## Analysis of RF power optimization for SCM based free space optical systems for IM3

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The focus of this investigation is on the optimization of input RF power in SCM based FSO systems to minimize the average BER because the higher input RF power does not always provide the FSO systems with better performance due to the effect of the third-order intermodulation (IM3) product. Analysis results have demonstrated the optimum input RF power and the minimum average BER.

*Introduction*: A previously published paper [1] proposed SCM based FSO systems and derived the closed form of the average BER. The Log-normal and Exponential model are used for the weak and strong scintillation effect respectively. As shown in Fig. 1, increase of the input RF signal power does not always provide the high system performance because the power of a fundamental frequency component is proportional to the input RF power while the power of IM3 product is proportional to the cube of the input RF power. Therefore, it is necessary to analyze the optimum input RF power to minimize the average BER performance. Subsequently, the results of the numerical analysis are verified by a direct search method [2], [3].

*Derivation of the optimum value of the input RF power*: In [1], the closed form of the average BER considering the effect of IM3 product was derived as follows,

$$P_{b} \approx \frac{1}{\sqrt{\pi}} \sum_{i=1}^{N} w_{i} Q \left( \sqrt{\frac{GP_{RF} e^{2(\sqrt{2}\sigma_{z}x_{i}+\mu_{z})}}{HP_{RF}^{3}} e^{2(\sqrt{2}\sigma_{z}x_{i}+\mu_{z})} + 4kT + qRA^{2} e^{2(\sqrt{2}\sigma_{z}x_{i}+\mu_{z})}}} \right)$$
(1)

for the log-normal and for the exponential channels

$$P_b \approx \sum_{i=1}^{N} w_i |x_i| Q \left( \sqrt{\frac{\overline{\delta}^2 G P_{RF} x_i^4}{\overline{\delta}^2 H P_{RF}^3 x_i^4 + 4kT + qRA^2 \overline{\delta}^2 x_i^4}} \right)$$
(2)

where

$$G = \left(\frac{\pi L_{att}^2 A^2 R}{2V_{\pi}}\right)^2, \quad H = \frac{1}{128} \left\lfloor \frac{M - 1}{2} \right\rfloor \left(\frac{\pi^3 L_{att}^2 A^2 R}{V_{\pi}^3}\right)^2 \left(5 - 3\cos\left(\frac{2\pi LD\lambda^2 f_w^2}{c}\right)\right) \tag{3}$$

*N* is the order of approximation,  $x_i$  (i = 1...*N*) is the zeros of the *N*<sup>th</sup>-order Hermite polynormial and  $w_i$  (i = 1...*N*) is weight factors for the *N*<sup>th</sup>-order approximation,  $k = 1.38 \times 10^{-23}$ J/K is the Boltzmann constant,  $q = 1.6 \times 10^{-19}$  C is the electron charge, T = 300 K is the absolute temperature, *B* is the effective noise bandwidth, *A* is the optical carrier,  $V_{\pi}$  is the switching voltage of DD-MZM,  $L_{att}$  is the attenuation of DD-MZM, *R* is the responsivity, *M* is the number of users,  $\delta$  is scintillation coefficient representing the log-normal and the exponential distribution [4]-[6],  $\mu_k$  and  $\sigma_k$  are the mean and standard deviation of *K*,  $\delta = e^K$ , respectively, and  $\overline{\delta}$  is the expectation of  $\delta$ .

Under the Log-normal channels the differentiation of (1) with respect to the input RF signal power ( $P_{RF}$ ) is given by:

$$\frac{dP_b}{dP_{RF}} = \sum_{i=1}^{N} w_i \exp\left[\frac{-\xi_{1,i}P_{RF_{opt}}}{2(\xi_{2,i}P_{RF_{opt}}^3 + \xi_{3,i})}\right] \sqrt{\frac{\xi_{2,i}P_{RF_{opt}}^3 + \xi_{3,i}}{\xi_{1,i}P_{RF_{opt}}}} \frac{\xi_{1,i}\xi_{3,i} - 2\xi_{1,i}\xi_{2,i}P_{RF_{opt}}^3}{(\xi_{2,i}P_{RF_{opt}}^3 + \xi_{3,i})^2} = 0 \quad (4)$$

where

$$\xi_{1,i} = G \exp^{2(\sqrt{2}\sigma_k x_i + \mu_k)}$$
  

$$\xi_{2,i} = H \exp^{2(\sqrt{2}\sigma_k x_i + \mu_k)}$$
  

$$\xi_{3,i} = 4kTB + qRA^2B \exp^2\left[\sqrt{2}\sigma_k x_i + \mu_k\right].$$
(5)

Finally, we can obtain the approximately optimum input RF power under the Log-normal turbulence channel as follows:

$$P_{RF_{opt}} \approx \sum_{i=1}^{N} w_i \left(\frac{\xi_{3,i}}{\xi_{2,i}}\right)^{1/3}.$$
 (6)

Employing the same process as the Log-normal channel, the differentiation of (2) with respect to the input RF signal power under Exponential channels is as follow:

$$\frac{dP_{b}}{dP_{RF}} = \sum_{i=1}^{N} w_{i} \left| x_{i} \right| \exp \left[ \frac{-\zeta_{1,i} P_{RF_{opt}}}{2(\zeta_{2,i} P_{RF_{opt}}^{3} + \zeta_{3,i})} \right] \sqrt{\frac{\zeta_{2,i} P_{RF_{opt}}^{3} + \zeta_{3,i}}{\zeta_{1,i} P_{RF_{opt}}}} \frac{\zeta_{1,i} \zeta_{3,i} - 2\zeta_{1,i} \zeta_{2,i} P_{RF_{opt}}^{3}}{(\zeta_{2,i} P_{RF_{opt}}^{3} + \zeta_{3,i})^{2}} = 0$$
(7)

where

$$\zeta_{1,i} = G\overline{\delta}^2 x_i^4$$

$$\zeta_{2,i} = H\overline{\delta}^2 x_i^4$$

$$\zeta_{3,i} = 4kTB + qRA^2 B\overline{\delta}^2 x_i^4.$$
(8)

The approximate optimum input RF power is represented as follows:

$$P_{RF_{opt}} \approx \sum_{i=1}^{N} w_i |x_i| 2^{1.54} \left(\frac{\zeta_{3,i}}{\zeta_{2,i}}\right)^{1/3}.$$
(9)

Numerical results: In this section, numerical results for the average BER and the optimum input RF signal power as a function of the number of users (*M*) are presented. A FSO system with a wavelength of 1550 nm, the switch voltage ( $V_{\pi}$ ) of 2.5 V, the DD-MZM insertion loss ( $L_{att}$ ) of 6 dB, the responsivity (*R*) of 0.8 A/W and the communication distance (*L*) of 2 Km is considered. In Fig. 2, the approximated optimum values of the input RF power (solid and dash lines) given by (6) and (9) are compared with a direct search method given by [2] and [3] (circles and squares). It is observed from Fig. 2 that (6) and (9) provide a good approximation and coincides with the results of the direct search method. For example, under the Lognormal channel and for M = 128 the maximum error is between the approximated  $P_{RF}$  of 15.71 dBm and the direct searched  $P_{RF}$  of 15.64 dBm is very small (i.e. < 0.1 dBm). In Fig. 3, the approximated optimum value of the input RF power that achieves the minimum average BER given by (1) and (2) are respectively represented for a given number of users  $M \in \{8, 16, 32, 64, 128\}$ . For Fig. 3 the Log-normal distribution with SI = 0.25,  $\overline{\delta} = 0.5$ ,  $\sigma_k = \sqrt{Log[SI+1]}$  and  $\mu_k = Log[\overline{\delta}] - \sigma_k^2 / 2$  and the exponential channel with SI = 1 and  $\overline{\delta} = 3$  are used.

*Conclusion*: In this letter, the numerical analysis of the optimum input RF power of the FSO system has been derived as a function of the number of users under two turbulence channels considering IM3 products. First, the verification of the numerical analysis has been shown by comparing with the directed searched method. Then, for a given number of users, the optimum input RF power which gives the minimum average BER has been presented.

*Acknowledgment*: This research was supported by the World-Class University Program funded by the MEST though the NRF (R31-10026).

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## **Figure captions:**

Fig. 1 Average BER as a function of the input RF power according to three user types (M = 8, 16 and 32).



Fig.2 Comparison of the approximated optimum RF power with the directly searched value.

- - - Approximated P<sub>RF</sub> given by (6) under the Log-normal channel

Approximated  $P_{RF}$  given by (9) under the Exponential channel



Directly searched under the Log-normal channel

Directly searched under the Exponential channel

Fig. 3 Average BER as a function of the number of users under the Log-normal channel (with SI = 0.25) and the Exponential channel (with SI = 1).



Figure1



Input RF power (dBm)





Number of users (M)





Number of users (M)