvantages of Shewhart charts when small sample sizes are used.

Chapter 7

Conclusion, Recommendations and Future Work

Generally, Shewhart Control charts are considered to be a good statistical tool to monitor quality characteristics. Such control charts are effective when large sample size is used (more than 30), however, in some manufacturing environments where low production volumes and high cost exists Shewhart control charts are being undesirable to use due to the financial aspect associated with the use of large sample sizes. It has been a need for the industry to provide an alternative of Shewhart control chart, and the need of using small sample sizes to reduce inspection time and cost. Knowing that the alternative method should provide an acceptable level of accuracy and sensitivity in detecting variations. Based on this existing problem, this research was carried out to provide a solution to the problem of using Shewhart control charts with small sample sizes.

Weibull distribution has showed a useful prediction of reliability aspects with small sample sizes, and using Weibull analysis as a quality tool not a reliability tool is the basic hypothesis of this research. Therefore, the research will be based on the following hypothesis: "It is suggested that remodelling small Weibull samples to accommodate populations will produce data suitable for measuring non-conformance".

In order to test the research hypothesis, two main aims were set. These aims guided the research process to find a solution for the present industrial problem. The aims were achieved in this research, and the following points address the research aims and the way they were handled in the process of testing the hypothesis:

- Identifying how small samples affect statistical analysis to monitor processes with the use of Weibull data. This was meet by a critical literature review for the existing knowledge and spotting areas of concern regarding small sample use in quality control analysis. Also, statistical quality control tools were examined and clearly understood to establish the basic fundamentals of each tool and generate a clear view of the use of each tool. Then, small sample size behaviour was investigated to tackle the problem associated with the use of small sample size with conventional Shewhart control charts, and understand the mathematical properties of small sample size with Weibull analysis. Moreover, a clear ideas of small sample size behaviour with Shewhart control chart and Weibull distribution were flourished after a deep search in the scope areas accompanied with this aim.
- A second aim, which was brought forward when testing the hypothesis, is proposing a method of using Weibull analysis for statistical control of low

volume processes. Such aim is achieved by understanding the problem in the existing Shewhart control charts and Weibull analysis with small sample size. Afterward, a research based on the difficulties encountered by the use small sample size with Control charts and Weibull analysis were the foundation to find an acceptable model to omit the obstacles in the way of using Weibull analysis with small sample to construct new control chart, which serve the industry goals and achieve success in monitoring quality in a low cost manner.

The aims discussed previously were accomplished by setting the research objective, by which these objective provided a clear and confidant believe to accept the hypothesis of the research and to provide a strong considerations about the success of the solution to the research problem. This is guided by the research methodology (Chapter 3), which lighted the dark roots of the research problem. The objective can be summarised in four main principles:

- Establishing the principal limitations of small samples for process control. This objective was meet in chapter two, when a critical review has been discussed and showed the lack of certainty of using small sample sizes with Shewhart control charts. Also, it was shown in chapter 4, by primary investigation, that control charts limits are dependant on factor values, D_3 and D_4, and such values are functions of the sample size (Figure 4:14).
- To determine the relationship of Weibull for controlling the process. This essential objective has been examined and investigated thoroughly. Weibull knowledge background was fulfilled by a critical literature review. Then Weibull analysis nature has been clarified when associated with small sample size. Also, research led to a primary finding, on the limitation of Weibull

shape parameter estimation when small sample is use, and it was explained how Weibull shape parameter has a restricting lower value, by which the estimation of shape parameter start with (Figure 4:15). To understand why Weibull shape parameter estimated value exceeds the theoretical value, four main techniques (Weibull probability plot, rank regression on x, Maximum Likelihood estimation and Unbiased shape parameter estimation) were used to establish accurate explanation of such difference. It was figured out that the accuracy increase when using Unbiased estimation of Weibull shape parameter. Many sample sizes where used and it was noticed that when sample size tend to decrease prediction error increase, as the manufacturing data have an offset location parameter (Gamma parameter).

- The major objective, which was acceptably achieved with confident, is developing a Weibull model for the process. This objective was comprehensively covered in this research and led constructing a new model of control charts based on Weibull distribution with sample size of 7. This model is established after many findings. Conventional offset obstacle accompanied with Weibull analysis was bypassed by implementing the deduction method, this method was able to formulate the relationship between the percentage deduction value and parent population shape parameter (Equation5.2). Afterwards, a applicable choice of estimation technique was found (RRX). With the present of such findings and conclusions a new model was emphasised. The new Weibull deduction rank regression on x control charts were formed, and the limits of averages and ranges were derived.

- A final objective to achieve the aims of this research and proof the success of the research hypothesis was to generate and check a charting process control with small samples. Implementing experimental data, conducted from a single lap shear test, satisfied this objective. The new Weibull deduction method with rank regression on x estimation technique provided superb results. Weibull deduction average control chart was able to constrain the strength data with accurate limits. These limits were tight and provide high level of accuracy and can easily detect variation contrary of the Shewhart control charts. Also by using the new Weibull deduction charts, a significant conclusion were easily drawn and a clear decision was made regarding the strength of the joints when using untreated surface joints with adhesive material.

These logical reasonable procedure, which was carried our in the current work verified and approved the main hypothesis in this research. Therefore, the main contribution to knowledge was the validation of Weibull analysis with small sample in process manufacturing was successfully proven and the idea of creating new control charts based on Weibull distribution showed a great deal of accomplishment to solve the existing problem of using small sample sizes in monitoring quality in a low volume and high production costs in manufacturing atmosphere

It is highly recommended by the result of this research that the new Weibull deduction control charts will be able to replace the conventional Shewhart chart when using small sample size, and the new charts will provide a high level of confidence, as they compensate the lacks occurring when small sample sizes are used.

It is advised that further future work maybe carried out to ensure new stage of development of the new Weibull deduction control charts. The future work can be summarised in two ideas; firstly, establishing a confidence interval bond for the deduction model and gamma. Secondly, testing the new Weibull deduction control charts for different sample size. Sample size below 7 will be an interesting area of future research.

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Appendix A

Software Packages

In this research Five main commercial software where used to:

1. MiniTab .
2. Weibull Smith – By Wes Fulton
3. SPSS.
4. Weibull ++ - By ReliaSoft
5. Curve Expert – By Daniel Hymas

Appendix B

Primary Investigation Analysis Results

Wire Diameter (mm)

Beta =4.5

	1	2	3	4	5	6	7	8	9
1	7.25	7.5	7.25	6.5	7.25	7	7.25	7.5	6
2	7.25	6.75	7	6.5	6.75	7.25	7	6.5	7
3	6.25	6.75	6.5	7	6.75	6.25	6.5	7.5	6.75
4	7.25	6.75	7.5	7	7.25	7	6.75	7.25	6.5
5	7.5	7.5	6.25	7	6.25	7	6.75	6.5	7.5
6	6.5	7.25	7	6.75	7.25	6.75	6.75	7.5	6.75
7	7	7	6.75	5.75	6.5	7	7	6.75	7.25

Average	7	7.071429	6.892857	6.642857	6.857143	6.892857	6.857143	7.071429	6.821429
Standev	0.456435	0.345033	0.429562	0.453163	0.404587	0.318105	0.243975	0.472456	0.494012

Rank Regression on X (RRX)

	16.85	24.94	17.3939	16.7885	18.9632	24.7664	32.2403	17.3715	14.868
	7.2	7.2099	7.0856	6.8406	7.033	7.0293	6.9625	7.2675	7.0428

Max. Likelihood Est. (MLE) - UnBiased Beta & Eta

	16.82	16.02	17.86	16.4	18.33	21.64	26.61	20.88	14.34
	7.253	7.309	7.12	6.905	7.166	7.082	7.033	7.127	7.447

RRX Beta	16.85	24.94	17.3939	16.7885	18.9632	24.7664	32.2403	17.3715	14.868
MLE Beta	16.82	16.02	17.86	16.4	18.33	21.64	26.61	20.88	14.34

Biased Factor(Gn) is 1-(1.346/n)-(0.8334/n^2) :Where n=7 Gn=0.79070614

*Biased beta = MLE Beta * Gn*

Bias Beta	13.29968	12.66711	14.12201	12.96758	14.49364	17.11088	21.04069	16.50994	11.33873

10	11	12	13	14	15	16	17	18
6.75	6.5	6.5	6.75	7.75	7.5	7	7.25	7
7.25	7.75	6.75	7.5	6.25	6	7.25	7.5	7
7	6.5	6.75	6.75	7.25	7	7.25	7	7.5
6	7.25	6.25	7	7	7.75	7.25	7.5	8
6.75	7.25	7	6.5	6.5	6.75	6.25	7	5.75
7	6.25	7.25	7.75	6.5	7	7.75	7.5	6.5
7.25	6.75	6.75	6.75	7.25	7.75	7	6.75	6.75

6.857143	6.892857	6.75	7	6.928571	7.107143	7.107143	7.214286	6.928571
0.429562	0.537299	0.322749	0.456435	0.534522	0.626783	0.453163	0.303746	0.717552

avg.avg= 6.938492

17.5894	14.8632	23.2864	18.9109	14.7019	12.1865	17.5921	26.6716	10.4282
7.0459	7.1146	6.8923	7.1781	7.1544	7.3845	7.3025	7.3476	7.2409

25.5	18.47	18.33	14.24	13.63	13.92	17.65	16.57	15.93
7.054	7.026	6.896	7.167	7.191	7.274	7.421	7.457	7.315

17.5894	14.8632	23.2864	18.9109	14.7019	12.1865	17.5921	26.6716	10.4282
25.5	18.47	18.33	14.24	13.63	13.92	17.65	16.57	15.93

20.16301	14.60434	14.49364	11.25966	10.77732	11.00663	13.95596

10	11	12	13	14	15	16	17	18
6.75	6.5	6.5	6.75	7.75	7.5	7	7.25	7
7.25	7.75	6.75	7.5	6.25	6	7.25	7.5	7
7	6.5	6.75	6.75	7.25	7	7.25	7	7.5
6	7.25	6.25	7	7	7.75	7.25	7.5	8
6.75	7.25	7	6.5	6.5	6.75	6.25	7	5.75
7	6.25	7.25	7.75	6.5	7	7.75	7.5	6.5
7.25	6.75	6.75	6.75	7.25	7.75	7	6.75	6.75

6.857143	6.892857	6.75	7	6.928571	7.107143	7.107143	7.214286	6.928571
0.429562	0.537299	0.322749	0.456435	0.534522	0.626783	0.453163	0.303746	0.717552

avg.avg= 6.938492

17.5894	14.8632	23.2864	18.9109	14.7019	12.1865	17.5921	26.6716	10.4282
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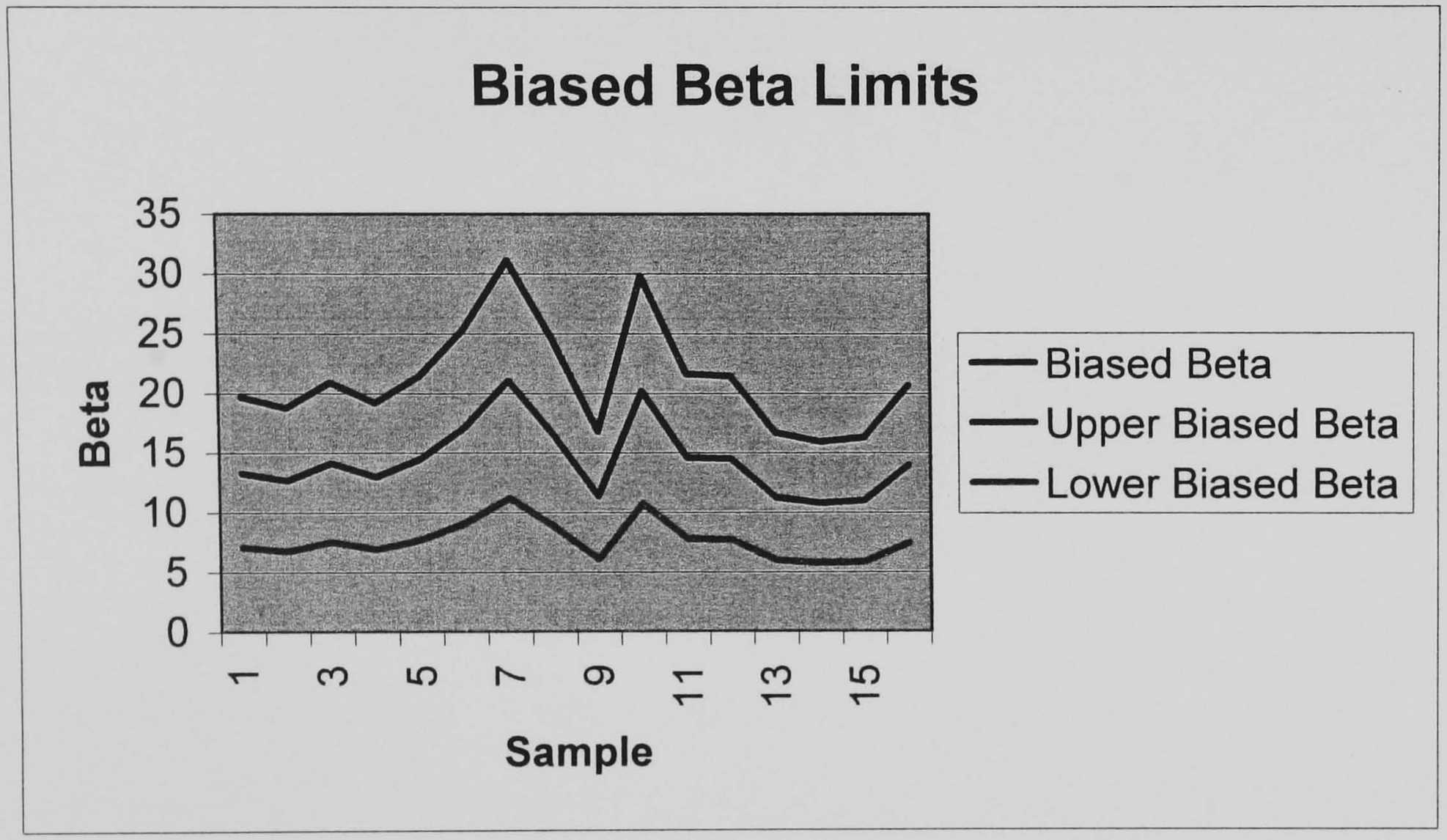
7.054	7.026	6.896	7.167	7.191	7.274	7.421	7.457	7.315

17.5894	14.8632	23.2864	18.9109	14.7019	12.1865	17.5921	26.6716	10.4282
25.5	18.47	18.33	14.24	13.63	13.92	17.65	16.57	15.93

20.16301	14.60434	14.49364	11.25966	10.77732	11.00663	13.95596

Bias Beta	13.29968	12.66711	14.12201	12.96758	14.49364	17.11088	21.04069	16.50994	11.33873
Upp.Beta	19.67418	18.73843	20.89066	19.18291	21.44041	25.31208	31.12544	24.42312	16.77335
Low.Beta	7.090336	6.753103	7.52874	6.913289	7.726865	9.122169	11.21723	8.801797	6.044912

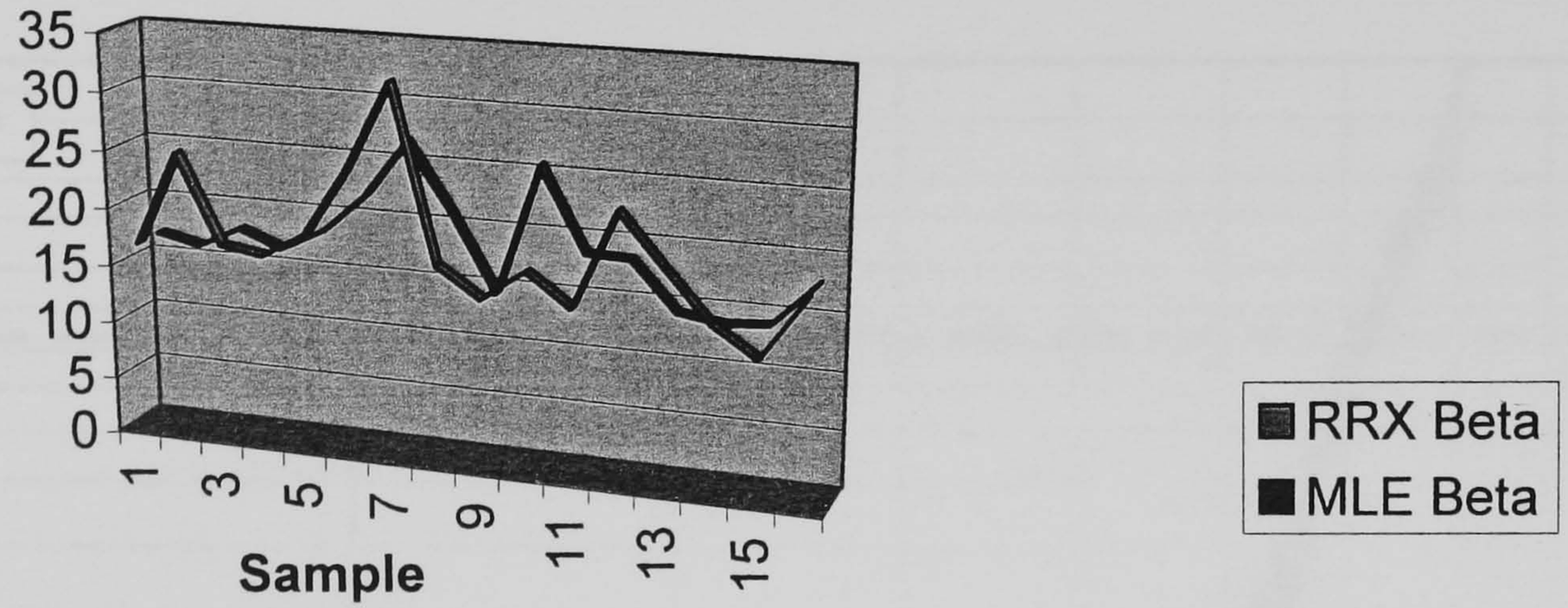


20.16301	14.60434	14.49364	11.25966	10.77732	11.00663	13.95596

avg= 14.36318

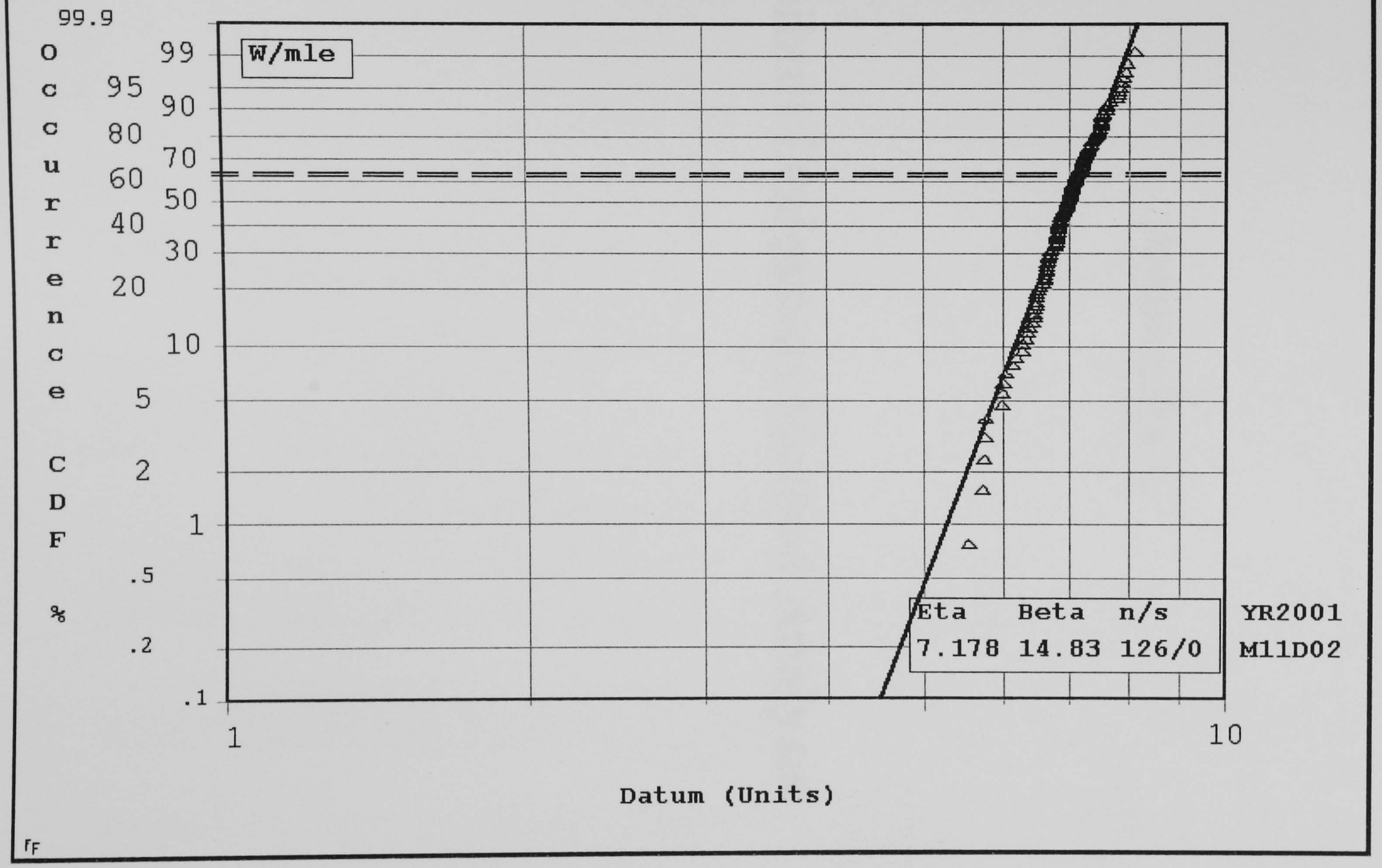
29.82709	21.60417	21.44041	16.65638	15.94287	16.28208	20.64502
10.74932	7.785881	7.726865	6.002758	5.745617	5.867865	7.440216

RRX Beta Vs. MLE Beta



	1	2	3	4	5	6	7	8	9	1	1	1	1	1	1	1
RRX Beta	17	25	17	17	19	25	32	17	15	18	15	23	19	15	12	18
MLE Beta	17	16	18	16	18	22	27	21	14	26	18	18	14	14	14	18

B=4.5 overall data



Appendix C

Weibull Deduction Method Analysis

Wire Diameter (mm) *Beta=4.5 - 5.147*

	1	2	3	4	5	6	7	8	9
1	2.103	2.353	2.103	1.353	2.103	1.853	2.103	2.353	0.853
2	2.103	1.603	1.853	1.353	1.603	2.103	1.853	1.353	1.853
3	1.103	1.603	1.353	1.853	1.603	1.103	1.353	2.353	1.603
4	2.103	1.603	2.353	1.853	2.103	1.853	1.603	2.103	1.353
5	2.353	2.353	1.103	1.853	1.103	1.853	1.603	1.353	2.353
6	1.353	2.103	1.853	1.603	2.103	1.603	1.603	2.353	1.603
7	1.853	1.853	1.603	0.603	1.353	1.853	1.853	1.603	2.103

Average	1.853	1.924429	1.745857	1.495857	1.710143	1.745857	1.710143	1.924429	1.674429
Standev	0.456435	0.345033	0.429562	0.453163	0.404587	0.318105	0.243975	0.472456	0.494012

Rank Regression on X (RRX)

	4.0647	6.79	4.17	2.97	4.46	5.69	7.94	4.5	3.3
	2.0367	2.05	1.92	1.68	1.87	1.88	1.81	2.1	1.87

Max. Likelihood Est. (MLE) - UnBiased Beta & Eta

	5.7018	6.693	5.0848	4.6643	5.4873	8.3171	8.1399	5.4005	4.2688
	2.0152	2.0631	1.9036	1.6413	1.86	1.8586	1.8107	2.0984	1.8445

RRX Beta	4.0647	6.79	4.17	2.97	4.46	5.69	7.94	4.5	3.3
MLE Beta	5.7018	6.693	5.0848	4.6643	5.4873	8.3171	8.1399	5.4005	4.2688

Biased Factor(Gn) is 1-(1.346/n)-(0.8334/n^2) :Where n=7 Gn=0.79070614

*Biased beta = MLE Beta * Gn*

10	11	12	13	14	15	16	17	18
1.603	1.353	1.353	1.603	2.603	2.353	1.853	2.103	1.853
2.103	2.603	1.603	2.353	1.103	0.853	2.103	2.353	1.853
1.853	1.353	1.603	1.603	2.103	1.853	2.103	1.853	2.353
0.853	2.103	1.103	1.853	1.853	2.603	2.103	2.353	2.853
1.603	2.103	1.853	1.353	1.353	1.603	1.103	1.853	0.603
1.853	1.103	2.103	2.603	1.353	1.853	2.603	2.353	1.353
2.103	1.603	1.603	1.603	2.103	2.603	1.853	1.603	1.603

1.710143	1.745857	1.603	1.853	1.781571	1.960143	1.960143	2.067286	1.781571
0.429562	0.537299	0.322749	0.456435	0.534522	0.626783	0.453163	0.303746	0.717552

avg.avg= 1.791492
st.dev avg 0.444397

3.79	3.66	5.32	5.05	3.62	2.9	4.39	7.42	2.24
1.89	1.93	1.73	2	1.97	2.21	2.14	2.19	2.04

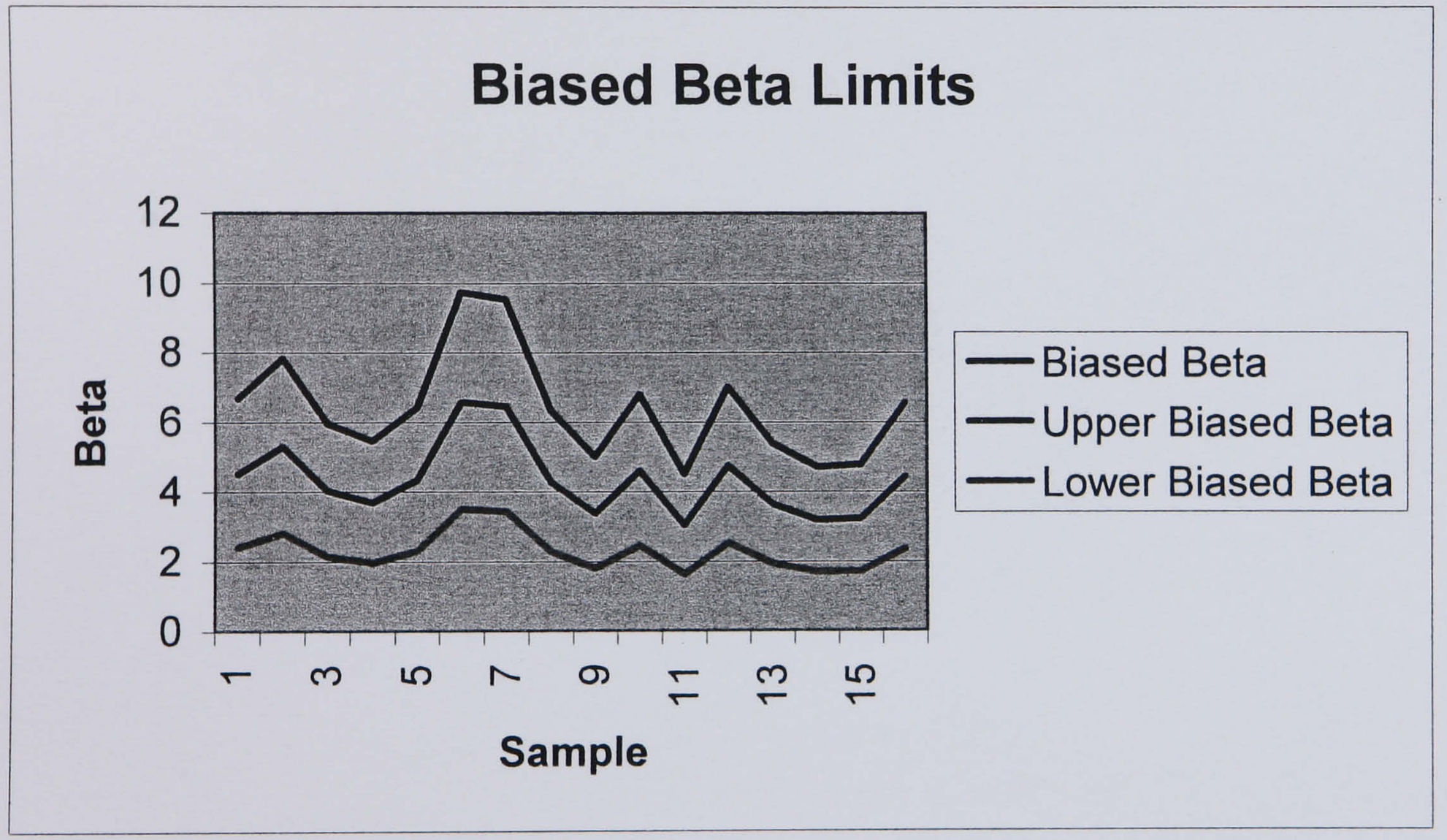
5.8108	3.8402	5.9955	4.5936	4.0239	4.083	5.59766	9.1104	2.9811
1.8551	1.9338	1.7271	2.0256	1.9693	2.1668	2.1215	2.1885	1.9958

3.79	3.66	5.32	5.05	3.62	2.9	4.39	7.42	2.24
5.8108	3.8402	5.9955	4.5936	4.0239	4.083	5.59766	9.1104	2.9811

avgB-RRX 4.570817
avgB-MLE 5.544092

Bias Beta	4.508448	5.292196	4.020583	3.688091	4.338842	6.576382	6.436269	4.270209	3.375366

Upp.Beta	6.669336	7.828733	5.947638	5.455783	6.418438	9.728426	9.521157	6.316909	4.993171
Low.Beta	2.403548	2.821381	2.143457	1.966198	2.313127	3.506007	3.43131	2.276538	1.799478



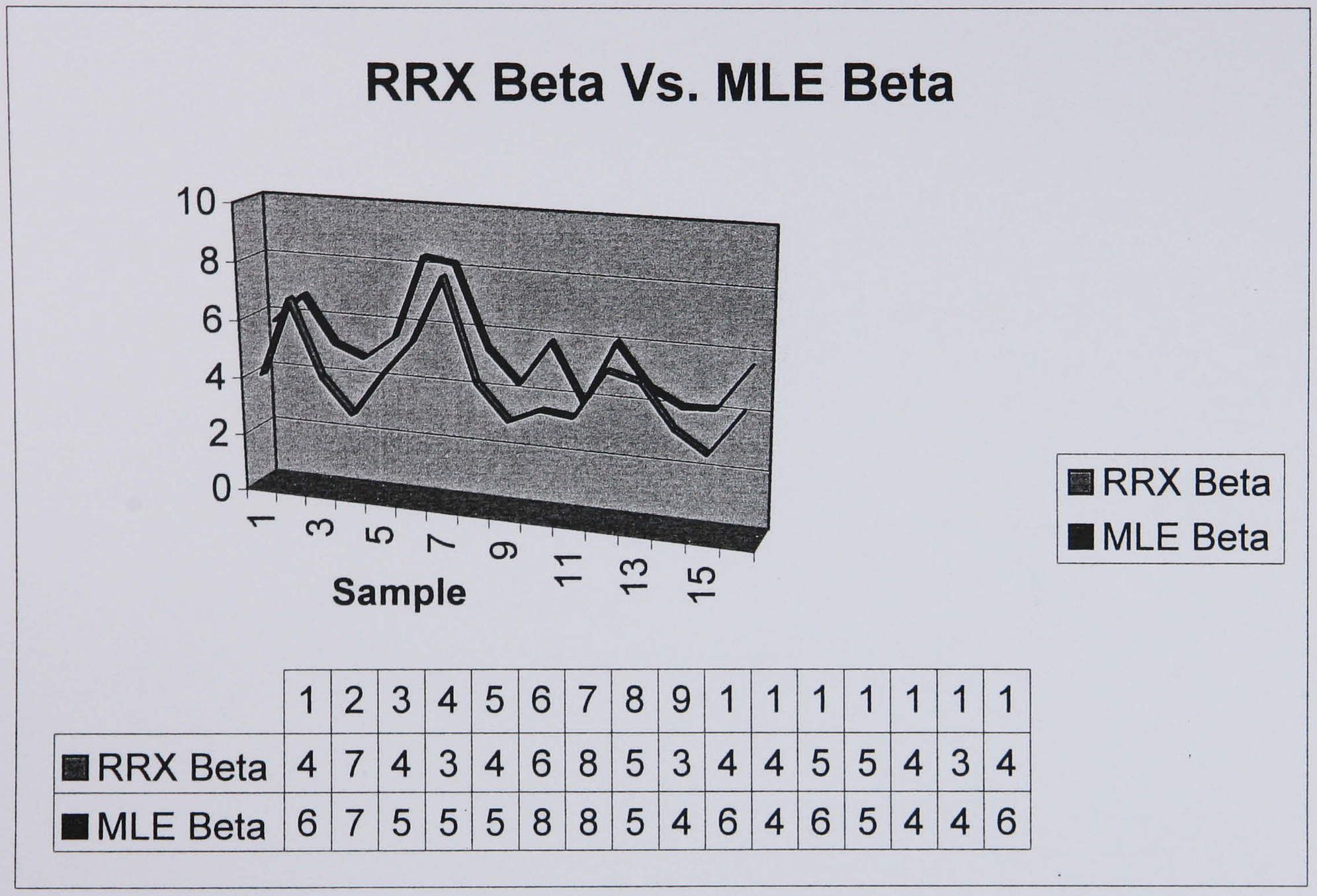
4.594635	3.03647	4.740679	3.632188	3.181722	3.228453	4.426104

avg= 4.334165

6.796833	4.491842	7.012874	5.373086	4.706714	4.775843	6.547525
2.449496	1.618806	2.527355	1.936395	1.696243	1.721156	2.359649

Bias Beta	4.508448	5.292196	4.020583	3.688091	4.338842	6.576382	6.436269	4.270209	3.375366

avgB-Bias 4.383748



	1	2	3	4	5	6	7	8	9	1	1	1	1	1	1	1
RRX Beta	4	7	4	3	4	6	8	5	3	4	4	5	5	4	3	4
MLE Beta	6	7	5	5	5	8	8	5	4	6	4	6	5	4	4	6

4.594635	3.03647	4.740679	3.632188	3.181722	3.228453	4.426104	7.203649	2.357174

Appendix D

Published Papers

Weibull Analysis used as Statistical Process Control

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Summary

In this paper, Weibull analysis will be examined and used to establish a durable understanding of the implication of replacing Statistical Process Control methods with Weibull method regarding using small sample size (n<6). SPC failed to control and detect variations with small sample size, therefore, Weibull techniques will be a useful method to use specially in high cost inspection items to reduce time and cost. Weibull beta parameter will be examined and a clear understanding of the relation between the shape factor and the sample size will be studied in order to constrain the limits for the shape factor, which will be contributed with specified sample size.

Introduction

Quality control charts are effective Statistical Process Control (SPC) methods to control quality by detecting any distortion or non-conforming criteria within the manufacturing [1]. Control charts ($\overline{X}$, R charts) measure quality characteristics through sampling. Sample size is a vital parameter when using such charts; large sample size (usually n>30) is needed to obtain an acceptable prediction with a satisfactory level of confidence. Due to some restrictions in the inspection and controlling quality, small samples are preferable to reduce the cost of destructive inspection especially in high cost elements, also not to waste time, which is highly contributed to the overall cost.

Theories now exist to develop new mathematical or numerical models, which is analysed and tested to replace the conventional SPC method, as these conventional methods give predictions on samples not on the overall population. Taking into considerations that these methods will be analysed on small sample size where the conventional SPC methods failed to establish a high level of confidence solution concerned with the criteria of choosing small sample size (n ≤6). Further on, these techniques will be developed to successfully provide an advance durable model, which will be developed -to replace SPC- to overcome any manufacturing quality and unreliability problem. This model aims to satisfy the quality standards, manufacturing specifications and provide a profitable, confident and reliable method of establishing reliability using small sample size.

Theory

Monitoring the level of achieving desired specification within the manufacturing is a requisite aspect to control quality. Two main hazardous criteria should be tackled and omitted from any production process, as they represent a risky situation on quality. These two criteria are: (1) deviations from target specifications, and (2) excessive variability around target specifications. In 1920's, Dr. Walter A. Shewhart set elaborated charts, which test, monitor and control the variability within a process. Shewhart developed three main control charts to detect various variabilities and distortions in the process. These charts are:

1- Shewhart control charts for measurable quality characteristics (know as Variables charts)-
   i- $\bar{X}$, R chart (average and range chart).
   ii- $\bar{X}$, *s* chart (sample average and standard deviation chart).
2- Shewhart control chart for fraction rejected (*p* chart).
3- Shewhart control chart for number of non-conformities (*c* chart).

The most commonly used charts in manufacturing are $\bar{X}$, R charts. $\bar{X}$, R charts are charts to measure the variability, which means when a record is made of actual measured quality characteristic; then the quality is said to be expressed by Variable. Specification of variables may have limits of control (UCL-Upper Control Limit, LCL-Lower Control Limit). Figure 1 shows $\bar{X}$, R chart based on standard SPC formula for calculating control limits [2]

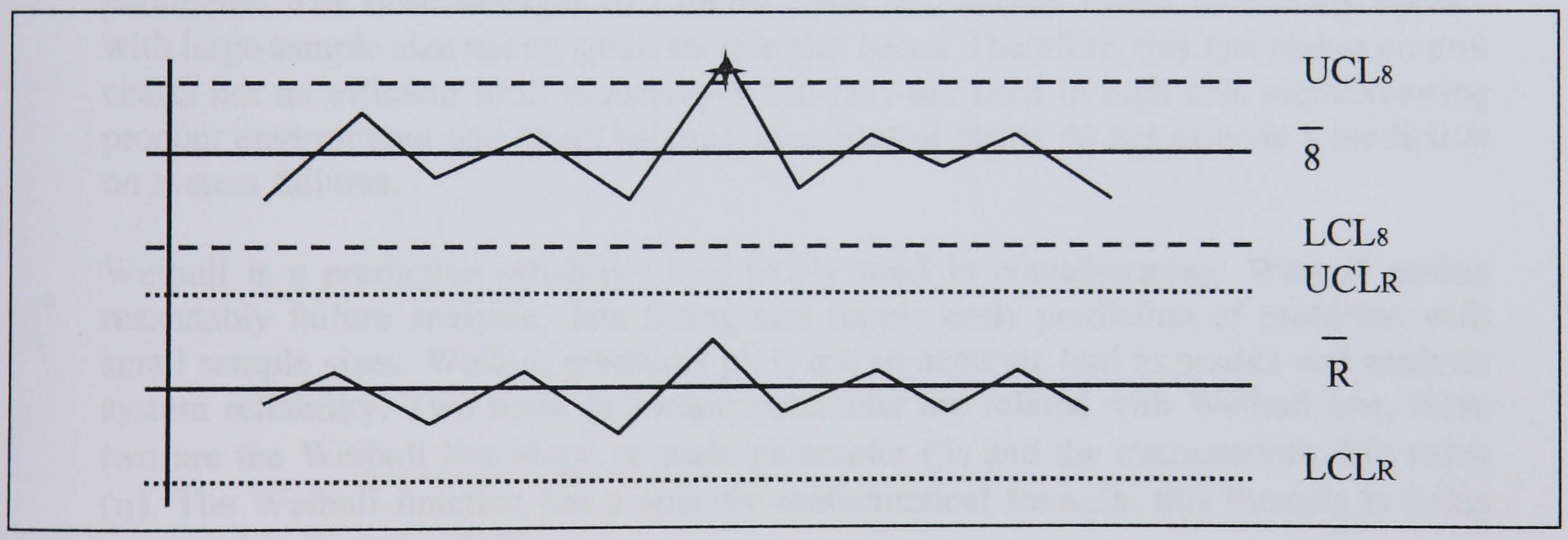

Figure 1 - *$\bar{X}$, R chart*

$\bar{X}$, R charts can be a useful tool to control a process quality, they indicate lack of control if any point is out of the boundary limits. Therefore, the system will not be a constant-cause system, because causes of variations are present (as it can be seen in the $\bar{X}$ chart in figure 1 at the star point, which is out of the UCLx).

Noting from the equation of UCLR and LCLR both limits depends on D3, D4 factor. D3, D4 have various numerical values depending on the sample size. From figure 2 D4, D3 converge to specified numerical value at large sample size. Therefore, the result of

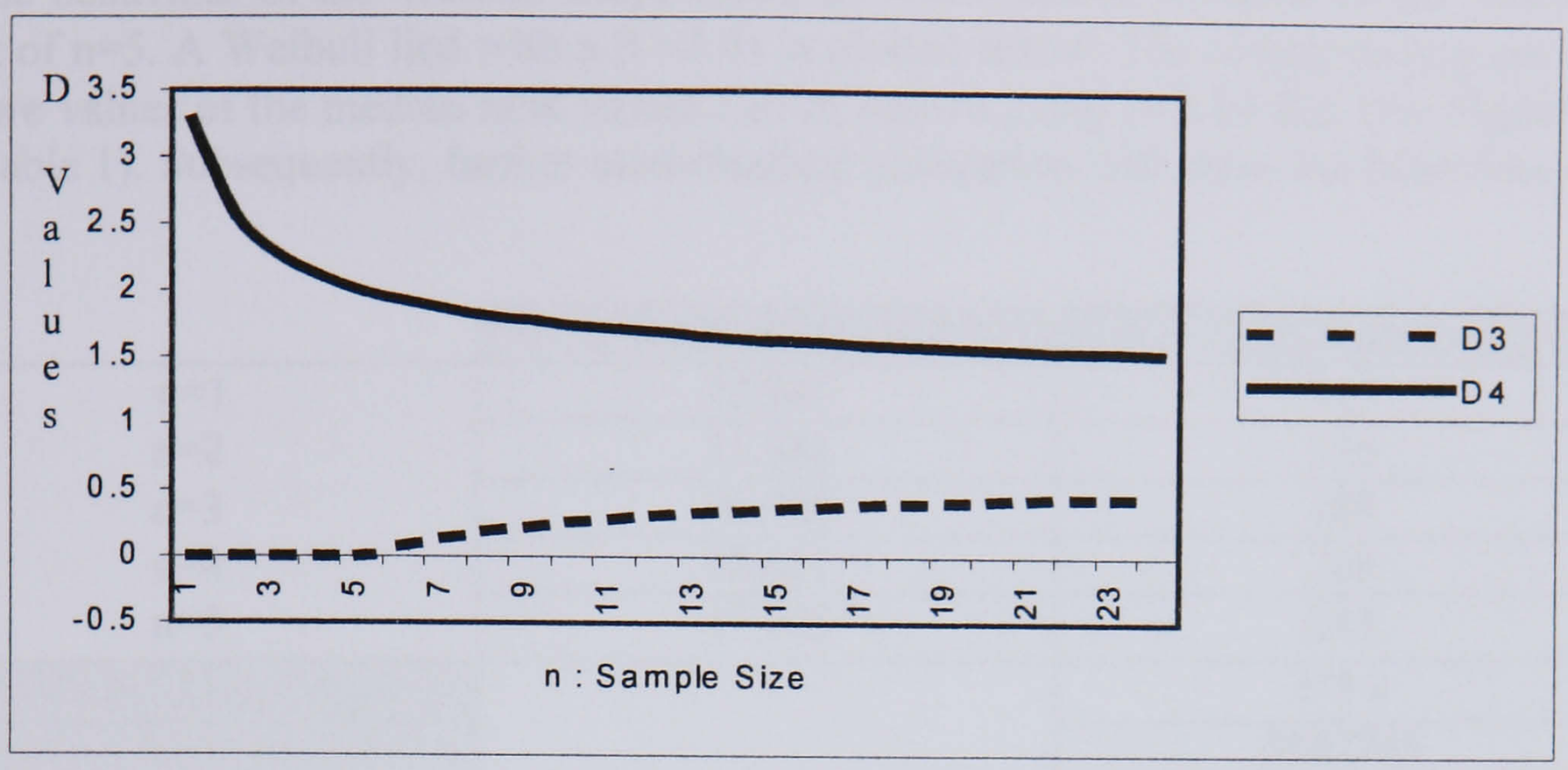

***Figure 2** – D3, D4 curves*

UCLR and LCLR calculations will be achieved with high confidence and approved certainty.

Control charts provide a true analysis of the process or system. It keeps superintendence on variable and acquaints any variability of specified variable within the system. Control charts are successful tools to respond to any fluctuation with in the system parameter. The disadvantages of Control chart are, control charts effectively operate with large sample size not on small sample size bases. Therefore, this fact makes control charts not an efficient tool; especially when they are used in high cost manufacturing product environment and small batches; also control charts do not provide a prediction on system failures.

Weibull is a predictive reliability tool newly used in manufacturing. Weibull assists reasonably failure analysis, data fitting and supply early prediction of problems with small sample sizes. Weibull graphical plots are an accurate tool to predict and analysis system reliability. Two main important parameter are related with Weibull line, these two are the Weibull line slope or scale parameter (β) and the characteristic life value (η). The Weibull function has a specific mathematical formula; this formula is being presented in Equation 1[3].

$$R(t) = 1 - F(t) = e^{-(t-\gamma\ /\eta-\gamma\ )^{\beta}}, \qquad \gamma \text{ is location parameter} \qquad \textbf{\textit{Eq. (1)}}$$

Application of Weibull

Data which is normally distributed will have a shape factor (β) of 3.44. Nevertheless, small sample sizes produce various β values, which can be explained from a comparison with D3, D4 constants used in SPC, see Figure 2 [4]. In this paper, small sample size will be discusses with respect to Weibull techniques to achieve a durable understanding

of the behaviour of the Weibull shape factor β. This research is based on the median rank of n=5. A Weibull line with a β =3.44 is plotted below. The corresponding age of failure values of the median rank values can be known using β=3.44 line (see Figure 3 & Table 1). Subsequently, further mathematical calculation will show the behaviour of β.

	Median Rank	Age Failure
n=1	12.945	113
n=2	31.381	150
n=3	50.000	180
n=4	68.619	208
n=5	87.055	245
8		179.2
σ		50.97745

***Table 1-** Median rank Vs. Age failure for n=5*

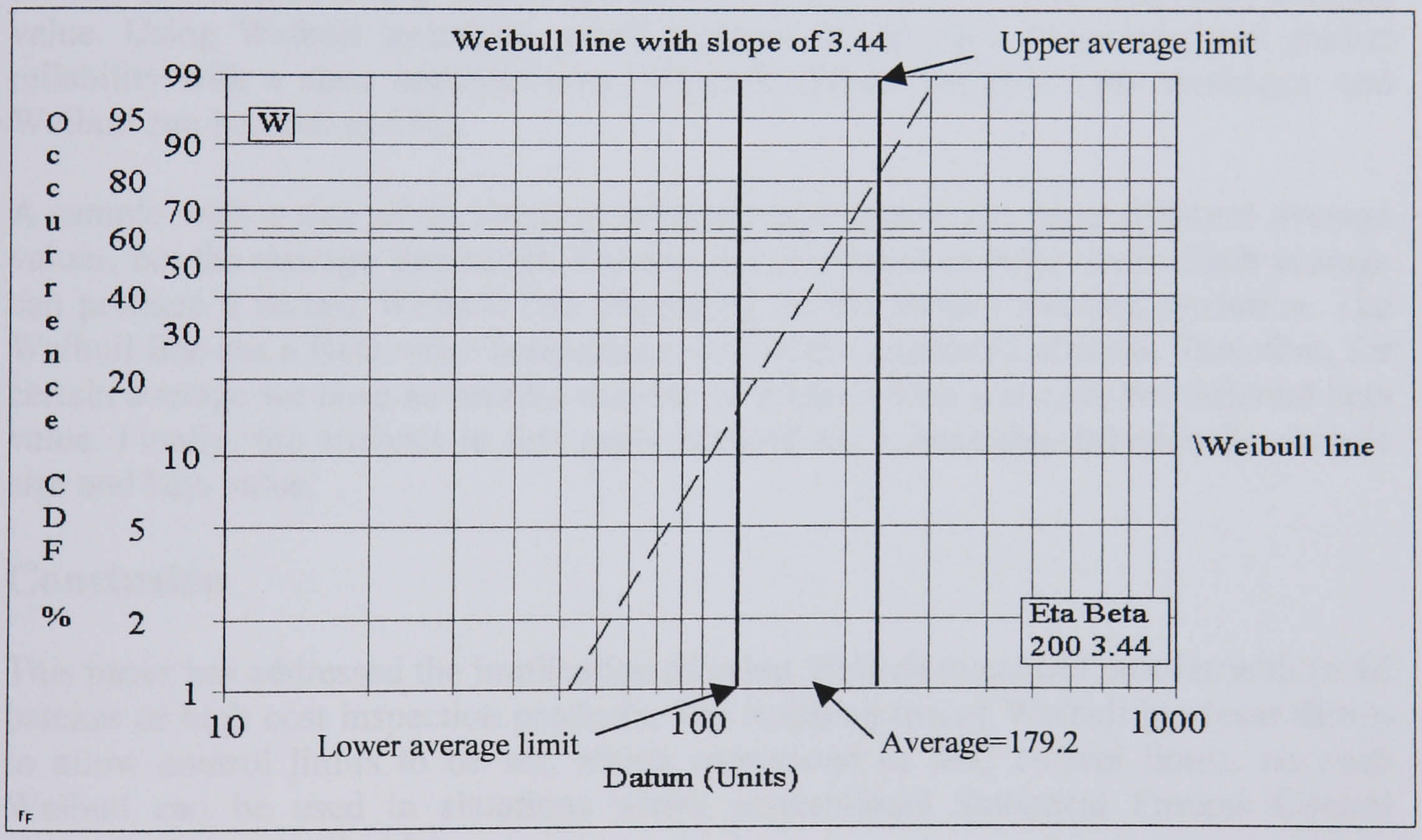

***Figure 3** – Weibull analysis plot*

For a given σ, $\overline{R}$ can be calculated ;

$$\sigma = \overline{R}/d_2 \Rightarrow \overline{R} = d_2 \times \quad = 50.977 \times 2.326 = 118.573 \qquad \cdots \textit{Eq. 2}$$

Which produces average control limits

$$\overline{X} \pm \left(\overline{R}/2\right) = 179.2 \pm \left(118.573/2\right) = 238.7865 \cong 239 \;\&\; 120.2135 \cong 120 \qquad \cdots \textit{Eq. 3}$$

From Weibull for a given confidence :

$$R(t) = e^{-\left(t/\eta\right)^{\beta}} \qquad \cdots \textit{Eq. 4}$$

With a Reliability of 98% $\rightarrow$ $t = 130$, $0.98 = e^{-\left(130/200\right)^{\beta}}$

The limiting β values for n = 5 will be :

$\therefore\ \beta = 9.057784124 \cong 9.06$

$\Rightarrow \beta \in [\,9.06\,, \infty\,]$ for a sample size of 5 (See D_3 for n = 5)

In figure 3, an average line has been plotted showing the upper and lower average limits. Assuming a reliability of 98%, beta can have various values but these values must belong to calculated β range, which is [9.06, ∝]. Therefore, it has been proved that for any given sample size with a specified σ, β can has a specific minimum limiting value. Using Weibull technique, small samples can be used to achieve and predict reliability with a clear understanding of the limitation conditions this technique, and Weibull can be used as SPC.

A sample with a size of 5, which is normally distributed, can have different average values, but the average should fall between the calculated average limits. Each average can produce a certain Weibull line depending on the sample standard deviation. The Weibull line has a Beta value constrained with in the calculated interval. Therefore, for certain average we have an infinite number of Weibull lines and each has different beta value. Finally, the analysis in this paper showed the relationship between the sample size and beta value.

Conclusion

This paper has addressed the implication of using Weibull to control process with small batches or high cost inspection products. The resulting use of Weibull has been shown to allow control limits to be set, which correspond to SPC control limits. As such Weibull can be used in situations where conventional Statistical Process Control methods are not applicable.

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Measurement of adhesive strength in sheet aluminium/epoxy joints by crack propagation techniques

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Keywords: Crack, Aluminium/Epoxy, Joint, Lap shear, BWT.

ABSTRACT

Adhesive bonding can replace conventional joining techniques such as mechanical fastening, soldering, brazing and welding in appropriate manufacturing situations. Advanced adhesive technologies can offer improvements in productivity, cost, strength and durability. The greatest drawback to the use of organically based adhesives is that they are still suspect in hostile environments and at high temperatures. Surface treatments of aluminium alloy sheet with siloxane are known to enhance bond strength of joints constructed of aluminium sheet material with organic adhesives. The present paper tests strength of siloxane surface treated aluminium alloy sheet with a rubber-toughened epoxy (Cytec FM73) as the adhesive. The Boeing wedge test is used with a range of temperatures, exposure times and atmospheric relative humidities to model hostile environments. Crack growth at the interface over time shows good strength retention. The results are also compared with lap shear tests to show general agreement. Results of the mechanical tests are related to the mode of failure of the joint.

1. INTRODUCTION

Metal to metal joining of 2024-T3 aluminium with epoxy based adhesives have been used in a variety of aerospace applications (1). Strength tests to date have shown that durability of aluminium joints depends on several factors including the type of alloy, the pre-treatment, the primer if used, the adhesive and the environment to which the structures are exposed(2). Many surface treatments have been developed to increase the initial strength and durability of bonds to aluminium alloys (3). Recent investigations into methods for improving the strength of adhesive bonds have used a polyether siloxane as part of the surface pretreatment (4). These investigations using FM73 film adhesive indicated an increase in strength retention particularly for short-term durability tests.

In the present work, polyether siloxane only and no treatment conditions are studied using Boeing Wedge Test investigation (BWT). Prepared BWT specimens using 2024-T3 aluminium with FM73 adhesive are assessed after exposure to a harsh condition. Standard investigations are carried out at 33%, 50% and 96% RH (Relative Humidity), performed at 20°C and 35°C to analyse strength retention behaviour (3). These results are also compared with lap-shear strength test data; where specimens were exposed to 1000 hour durability in 50°C at 96% RH.

2. EXPERIMENTAL

2.1. Materials

The adherend material was 2024-T3 unclad aluminium, solution heat-treated, cold worked and naturally aged to a stable condition). The 3mm thick aluminium sheet was cut into strips of 150mm x 25mm. The adhesive used was Cytec FM73, a toughened single part epoxy adhesive supplied as a 0.25mm thick film and cured at 120°C under a load of 200KPa (The FM73 film adhesive is a general purpose aerospace epoxy to be used from -55°C to 82°C). The siloxane used was a polyether type supplied by Th.Goldschmidt AG.

2.2. Specimen preparation and measurement

All samples were degreased using acetone prior to preparation/assembly. Siloxane was deposited onto the aluminium surface by flooding the specimen to ensure complete coverage, followed by removal of excess siloxane with a lint-free disposable cloth. Each set of aluminium samples for particular test regimes of adhesive type and durability was prepared together in batches of six samples. Bond line thickness was controlled using two 0.1mm thick steel wires across the adhesive area prior to assembly. All joints during the adhesive curing process were subjected to a load of 200 kPa that was applied by a compression spring incorporated within the clamping system (5). Assembled specimens were cured as recommended by the manufacturers. Specimens were allowed to cool to room temperature, prior to being placed in an environment chamber for durability testing and then joint strength testing. All specimens were tested in a harsh environment using a range of saturated salt solutions in a closed system at different temperatures. Potassium sulphate was used to obtain 96%RH, with Sodium dichromate and Magnesium chloride to obtain 50% and 33% RH respectively.

2.2.1. BWT investigations

Standard specimens were constructed (6), which were made of 150mm x 25mm x 3mm strips as shown in Figure 1. The adhesive was placed in between the substrates leaving a 19mm gap for wedge insertion. The crack length was measured which gave the strength retention data of the joint as a function of time.

2.2.2. Lap-shear investigations

Joints were assembled with a 12.5mm overlap, then stacked on a special clamping jig using guide pins on a metal base to control dimensional changes. This method minimised alignment variation as the load was applied during the curing process-See Figure 2. Maximum lap-shear strengths of the joints were measured with a constant cross-head speed of 0.42 mm/s (1 inch per minute).

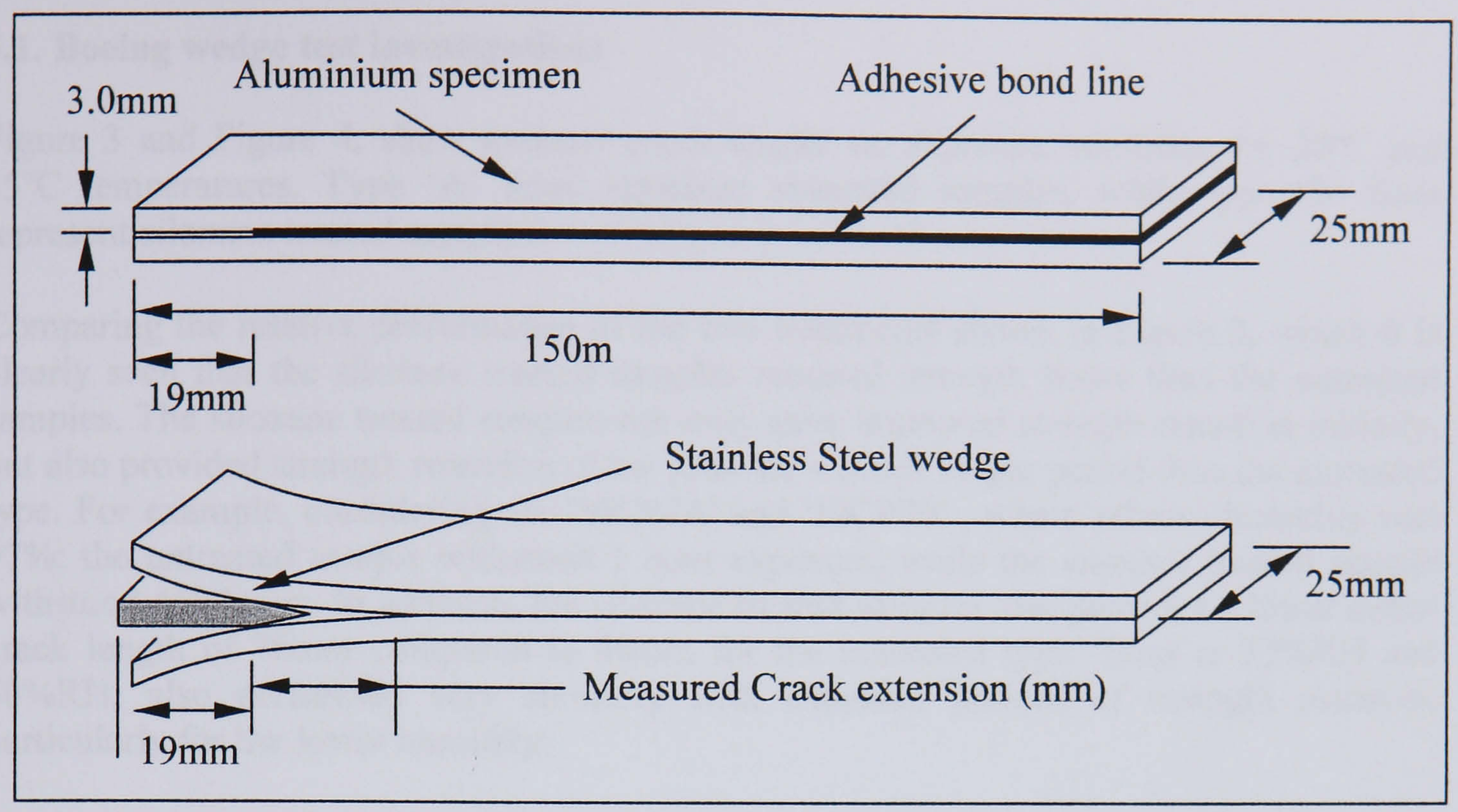

Figure 1 Boeing wedge test specimen (6)

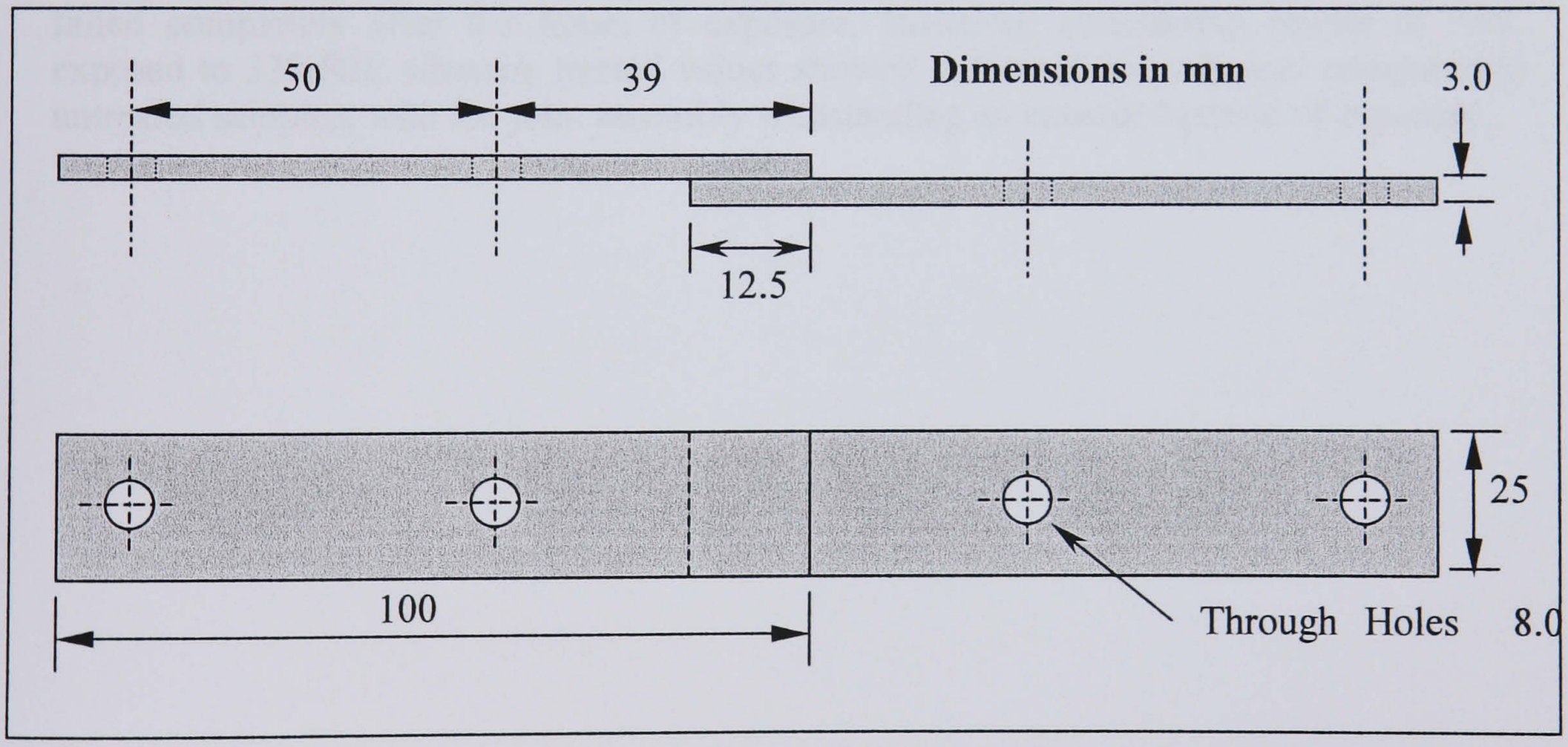

FIGURE 2 Lap Shear Test

3. RESULTS AND DISCUSSION

3.1. Boeing wedge test investigations

Figure 3 and Figure 4, show average crack length vs. exposure condition for 20°C and 35°C temperatures. Type 'A' lines represent untreated samples, while type 'B' lines represent siloxane treated samples.

Comparing the relative performance of the two treatments shown in Figure 3, which it is clearly seen that the siloxane treated samples retained strength better than the untreated samples. The siloxane treated samples not only gave improved strength retention initially, but also provided strength retention of the joint for a much longer period than the untreated type. For example, considering the '20C97A' and '20C97B' , where relative humidity was 97%; the untreated sample withstood 1 hour exposure, while the siloxane treated sample withstood 450 hours. In addition, the siloxane treated samples also provided a lower initial crack length of 76mm compared to 90mm for the untreated type. Tests at 33%RH and 50%RH, also performed very similarly with extended periods of strength retention particularly for the lower humidity.

Figure 4 clearly shows a significant difference between the two treatments. Similar to the tests performed at 20°C, siloxane treated samples show improved strength retention compared to the untreated type. Raising temperature from 20°C to 35°C, produced an aggressive atmosphere which at high humidity conditions caused the joints to fail much sooner. This is clearly seen comparing the results of '35C96A' and '35C96B', where siloxane treated samples lasted 1 hour before complete failure and the untreated samples failed completely after 0.3 hours of exposure. However, considering results of 35°C exposed to 33%RH; siloxane treated values showed a distinct improvement compared to untreated samples, with the joint assembly withstanding an extended period of exposure.

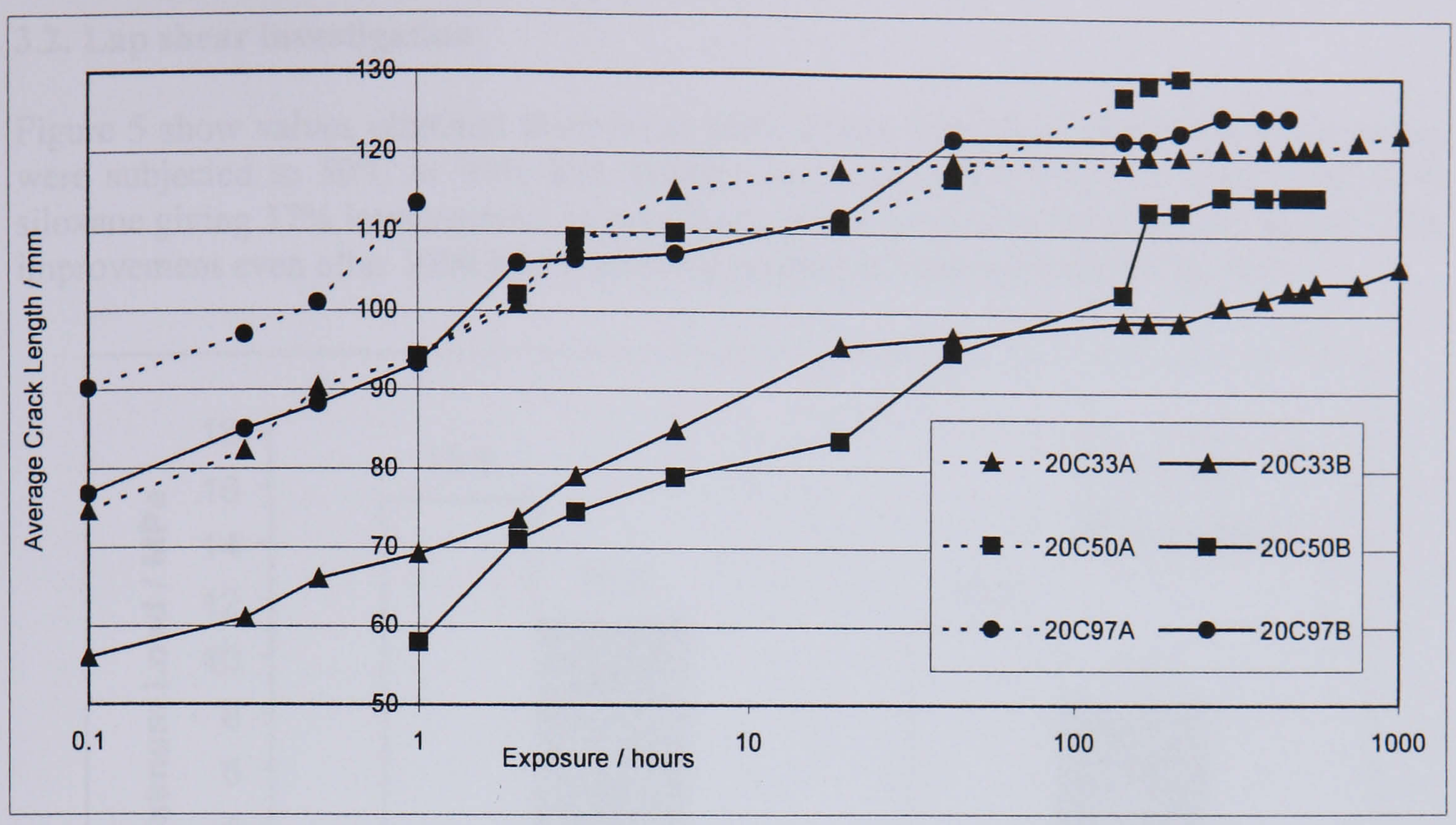

Figure 3 Crack length vs. exposure results at 20°C temperature

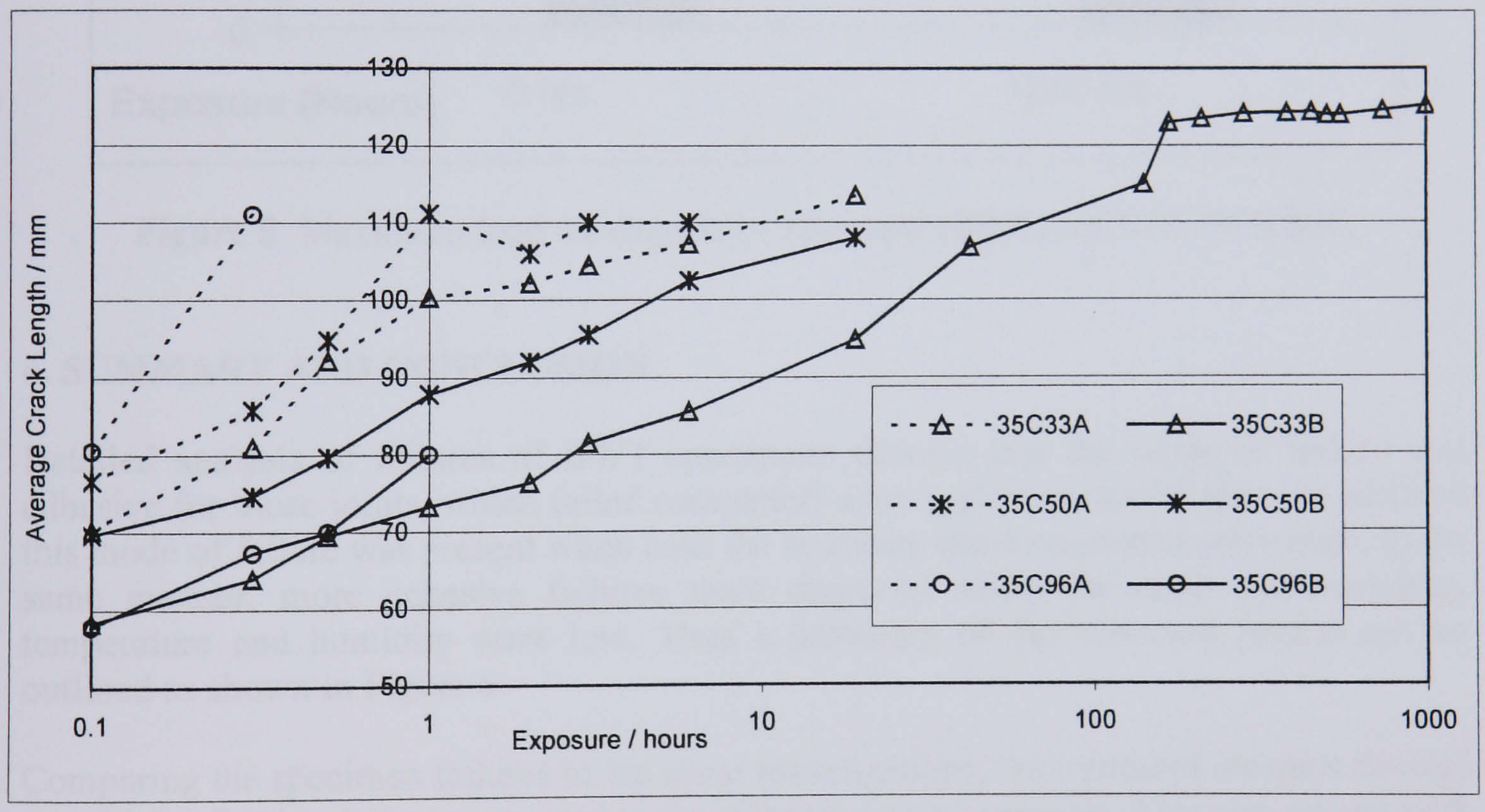

Figure 4 Crack length vs. exposure results at 35°C temperature

3.1.1. Failure analysis of wedge test specimens

For the Purpose of this paper, pairs of failed specimens of Boeing wedge tests samples picked randomly, from both 20°C and 35°C exposures showed high adhesive failure with increased temperature and humidity. Mixed mode failures and cohesive failure were also observed for some conditions of exposure and surface treatment. Extended exposure at low temperature or low humidity also showed a build up of oxide. Although increased humidity showed a clear adhesive failure, certain areas of the failed specimens (especially at low temperature) show a mixed mode failure of cohesive and adhesive; or clearly, a cohesive failure.

3.2. Lap shear investigation

Figure 5 show values obtained from these tests, where specimens after adhesive bonding were subjected to 50°C at 96% RH. Results show the FM73 adhesive when used with siloxane giving 37% improvement at zero hours compared to untreated specimens and 27% improvement even after 1000 hours, showing agreement with Boeing wedge tests.

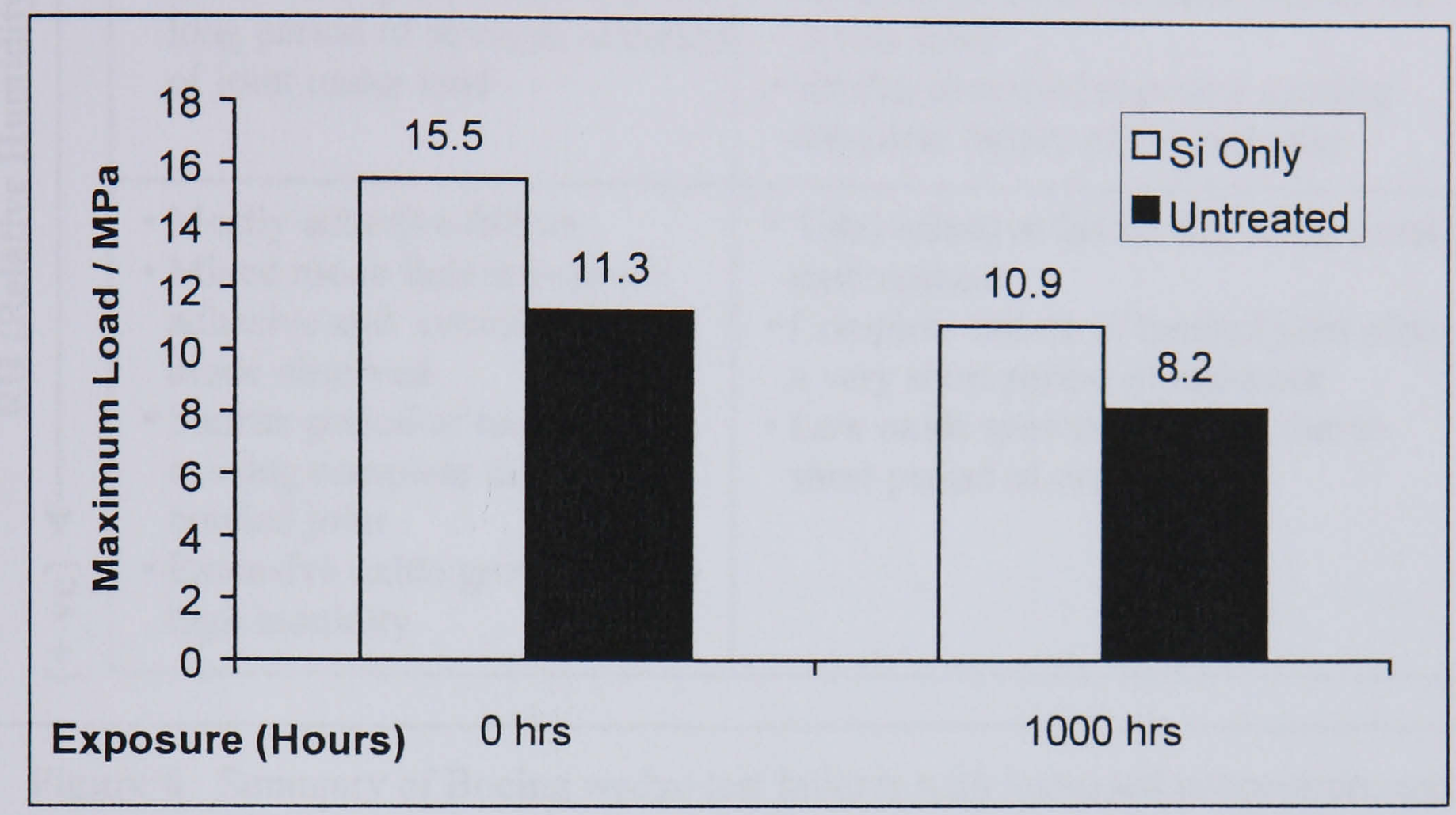

Figure 5 Maximum load vs. exposure (0 hrs and 1000 hrs at 50°C 96% RH)

4. SUMMARY AND CONCLUSION

Detailed analysis of failures of BWT specimens showed that the mode of failure was adhesive for those joints, which failed completely after a short period of time. In addition this mode of failure was present when both the humidity and temperature were high. In the same manner, more cohesive failures were observed when the same two variables, temperature and humidity were low. Thus a summary of the observed results can be outlined as shown in Figure 6.

Comparing the specimen failures in lap shear investigations, the untreated samples showed very high adhesive failure compared to the siloxane treated samples. This also agreed with the relevant measured results of decreased crack growth and extended periods of exposure prior to complete failure, for siloxane treated samples in the BWT investigation; and provides useful evidence to show that siloxane treated samples improve surface treatment characteristics. The low rate of crack growth, joints withstanding extended periods of exposure under tensile load and increased cohesive failure of joints all lead to the siloxane treated samples producing joints of high strength and durability. Additional testing is needed to identify the best composition of adhesive and siloxane to obtain a successful procedure for adhesive bonding of aluminium.

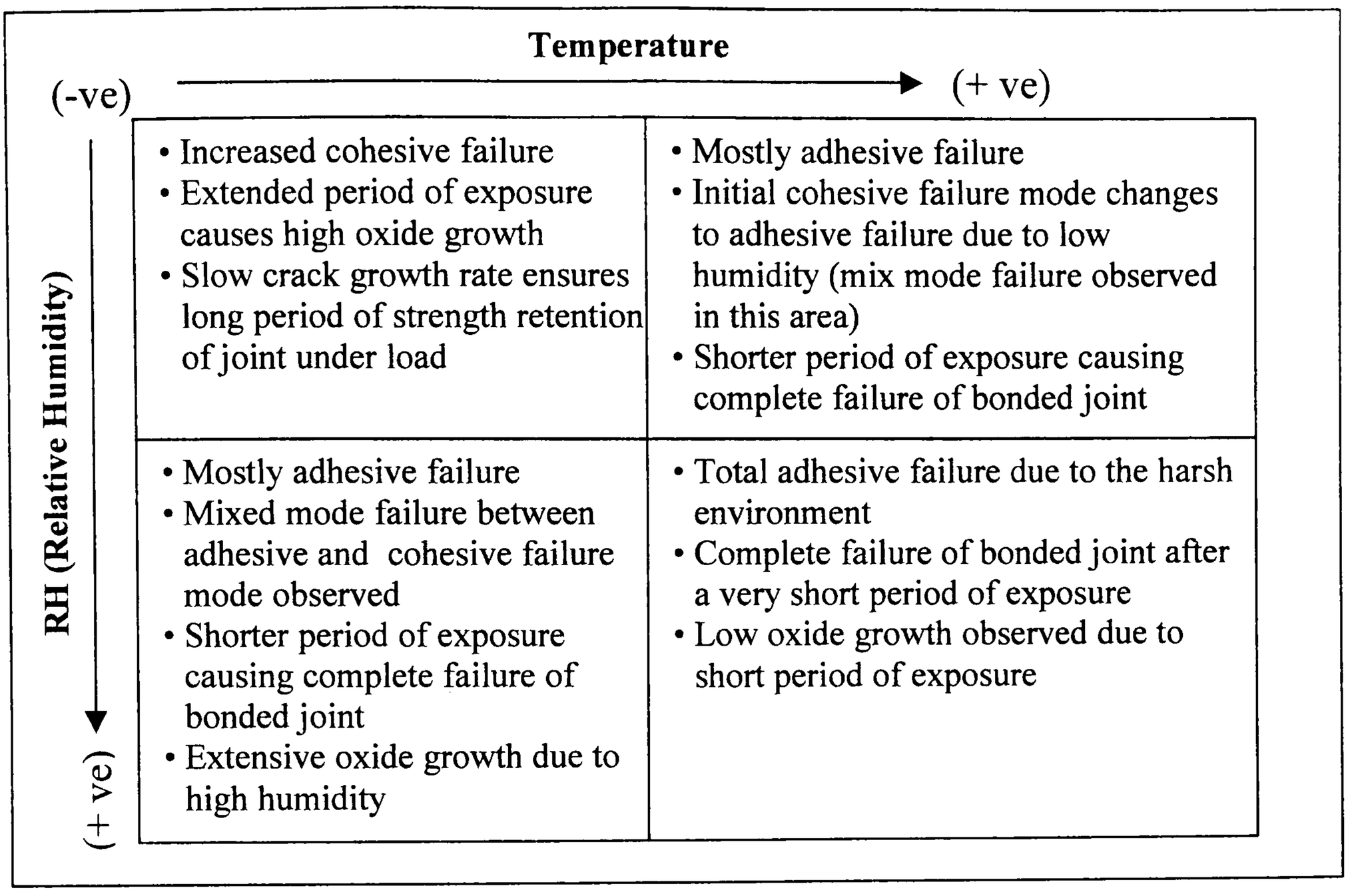

Figure 6 Summary of Boeing wedge test failures with increased temperature and humidity

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A Comparison of Weibull Parameter Estimation Techniques in Small Sample Inspection

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ABSTRACT

Small sample inspection in manufacturing has traditionally relied on conventional statistical analysis to predict the parent population characteristics [1]. The theory of t-distributions and the central limit theorem assumes samples will have near normal distributions, which cannot be guaranteed. What the conventional approach has failed to address is the implications of the small sample inspection and predicting function parameters with acceptable accuracy. Weibull analysis applied to sample inspection can be shown to allow predicting the critical parameters values. Nevertheless, small samples implementation in Weibull analysis shows a clear lack of predicting such parameter in reasonable accuracy, which does not correspond with the theory behind Weibull distribution. Small samples will be used to achieve a clear understanding of the estimation of Weibull shape parameter and calculate accurate confidence intervals constraining the estimation range.

1 INTRODUCTION

Often, a single or a best estimate of a process parameter is needed. Also it is important to determine an interval or range, in which this single point estimate has a high probability that the parent process universe will fall in. In other words, sampling is required in order to predict the population parameter by estimating this parameter using the sample collected and this estimate will reflect the overall population parameter. Confidence limits are a good way to narrow the estimation with in certain parameter and it achieved a durable accuracy of this prediction. The confident interval has two limits, Upper Limit and Lower limit. In probability, having a 95% confidence limit means there is 0.025 chance that interval will not include the population parameter value because the interval fell below it, and 0.025 chance that the interval will not contain the population parameter value as the interval fell above it. Thus the confidence interval is a balanced interval. Sample size is a vital issue when calculating the confidence interval. The accuracy of the estimation depend on the sample size, it is proportionally related with the accuracy, which means, if the sample size if high the accuracy of the interval estimation is high as the interval converge to a smaller range of probabilities.

Weibull distribution is an effective way to be used in sampling, it can predict the lifetime and failure occurrence within a process, therefore, it has many advantages over the conventional statistical control methods. Consequently, Weibull is a predictive reliability tool used in manufacturing. As Weibull facilitate failure calculations, data fitting and supply early prediction of problems with small sample sizes. Two main important parameters are associated with Weibull distribution; these two are the Weibull line slope or scale parameter

$$R(t) = 1 - F(t) = e^{-(t-\gamma/\eta-\gamma)^{\beta}}, \qquad \gamma \text{ is location parameter} \qquad \textit{... Eq. (1)}$$

(β) and the characteristic life value (η). The Weibull function has a specific mathematical equation; this formula is being presented in Equation 1[2]

2 THEORY

Weibull distribution theory dictates that a normally distributed data has a Weibull shape parameter of 3.44. However, in practice that does not apply especially when a small sample size is used. Knowing, that manufacturing strategy tends to have the small sample size testing approach in order to save money and time. Using the confidence limits may help solving the small sample size problem and provide a reasonable estimation of the parameter. Data normality can be tested through many methods; one of these methods is Ryan-Joiner normality test. Generally, it is not that accurate to use this technique with small sample but in this research, this low accuracy will be accepted for such research purpose. If x is a normal random variable, then the probability distribution of x is [3]

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \qquad -\infty < x < \infty \qquad \textit{...Eq. (2)}$$

2.1 Confidence intervals for population standard deviation and mean

Confidence limits for a universe variance or standard deviation are most easily obtained when the universe is normal. The sample values of $\frac{(n-1)s^2}{\sigma'^2}$ form a X^2 - distribution with $v = n-1$, Hence it has the following:

$$\text{Prob.}\left(x^2_{0.975} \le \frac{(n-1)s^2}{\sigma'^2} \le x^2_{0.025} \right) = 0.95 \qquad \textit{...Eq.(3)}$$

but it can be written as

$$\left[\frac{(n-1)s^2}{x^2_{0.025}} \le \sigma'^2 \le \frac{(n-1)s^2}{x^2_{0.975}} \right] = 0.95 \qquad \textit{... Eq. (4)}$$

Hence $\frac{(n-1)s^2}{x^2_{0.025}}$ and $\frac{(n-1)s^2}{x^2_{0.975}}$ are the lower and upper 0.95 confidence limits for σ'^2.

The confidence limit for the mean with a normal distribution with unknown mean and unknown variance is

$$\overline{X} - t_{\alpha/2,\,n-1}\frac{S}{\sqrt{n}} \le \mu \le \overline{X} + t_{\alpha/2,\,n-1}\frac{S}{\sqrt{n}} \quad \text{...Eq.(5)}$$

where $t_{\alpha/2,n-1}$ denotes the percentage point of the t - distribution with n - 1 degrees of freedom such that $P\{t_{n-1} \geq t_{\alpha/2,n-1}\} = \alpha/2$.
Therefore, the sample average should be calculated and then substituted within the mean confidence limits.

2.2 Maximum Likelihood Estimation (MLE)

Likelihood function is one of the common methods exploited in estimating Weibull distribution parameters. The likelihood function has many sub functions, which serve the estimation methods, these sub-functions are: marginal, partial and maximum likelihood methods.

It can be deduced that the likelihood function is the joint probability of an observed sample as function of unknown parameter. It is more convenient to calculate the logarithmic values of the likelihood function that to calculate the function itself. Plotting the likelihood function will be greatly simplified since the likelihood are normally calculated by multiplying the probabilities of independent events and by considering the logarithm of the function it can eliminate the constant term of the logarithm. The likelihood function usually has a maximum at specific values of the distribution parameters. These values of parameters are more likely to give rise to the data that other values. Therefore, using a maximum likelihood method will provide a best single point estimate in predicting a parameter of the needed function.

Maximum likelihood method objective is to determine the best estimates of certain function parameters. Establishing the likelihood function for the data and obtaining its logarithmic expression can reach such objective. This expression is then differentiated with respect to the parameters, and the resulting equation is set to equal to zero. The resulting equations are then solved simultaneously to obtain the best estimates of the parameters that maximize the likelihood function and such estimate is called the Maximum Likelihood Estimate (MLE).

The probability density function (p.d.f.) of the Weibull distribution is given by

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \exp\left[-\frac{t}{\eta}\right]^{\beta} \text{ ...Eq.(6) where } F(t) = 1 - \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] \text{ and } h(t) = \frac{\beta}{\eta^{\beta}} t^{\beta-1} \text{ ... Eq.(7)}$$

Using the maximum likelihood procedure, it can be shown that $\hat{\beta}$ is the solution of Equation 8[4]

$$\frac{\sum_{i=1}^{n} t^{\hat{\beta}_i} \ln t_i}{\sum_{i=1}^{n} t^{\hat{\beta}_i}} - \frac{1}{\hat{\beta}} = \frac{1}{n}\sum_{i=1}^{n} t_i \qquad \text{...Eq.(8)}$$

3 CONFIDENCE INTERVALS APPLICATION

A rod diameter is being measured and 10 samples are being collected. Each sample has a sample size of 5 readings -See Table 1. By using these samples, the population mean and variance (standard deviation) will be estimated with a 95% confidence interval. A normality test was used to check for normality characteristic see figure 1. Also a Weibull plot (using WinSmith software- refer to figure 3) will allow the calculation of beta value of the sample. Table 2 shows the resulted parameters of the rod diameter testing.

Table 1 Rod diameter testing samples

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
	0.65	0.75	0.75	0.60	0.90	0.60	0.75	0.60	0.65	0.60
	0.70	0.85	0.80	0.70	0.75	0.75	0.80	0.70	0.80	0.70
	0.65	0.75	0.80	0.70	0.65	0.75	0.65	0.80	0.85	0.80
	0.65	0.85	0.70	0.75	0.85	0.85	0.75	0.75	0.85	0.80
	0.85	0.65	0.75	0.65	0.80	0.70	0.75	0.75	0.75	0.65

Using the known statistics the standard deviation and the mean of each sample can be calculated and the result can be seen in the table 2. This table also shows the upper and lower limit of the standard deviation and the mean, which are been calculated by the equation 2, 3, 4 and 5. Noting that the X value (ν=4) 0.975 and 0.025 are from X-distribution table = 0.48 and 11.14 respectively; also, that the 95% accuracy is used which means that the alpha factor (α) is 0.05, therefore, 100(1-α)%=95%. In the calculation of 95% mean confidence limits, it can be seen that (α) is 0.05; and from the t-distribution tables, the value of $t_{0.025,\,4}$ is equal to 2.776. Refer to table 2 for the variance estimation of the samples.

Table 2 Rod diameter parameters

	σ^2	σ^2 Upper	σ^2 Lower	μ	μ Upper	μ Lower	Beta	Beta Unbiased	Upper Beta	Lower Beta
1	0.0075	0.062	0.0026	0.7	0.820	0.5796	10.57	7.37	11.19	3.07
2	0.007	0.058	0.0025	0.77	0.886	0.6538	12.02	8.38	12.73	3.48
3	0.0018	0.014	0.0006	0.76	0.818	0.7019	25.02	17.45	26.50	7.25
4	0.0035	0.027	0.0016	0.68	0.759	0.6008	13.38	9.33	14.17	3.88
5	0.0095	0.077	0.0033	0.79	0.923	0.6565	11.12	7.76	11.78	3.22
6	0.0082	0.068	0.0029	0.73	0.856	0.6039	10.31	7.19	10.92	2.99
7	0.003	0.025	0.0010	0.74	0.816	0.6639	14.02	9.78	14.85	4.06
8	0.0058	0.047	0.0020	0.72	0.825	0.6147	13.98	9.75	14.81	4.05
9	0.007	0.058	0.0021	0.78	0.896	0.6638	11.62	8.10	12.31	3.37
10	0.008	0.066	0.0028	0.71	0.834	0.5858	9.77	6.81	10.35	2.83

3.1 Unbiased estimate of Weibull shape parameter

The MLE may be used to provide a point estimate of beta, but this calculated value is biased for a small n. Bain and Engelhardt [5] suggest the use of an unbiasing factor Gn. Using such factor, the unbiased estimation of the shape parameter is

$$\hat{\beta} = G_n \hat{\beta}_{MLE} \qquad \text{... } Eq.(9)$$

G_n can be computed using this approximation :

$$G_n = 1.0 - \frac{1.346}{n} - \frac{0.8334}{n^2} \qquad \text{where n is the sample size} \qquad \text{....Eq.(10)}$$

For n=5 the Unbiasing factor is: Gn=1-(1.346/5)-(0.8334/25)= 0.69744, While the unbiased Shape parameter β is shown in Table 2. Using Bain and Engelhardt technique gives us the following approximation for the upper and lower limit for the estimated unbiased Weibull shape factor. [6]

$$\hat{\beta}_L = \hat{\beta}\left[\frac{X^2_{(1-\alpha),df}}{c\,n}\right]^{\frac{1}{1+p^2}} \quad ...Eq.(11), \qquad \hat{\beta}_U = \hat{\beta}\left[\frac{X^2_{\alpha,df}}{c\,n}\right]^{\frac{1}{1+p^2}} \quad ....Eq.(12)$$

where c is the chi - squared factor = $2/[(1+p^2)^2\, p\, c_{22}]$ where c_{22} is asymptotic values for MLE

c= 0.822 for p equal to 1, and

$$X^2_{0.95,4} = 0.711 \text{ and } X^2_{0.05,4} = 9.488$$

Substituting these values in the equations the unbiased confidence limits are shown in figure 1 and table 2.

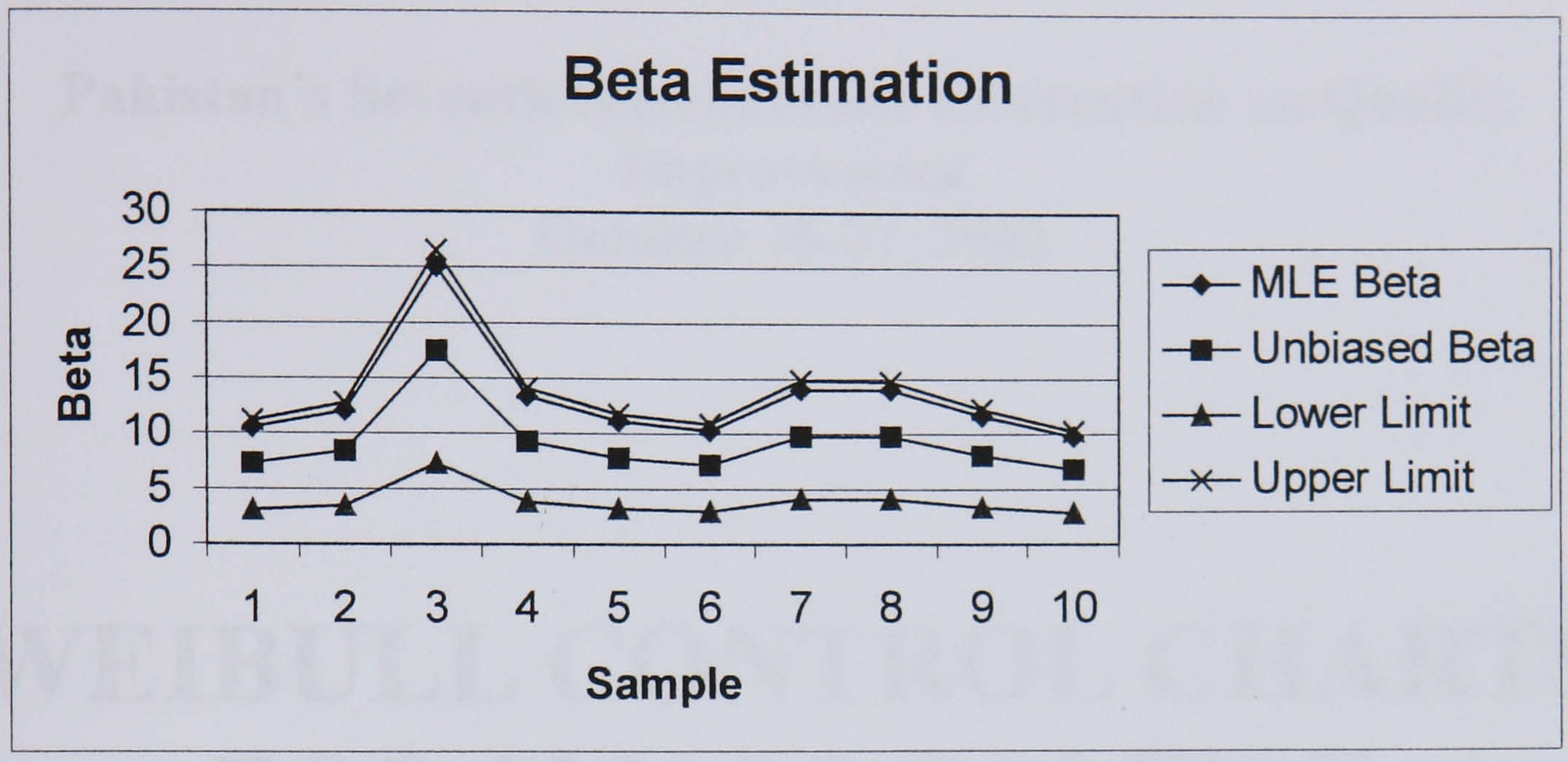

Figure 1 Weibull shape parameter estimation

4 CONCLUSION

Methods exist to determine population parameters with sample sizes that exceed 10, these methods have been shown here to be unreliable for smaller samples. This paper has explored using MLE and unbiased Weibull shape parameters for small samples to predict the parent population parameters from known populations. The conclusion drawn from this comparison is that MLE is not suitable for estimating population parameters with small samples. Whilst the Weibull method does offer an alternative solution it still does not fully explain the total variability.

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WEIBULL CONTROL CHARTS FOR SMALL SAMPLE INSPECTION

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Abstract

The drive for quality improvement has lead to the use of Statistical Process Control (SPC) techniques to monitor and maintain low reject levels. For high scale production large samples can be used to measure with a high level of confidence. However, when low volumes is required, or processes with high piece cost, it can be expensive to collect large samples for analysis. Statistical analysis will always offer higher confidence levels as samples sizes increase. Conventional Statistical Process Control tools show a lack in accuracy. Weibull distribution has always shown a clear and acceptable prediction of failure and life behaviour with small sample size batches. Using such distribution enables the accuracy needed with small sample size to be obtained. With small sample control charts generate inaccurate confidence limits, which are low. On the contrary, Weibull theory suggests that using small samples enable achievement of accurate confidence limits. This paper highlights these two aspects and explains their features in more depth. An outline of the overall problem and solution point out success of Weibull analysis when Weibull distribution is modified to overcome the problems encountered when small sample sizes are used.

1. Introduction

Inspection is an imperative technique to check the quality standards that have been set to attain the elite quality required to fulfil customer satisfaction. Inspection can be a useful way to examine the behaviour of a process and detect the variation that may occur within the process. Inspection can be made on the whole production lot, and at that time is called 100% inspection, as every item will be thoroughly checked. Such a technique is a time consuming procedure.

Sample inspection is considered a more effective and efficient way to inspect variation or non-conformance of a process. Thus, many factors affect such techniques, it is considered to be a modern method to detect default and assure quality. Sampling inspection should submit to different guidelines, which are (Evans et al, 1999):

1. Sample should be rational - Sample should reflect the population behaviour, a chosen sample ought to be homogeneous, as the non-conformance should be clear and appear between samples, while it need not be noticed within each sample. By this principle, spotting deviation within the process across a certain time period can be precisely predicted by mathematical formulas.
2. Sample size should be diminutive – as the size of a sample may be proportional to some financial aspects. Many managerial opinions support the idea of having small sample size always. The sample size is important when financial criteria are involved. Small sample are preferable specially when low volume size, and highly cost product are being inspected. If the inspection involve destructive testing the a company can not risk testing large size sample due to the financial impact, which will cause the retail price to increase and the competitive virtue will decrease.
3. Sampling frequency (rate of recurrence) - Using large sample size frequently with short time period lags will be desirable for inspectors to maintain high quality standards and detect every variation, which may occur in the process. But due to economical reasons this behaviour cannot be useful and cost effective. For that

reason, a balance should be imposed between the frequency of sampling and the cost of quality needed. Practically, this issue is determined by the experience of the inspector and the quality designer.

Typically, small size samples are desirable, as sample size has an economical impact. The breakeven point is the standard of quality required to achieve customer expectations. Hence, Recent global competitiveness has made companies look for a new strategy to increase their profit, gain market reputation, and strengthen their industry. Quality control (SPC) and reliability can ensure these goals for any company if they are used in a correct manner; they are regarded as effective tools when large sample size (n>50) (Montogomery1994) is being tested. The problems is that with small samples, which means when a high value low volume is being manufactured- such as military, satellite, and medical parts, and normally these parts have an expensive financial value. Within this type of manufacturing, safety and life cycle computation is the most vital element to ensure the success of such products. Using the conventional SPC control charts does not ensure the detection of variability and non-conformity due to sample size restrictions.

Weibull distribution has always shown a clear and acceptable prediction of failure and life behaviour with small sample size batches. Using such distribution enables the accuracy needed with small sample size to be obtained (Drapella et al, 1999). While, on small samples SPC Charts generate inaccurate confidence limits, which are low. Additionally, Weibull theory suggests that using small samples enable achievement of accurate confidence limits.

Small samples testing failed to show a conformance with conventional SPC techniques, as the confidence limits for averages and standard deviation are considered to be too wide. Hence, using such sizes will provide unsecured results with a lack in accuracy. Therefore, in this paper a new idea will be investigated and examined to use a reliability model such as Weibull to be used as a Statistical Process Control Model for the expensive, low volume production.

2. Shewhart Control Charts for Variable Data

As Ishikawa stated, "95 percent of quality related problems in the factory could be solved with seven fundamental quantitative tools" (Ishikawa, 1986). The fundamental statistical tools aid the researcher to examine, scan, monitor, and analyse the process. Shewhart Control charts are considered an effective tool to be used.

Control charts enhance the analysis of a process by showing how that process is performing over time. Therefore, combining these charts with an appropriate statistical summary will provide a clear understanding for those who are studying certain process, and enable them to make decisions concerning future production. Also, Control charts describe whether the process is in terms of current performance or not.

As, modern quality goal is to produce a product or a service that exhibits little or no variation if afforded. Variation -where no two items or services are exactly the same- exists in all process. Variation varies depending on the criteria of investigating them and tackling these variations. Variation has mainly three types (a) within piece variation (b) piece to piece variation (c) time to time variation. Normal variation within certain processes is studied by sampling the process. Control charts monitor the variation within the process and using statistical measurements process variation is recorded on different control charts, which show

changes in the process, allowing early detection of process changes, which reduce rework, scrap, process delays and money loss.

Control charts, like any other basic tools for quality improvement, are relatively simple to use. Control charts have three basic applications: (1) to establish a state of statistically controlled process, (2) to monitor a process when the process goes out of control, and (3) to determine process capability.

This paper concentrates on small samples of variable data, and their behaviour using the conventional Shewhart SPC charts. While the attribute data assume only two values, good or bad, pass or fail, so the attribute data. Attributes usually cannot be measured, but they can be observed and counted and are useful in many practical situation. Usually, attributes data are easy to collect, often by visual inspection. Many accounting records, such as percent scrapped, are readily available. However, one drawback in using attributes data is that large samples are necessary to obtain valid statistical results. For these reasons, the main interest in the current investigations is to understand the background knowledge of variable control charts such as $\overline{X}$, R Charts.

In 1920's, Dr. Walter A. Shewhart set elaborated charts, which test, monitor and control variability within a process. Shewhart developed control charts to detect various variability and distortion in the process. Shewhart Control charts, which is used in this paper is: $\overline{X}$, R chart (Average, range chart).

The first step in developing $\overline{X}$, R chart is to gather data. Usually, about 25 to 30 samples are collected. Samples between size 3 and 10 are generally used, with samples size of 5 being the most common. The number of samples is indicated by k, and n denoted the sample size. For each sample I, the mean is denoted $\overline{X_i}$ and the range by R_i are computed. The values are then plotted on their respective control charts. Next, the overall mean and overall average range calculations are made using equation 2.1 and 2.2, and these values specify the centre lines for the $\overline{X}$, R chart.

$$\overline{\overline{X}} = \frac{\sum_{i=1}^{k} \overline{X}_i}{k} \qquad \text{(Equation 1)}$$

$$\overline{R} = \frac{\sum_{i=1}^{k} R_i}{k} \qquad \text{(Equation 2)}$$

The average mean and average range are used to compute control limits for $\overline{X}$, R chart. Control limits are easily calculated using the Shewhart formulas, as shown in equation 3, 4, 5, and 6 (Ott el al, 2000).

$$\text{Upper Average Control Limit} = UCL_x = \overline{\overline{X}} + A_2\overline{R} \qquad \text{(Equation 3)}$$

$$\text{Lower Average Control Limit} = LCL_x = \overline{\overline{X}} - A_2\overline{R} \qquad \text{(Equation 4)}$$

$$\text{Upper Control Range Limit} = UCL_R = D_4\overline{R} \qquad \text{(Equation 5)}$$

$$\text{Lower Control Range Limit} = LCL_R = D_3\overline{R} \qquad \text{(Equation 6)}$$

Where the constants D_3, D_4 and A_2 depend on the sample size and can be found in special tables. Figure 1 shows a standard shape for $\overline{X}$, R chart.

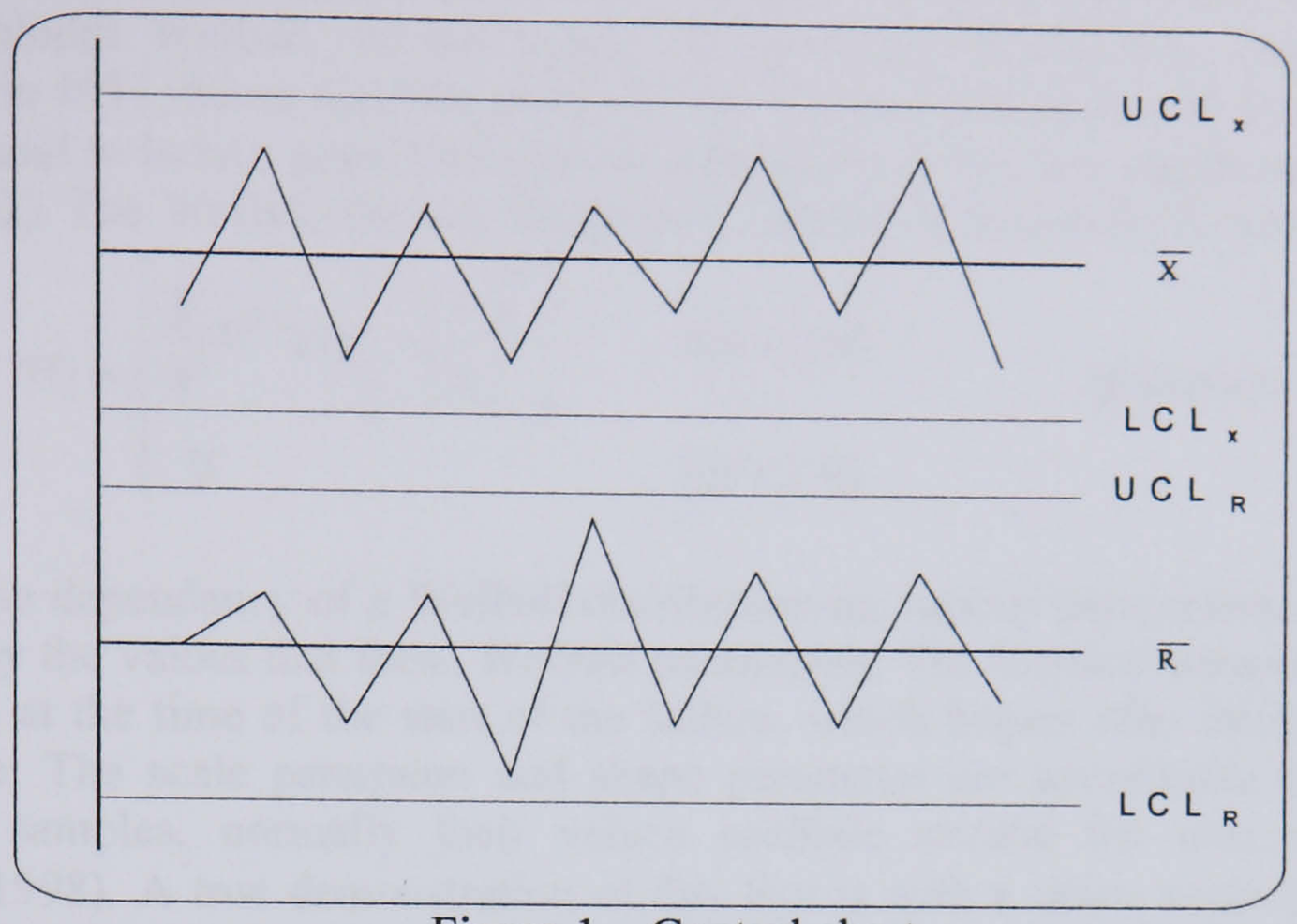

Figure 1 – Control chart

The control limits represent the range between which 99.73% of all points are expected to fall if the process is in statistical control. If any points fall outside the control limits or if any unusual patterns are observed, then some special cause has probably affected the process. The process should be studied to determine the cause. If special causes are present, then they are not representative of the true state of the statistical control and all the calculation for the centreline and control limits will be biased. The corresponding data points should be eliminated, and new values for the average of mean, average of range, and control limits should be computed.

3. Small Sample Size Effect on Conventional Control Charts

Commonly, using small sample size to generate control charts, which is a subset of quality control methods, implies dealing with samples obtained from a stable process, and these samples are then compared with some functions of the long-term parameters (e.g. mean, variance). If the sample has a very small size (less than six), and the process variation is relatively large, then the results acquired will be very rough. Therefore, the crucial issue in such situations is not the small size of the sample as the large size of the process variance.

Generally, Shewhart SPC charts can be effectively used with large sample size batches. When using small samples the probability of false notices can increase due to the rise of uncertainty with respect of small samples effect on the theory behind building up such control charts. An essential sampling disadvantage of control charts in small sample size methods is the risk of not detecting a non-conformance item (Fine, 1997). If a sample was deducted from a process and unfortunately, this sample did not contain a failed item (regarding specification), this item will be in the market as a passed item knowing that it is not, despite its high confidence. Nowadays, conventional SPC chart show a clear lack in complying with the trend of industry to cut its cost specially when using small sample size. SPC philosophy and model is an easy method to be adopted in manufacturing environment, therefore, many researchers are trying to adopt new adjustments to the conventional SPC chart to be used with the association of small sample size inspection and provide reasoning and confident results.

4. Usage of Small Samples with Weibull Analysis

Weibull distribution existed due to the unique research delivered by the Swedish Professor Waloddi Weibull. In his paper "A statistical Distribution Function of Wide Application" in 1951, he verified the ability of the Weibull distribution to be used with small sample sizes and to have a good flexibility to establish a good fit to reach reasonable results (Donson, 1962). The Weibull Density Function is defined as follow (O'Conner, 1993):

$$f(t) = \begin{cases} \frac{\beta}{\eta^{\beta}} t^{\beta-1} \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] & , \text{ for } t \geq 0) \\ 0 & , \text{ for } t < 0) \end{cases} \qquad \text{(Equation 7)}$$

Due to the dependency of a Weibull distribution on various parameters, its behaviour is constrained by the values that these Weibull parameters. The location parameter is normally equal to zero at the time of the start of the failure, which begins after initiating the part to operation life. The scale parameter and shape parameter are uncertainly calculated when using small samples, normally their values oscillate around the true unknown value (Abernethy, 1998). A true demonstration of this fact is with a shape parameter β=3.44, the Weibull plot approximates to a normal distribution. This is a theoretical value (i.e. a parameter) not an estimation value obtained from a sample. Hence, there is no expectation of an exact value of 3.44 for the shape parameter from a small size sample, which has been drawn from the normal distribution, especially if sample size is small.

In addition, different small samples, regardless of the distribution they come from, may provide widely varying point estimates. This is especially so when variance of parent distribution is large relative to the mean. Therefore, it can be seen that using small sample size is an uncertain method to predict quality and life behaviour for the manufactured product.

The nature of Weibull distribution distinguishes such distribution from others, by having different characteristic due to the altering of the shape parameter. The values of the shape parameter values vary the shape of the Weibull probability density function. As a result, Weibull distribution is a suitable distribution to be employed in various situations, by depending on the value of shape parameter; many distributions can be established (refer to table 1)

Beta	p.d.f. Shape
$\beta = 1$	Indicates Exponential distribution
$\beta = 2$	Indicates Rayleigh distribution
$\beta = 2.5$	Indicates Lognormal distribution
$\beta = 3.4$	Indicates Normal distribution
$\beta = 5$	Indicates peaked Normal distribution

Table 1 – Weibull shape parameter effect on p.d.f.

For that reason, Weibull is a good model to use, as it is a comprehensive method to cover most of the variation that may be involved in a process.

After the effective use of computers and the efficiency of the modern calculation devices and software, many models were developed (such as Monte Carlo, Maximum Likelihood Estimation MLE, and least square methods) to increase the accuracy of Weibull parameters estimation and to overcome the deficiencies encountered with the use of Weibull in manufacturing environment, specially using small sample size to test the performance of an item. The estimation can be point estimation or range estimation. The main focus in the present work will be on estimating shape parameter as Weibull scale parameter is mostly estimated by MLE method (Skinner et al, 2000), which ensures high confidence level using small sample sizes. On the contrary, Weibull shape parameter show no response with conventional estimation method to comply with these methods and enable an estimation of its value with reasonable confidence level.

5. Analysis

To analyse the current problem of small samples effect on control charts, a set of data, which has been taken from a single lap shear test, will be used to address the problems with conventional control charts and propose a suitable solution of such problem.

In Lap Shear Strength Test (LSST), the strength of a lap sheared adhesive bond will be stressed in shear to determine the strength of the joint, which have specific type of adhesive. The joint will be exposed to a concentric parallel force. The maximum shear force or stress rapture will be calculated. Lap shear specimens were prepared using 100mm x 3mm aluminium sheet (ASTM, 1996). In order to hold the specimens, two holes were drilled in the size of the clamp pin. Joints were assembled with a 12.5mm overlap, and then stacked on a special clamping jig using guide pins on a metal base to control dimensional changes. The adhesive was placed in between the substrates leaving a 19mm gap for wedge insertion. The crack length was measured which gave the strength retention data of the joint as a function of time. Maximum lap-shear strengths of the joints were measured with a constant cross-head speed of 0.42 mm/s (1 inch per minute) (Abughazaleh et al, 2001).

Figure 2 shows the specimen specification used in lap shear test, a set of 7 specimens were used in each test, and the shear stress was calculated for each test. Tests were performed under the room temperature and relative humidity. Test where done by using different adhesives (O-XD4600, Q-Citec FM73 and P-Araldite 2012), also the tests were done twice, first with untreated surface and the second with B-Si treated surface.

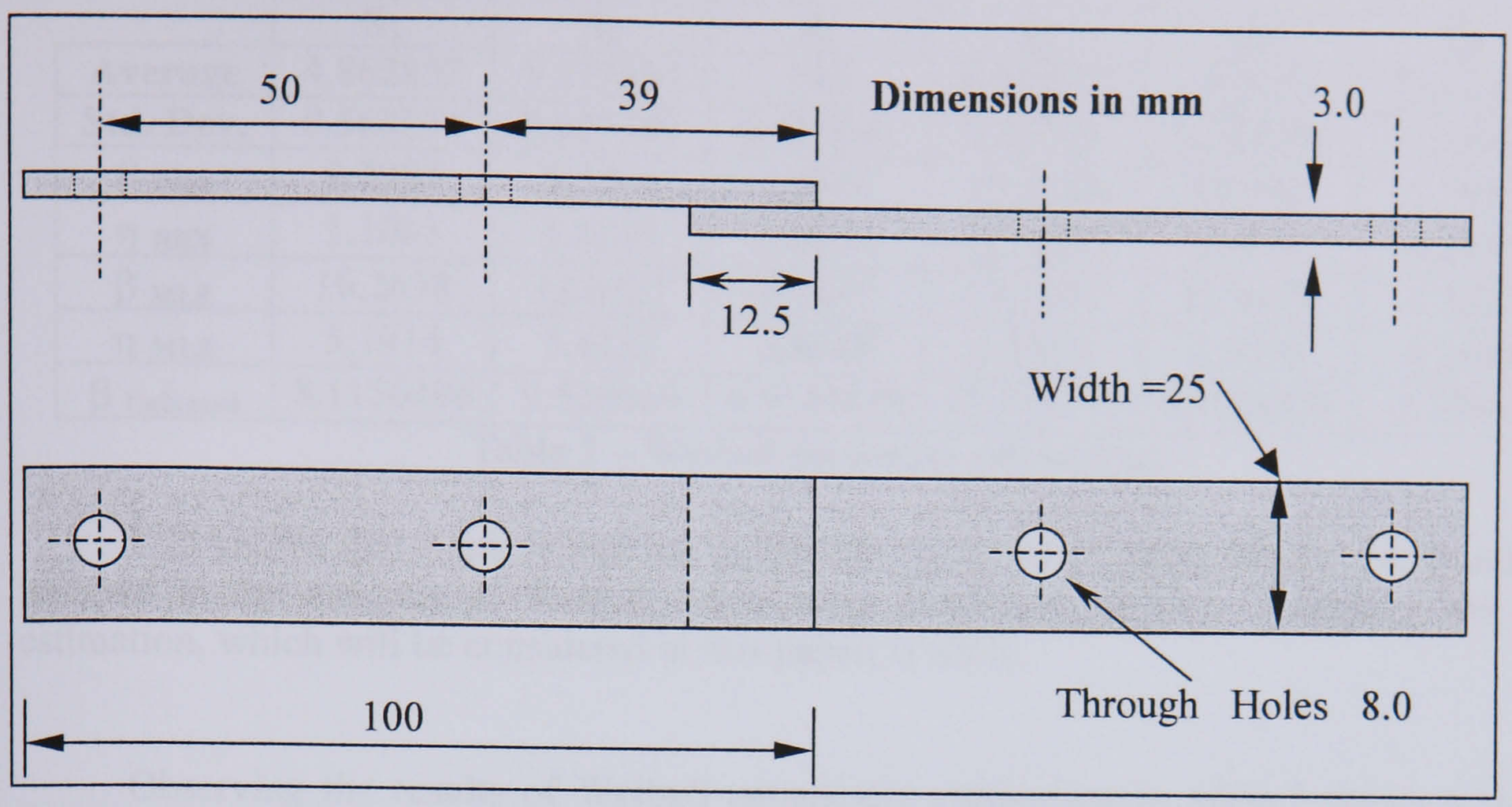

Figure 2 - Lap Shear Test

After performing the lap shear tests the following data were obtained, refer to table 2.

	S_1 O-XD4600	S_2 O-XD4600	S_3 Q-CitecFM73	S_4 Q-CitecFM73	S_5 P-Araldite 2012	S_6 P-Araldite 2012
	Un treated	**B-Si Only**	**Un treated**	**B-Si Only**	**Un treated**	**B-Si Only**
i_1	*4.62*	*5.37*	*3.165*	*4.72*	*0.834*	*0.15*
i_2	*4.06*	*5.41*	*3.045*	*5.55*	*0.914*	*0.313*
i_3	*5.03*	*5.89*	*4.18*	*5.095*	*0.99*	*0.592*
i_4	*5.4*	*5.6*	*4.035*	*5.12*	*0.85*	*0.0755*
i_5	*5.69*	*4.79*	*4.285*	*4.65*	*0.97*	*0.191*
i_6	*4.86*	*5.17*	*3.65*	*4.99*	*0.87*	*0.2203*
i_7	*4.38*	*3.98*	*3.19*	*4.86*	*0.676*	*0.22*

Table 2 – Lap Shear Test Results

The success of the Weibull analysis significantly depends on the accuracy of the parameters used. Many techniques have developed in recent years to study the behaviour of such parameter, and try to establish accurate point estimation for Weibull parameter. In this paper, the estimation of shape parameter will be dealt with; as such parameter is important in Weibull analysis, and the difference between theory and practice occur when using small sample size. The three main techniques are:

1. Weibull probability plot.
2. Least square technique (Regression analysis).
3. Maximum Likelihood Estimation.

Using the strength data in table 2, a prediction of the Weibull parameters can be calculated by using Weibull++ and WinSmith. Table 3 summarises the out coming results of such predictions based on different estimation techniques, which are: Rank regression on x, maximum likelihood estimation and Unbiased factor estimation (Abughazaleh et al, 2002).

	S_1	S_2	S_3	S_4	S_5	S_6
Average	4.862857	5.172857	3.65	4.997857	0.872	0.25168
Std. Dev.	0.568234	0.627341	0.523641	0.30179	0.104594	0.16664
β_{RRX}	9.2904	8.5984	7.9804	19.4164	8.7902	1.7686
η_{RRX}	5.1065	5.4505	3.8559	5.1227	0.9178	0.2830
β_{MLE}	10.2638	12.0521	8.1447	17.1691	11.6478	1.7635
η_{MLE}	5.1014	5.4132	3.8648	5.1367	0.9129	0.2848
$\beta_{Unbiased}$	8.1156496	9.529669	6.9144879	13.575713	9.2099870	1.39441

Table 3 – Weibull parameter estimations

It is known that Rank regression estimation on X (RRX) is an adequate estimation to be used in the analysis of Weibull when using small sample size. Therefore, the main estimation, which will be considered in this paper, is RRX.

Observing the results of Weibull parameters estimations in table 3 shows a lack of prediction and high level of variation. As these are sample sizes of 7 and the sample mean is normally distributed (Based on Central Limit Theorem), therefore, it is known that the parent population is normally distributed, with a shape parameter of 3.44. Using this fact, it is recommended to run a modified Weibull Method to achieve high level of confidence in shape parameters estimations.

It has been detected that Weibull shape parameter estimation may be accurate if deduction method is implemented (Abughazaleh, 2002). The detection methods try to shift the p.d.f. of the data, which have Weibull shape parameter estimation away from the true theoretical method (refer to figure 3) . This shift showed a constructive influence on the result of Weibull shape parameter estimation.

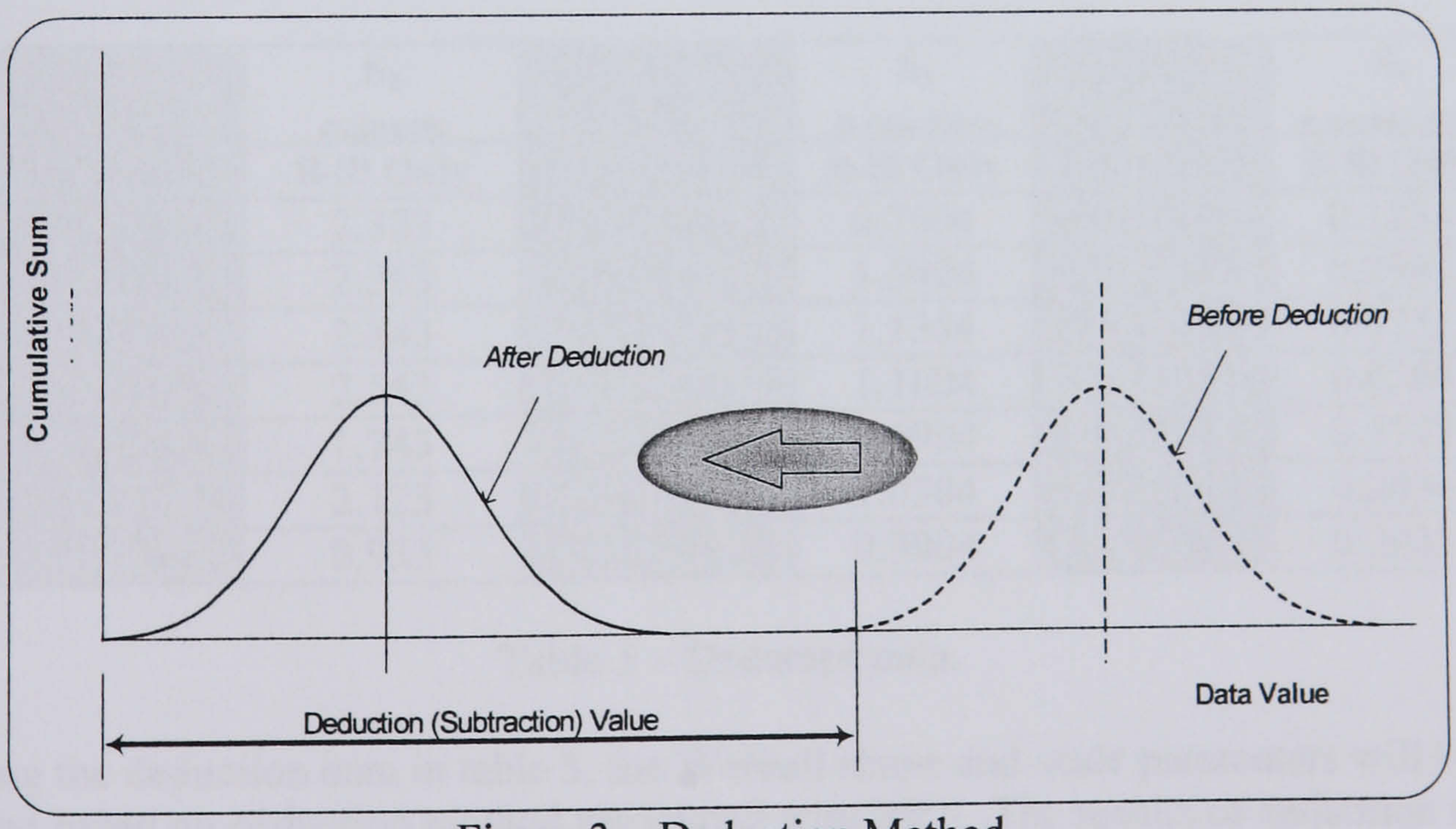

Figure 3 – Deduction Method

The deduction values are modelled by equation 8 (Abughazaleh, 2002), which allows the deduction value to be calculated based on sample size of 7.

$$\text{Deduction percentage} = a - b\,e^{-c\,n^{d}} \qquad \text{(Equation 8)}$$

Where:

$a = 0.99994$

$b = 0.10225931$

$d = 0.38041323$

$d = 1.5959549$

n= *Sample size.*

Also, having a parent population with a shape parameter of 3.77, and using the Deduction Model, then the deduction percentage can be calculated. The deduction percentage resulting from equation 8 is equal to 99.3288 %, table 4 shows the correspondent deduction values for deduction percentage of 0.993288. Also, table 5, shows the lap shear test data after the deduction.

Sample	Deduction Value
S_1	*2.9810*
S_2	*3.0470*
S_3	*2.0606*
S_4	*3.9596*
S_5	*0.5196*
S_6	*0.0167*

Table 4 – Deduction Values

	S_1 O-XD4600 Un treated	S_2 O-XD4600 B-Si Only	S_3 Q-CitecFM73 Un treated	S_4 Q-CitecFM73 B-Si Only	S_5 P-Araldite 2012 Un treated	S_6 P-Araldite 2012 B-Si Only
i_1	1.639	2.323	1.1044	0.7604	0.3144	0.1333
i_2	1.079	2.363	0.9844	1.5904	0.3944	0.2963
i_3	2.049	2.843	2.1194	1.1354	0.4704	0.5753
i_4	2.419	2.553	1.9744	1.1604	0.3304	0.0588
i_5	2.709	1.743	2.2244	0.6904	0.4504	0.1743
i_6	1.879	2.123	1.5894	1.0304	0.3504	0.2036
i_7	1.399	0.933	1.1294	0.9004	0.1564	0.2033

Table 5 – Deducted data.

Using the deduction data in table 5, the Weibull shape and scale parameters will be estimated based on deduction method associated with RRX, The results of deduction method estimation can be found in table 6.

	S_1	S_2	S_3	S_4	S_5	S_6
β_{RRX} (Deduction)	3.4028	3.0708	3.3406	3.9667	3.1113	1.5935
η_{RRX} (Deduction)	2.0961	2.3848	1.7621	1.1402	0.3948	0.2643

Table 6 – Weibull Deduction Parameters Estimations

6. Control charts Based on Weibull Deduction Model

Conventional control charts are based on normal distribution analysis, the common Shewhart average and range control charts is based on 3σ (standard deviation). Normal distribution has a unique property, which is the area under the normal curve equal to one. And it has predictable proportions of its total area within one, two and three standard deviations of the mean, regardless the magnitude of the standard deviation. Figure 4 shows the percentages of areas covered by 1σ, 2σ and 3σ.

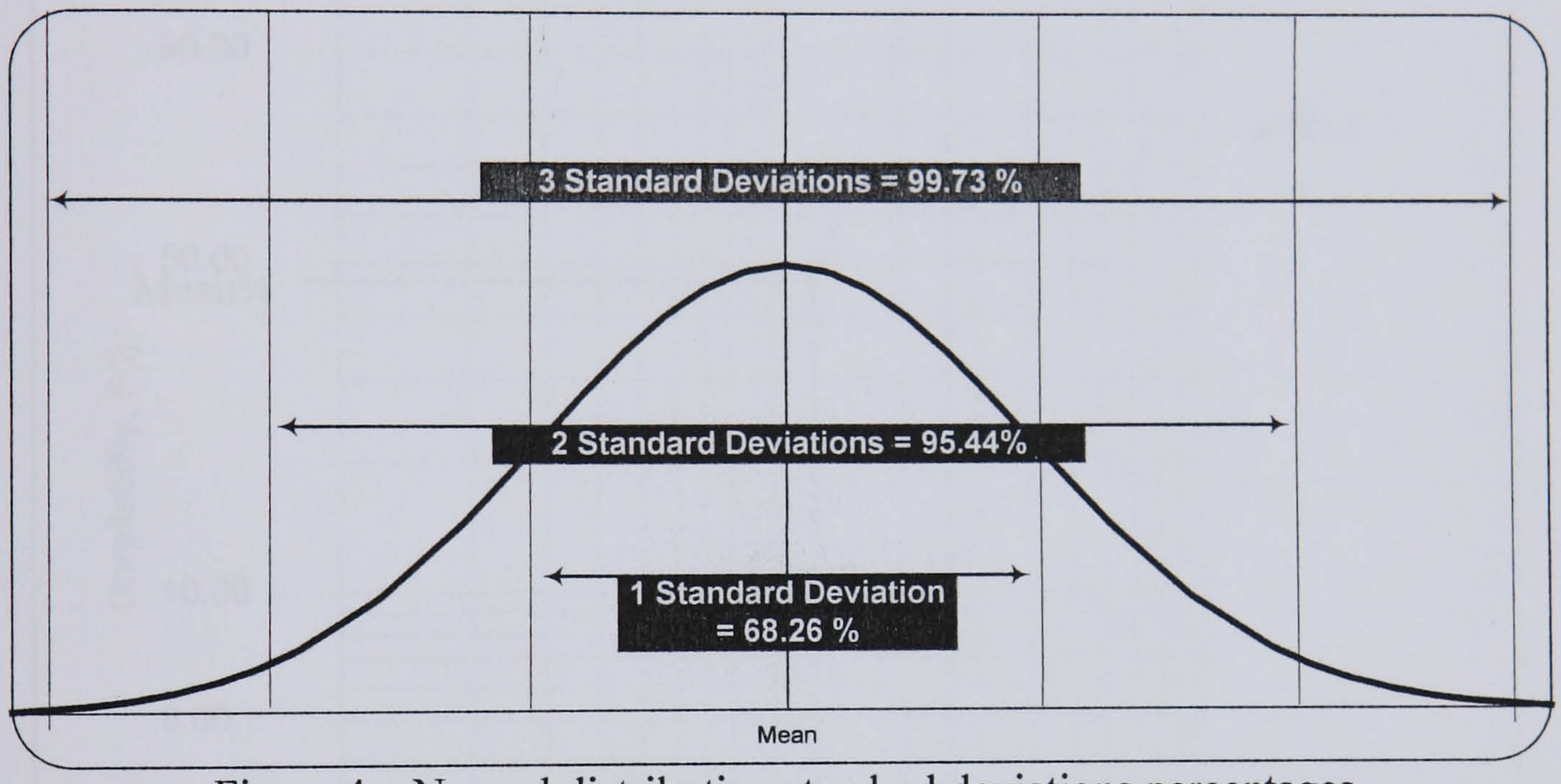

Figure 4 – Normal distribution standard deviations percentages

Using the principle of standard deviations percentages can be modified to Weibull distribution and develop values of the percentages based on Weibull probability plot.

When 3 standard deviations in this analysis, it can be found that Weibull has an upper and lower range limits as follow

Upper Range Limit = 50 % + 49.865 % = 99.865 % (Equation 9)
Lower Range Limit = 50 % - 49.865 % = 0.135 % (Equation 10)

Using such limits, a correspondent data values can be obtained by Weibull probability plot, which was originated using any test data after the use of deduction method to achieve accurate representative Weibull probability plot based on rank regression on x. Figure 5 represents the previous idea used to achieve the range limits based on Weibull analysis and deduction method.

From Figure 5, it can be seen that based on equation 9 and 10, upper and lower limits of the data used has been configured based on Weibull deduction method associate with RRX. The value are symbolised by P_{Upper} and P_{Lower}. Also the mean is predicted using the same technique. Using these predicted values control chart based on Weibull distribution can be constructed. The construction of Average and Range control chart can be described as follow: from the Values of Figure 5, it can be easily noticed that the average Weibull control chart limits are calculated as follow, (also refer to Figure 6)

$$\text{Average Weibull Centre Line} = \text{Mean} = \mu \qquad \text{(Equation 11)}$$

$$\text{Range} = P_{Upper} - P_{Lower} \qquad \text{(Equation 12)}$$

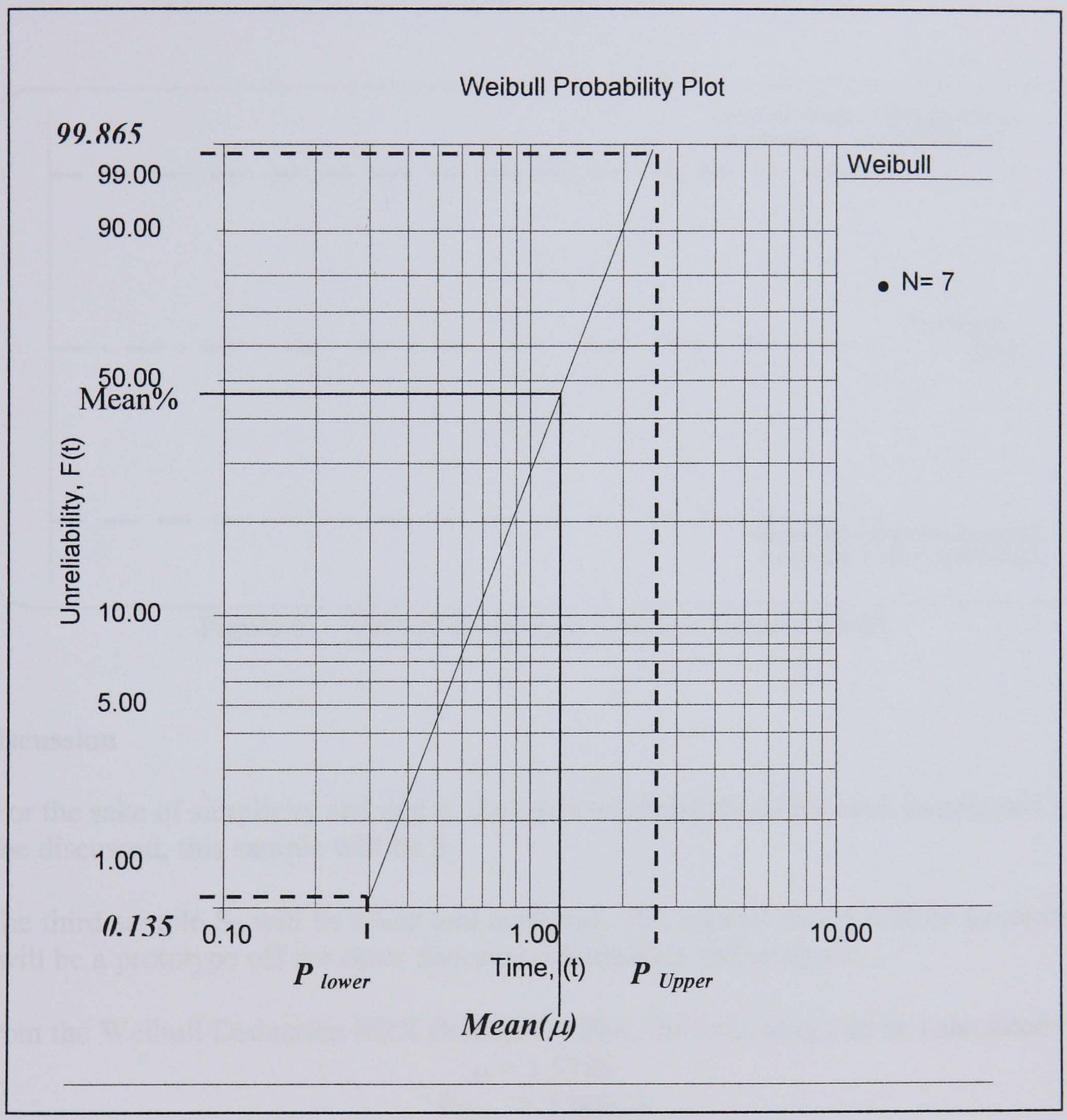

Figure 5 – Weibull Range Percentages (based on deduction and RRX)

$$\text{Range} = R = P_{Upper} - P_{Lower} = 3\sigma \qquad \text{(Equation 13)}$$

$$Therefore,\ \sigma = \frac{R}{3} = \frac{(P_{Upper} - P_{Lower})}{3} \qquad (Equation\ 14)$$

$$\text{Uppper Weibull Deduction Average Limit} = \mu + \frac{\sigma}{\sqrt{n}}$$

$$= \mu + \frac{(P_{Upper} - P_{Lower})}{3\sqrt{n}} = \mu + \frac{(P_{Upper} - P_{Lower})}{3\sqrt{7}}$$

$$UWDAL \cong \mu + 0.1259882\,(P_{Upper} - P_{Lower}) \qquad \text{(Equation 15)}$$

$$\text{Lower Weibull Deduction Average Limit} = \mu - \frac{\sigma}{\sqrt{n}}$$

$$= \mu - \frac{(P_{Upper} - P_{Lower})}{3\sqrt{n}} = \mu - \frac{(P_{Upper} - P_{Lower})}{3\sqrt{7}}$$

$$LWDAL \cong \mu - 0.1259882\,(P_{Upper} - P_{Lower}) \qquad \text{(Equation 16)}$$

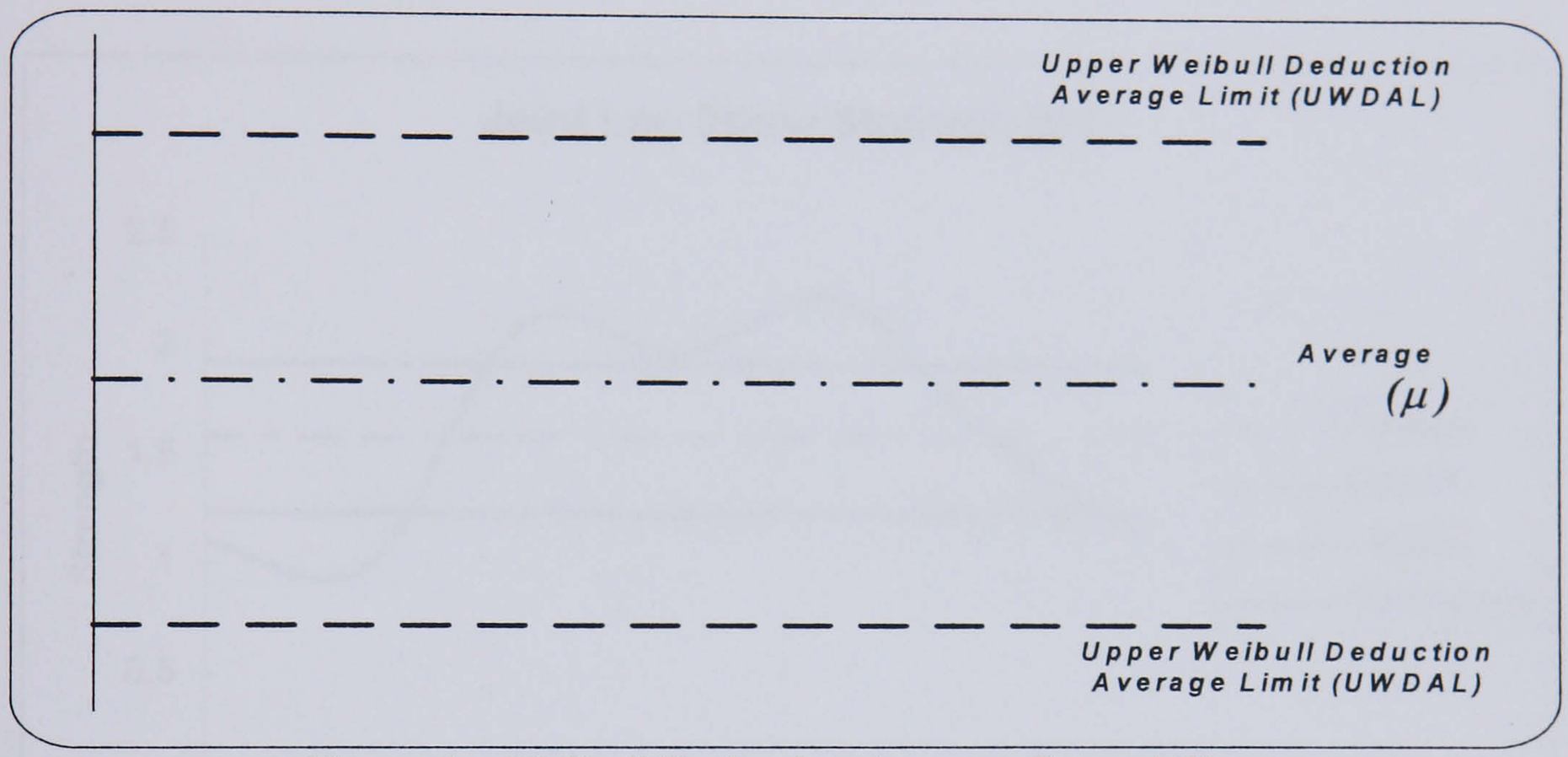

Figure 6 – Weibull Deduction Average Control Chart

7. Discussion

For the sake of simplicity and due to the retrieval calculations for each sample one sample will be discussed, this sample will be S_3.

The third sample S_3 will be taken and analysed. The control charts will be generated and this will be a prototype off the other Samples calculations and analysis.

From the Weibull Deduction RRX Probability Plot, the following can be calculated:

$$\mu = 1.5790$$
$$P_{Upper} = 3.00135$$
$$P_{Lower} = 0.2438$$

Using these values, the following Control charts terms can be calculated,

$$UWDAL = 1.926418761$$
$$LWDAL = 1.23158124$$
$$\sigma = 0.115806253$$

The Above control limits are deduction control limits, and in order to have the absolute control limit an addition value (Deduction Value) should be added. Therefore, the absolute control limits are:

$$\mu = 3.6396$$
$$\text{UWDAL} = 3.987018761$$
$$\text{LWDAL} = 3.29218124$$

The results obtained from the use of Weibull deduction technique can be employed graphically to generate a graphical representation to control charts based on Weibull distribution and rank regression on x for small sample size (n=7). A main graph can be generated, which is Weibull Deduction Average Control Chart. Figure 7 is a Weibull Deduction Average Control Chart, which show the behaviour of the average strength for a joint with Q-CitecFM73 adhesive and no surface treatment resulting form a single lap shear test.

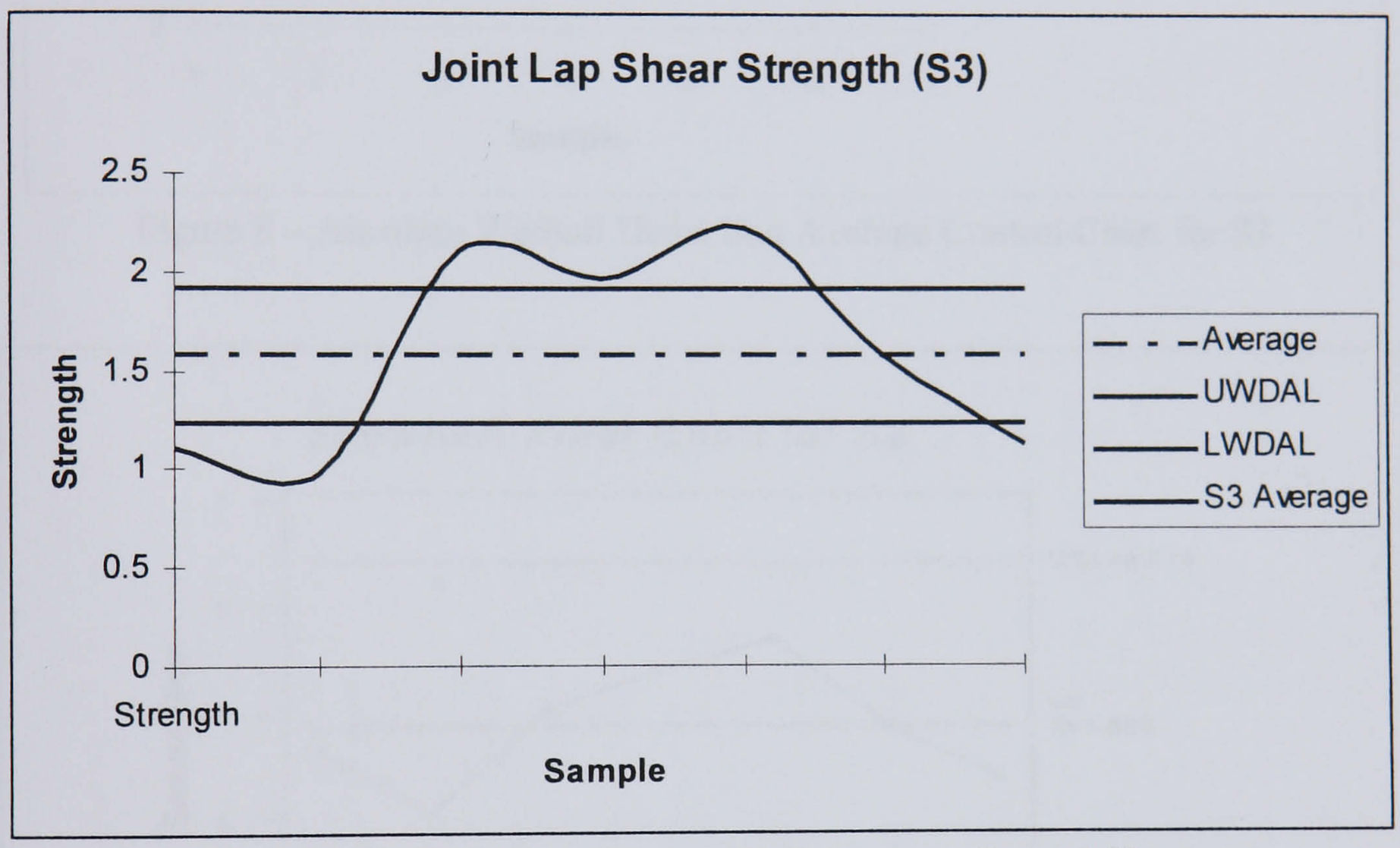

Figure 7 – Weibull Deduction RRX Average Control Chart

Also to make such analysis softer and easy to digest the Weibull Deduction Average Control Charts can be re-plotted by using absolute values, which is achieved by adding the deduction value to the limits and averages. Figure 8 shows Weibull deduction average charts based on absolute values. Figure 9 present a conventional average Shewhart chat for S3, and the difference between Shewhart and Weibull deduction chart is so apparent.

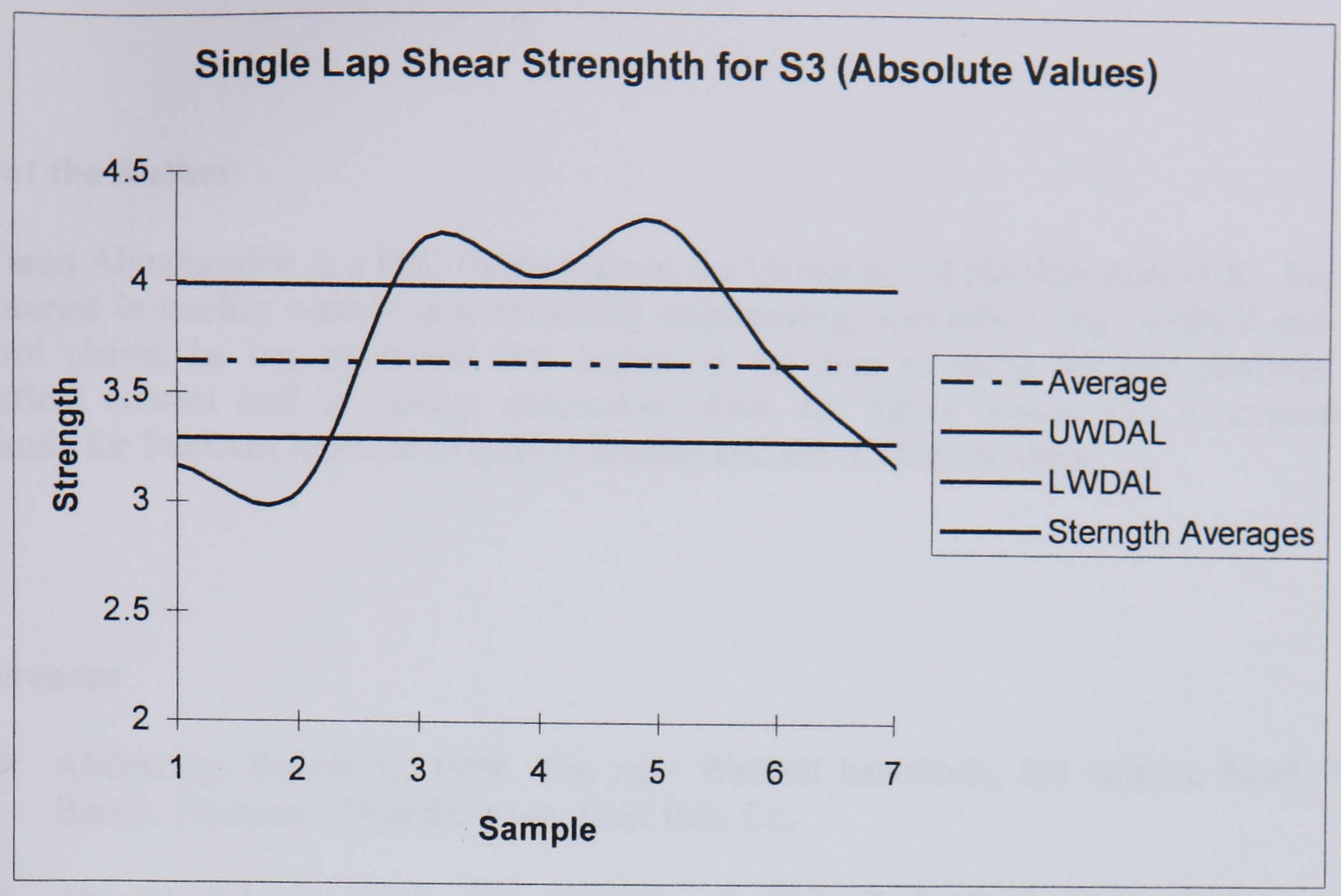

Figure 8 – Absolute Weibull Deduction Average Control Chart for S3

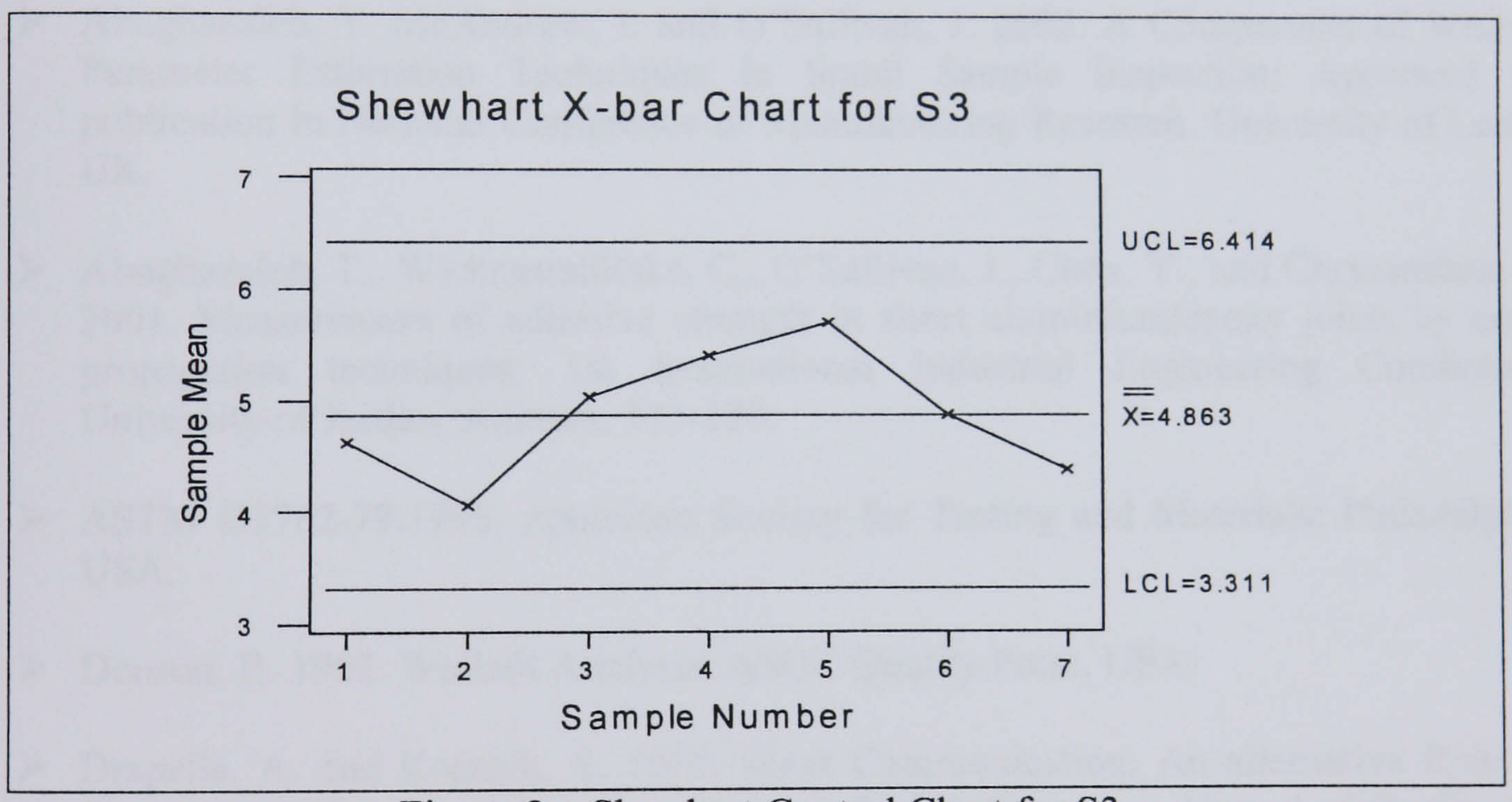

Figure 9 – Shewhart Control Chart for S3

8. Conclusion

It is clear that using a Weibull deduction based control charts overcome the problem of conventional Shewhart control charts associated with small sample size. Weibull deduction control charts can provide accurate control limits when using small sample and it can replace the conventional Shewhart control charts in small sample size inspection.

About the Author

Tareq Abughazaleh is a PhD Researcher in the University of Hertfordshire (UK), his field of interest is quality control and reliability engineering, specially using Weibull and SPC control charts, he has published few papers in the area of using Weibull analysis as a statistical control tool in quality inspection. Also, the author would like to express his gratitude for Pakistan institute of quality control and the ICQI committee.

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