

Average BER analysis of SCM-based free-space optical systems by considering the effect of IM3 with OSSB signals under turbulence channels

Wansu Lim¹, Tae-Sik Cho¹, Changho Yun², and Kiseon Kim¹

¹Department of Information and Communications, Gwangju Institute of Science and Technology (GIST), 261 Cheomdan-gwagiro(Oryong-dong), Buk-gu, Gwangju 500-712 Korea

²Korea Ocean Research and Development Institute, Ansan P.O.Box 29, 425-600 Korea

wansu99@gist.ac.kr

Abstract: In this paper, we derive the average bit error rate (BER) of subcarrier multiplexing (SCM)-based free space optics (FSO) systems using a dual-drive Mach-Zehnder modulator (DD-MZM) for optical single-sideband (OSSB) signals under atmospheric turbulence channels. In particular, we consider the third-order intermodulation (IM3), a significant performance degradation factor, in the case of high input signal power systems. The derived average BER, as a function of the input signal power and the scintillation index, is employed to determine the optimum number of SCM users upon the designing FSO systems. For instance, when the user number doubles, the input signal power decreases by almost 2 dBm under the log-normal and exponential turbulence channels at a given average BER.

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1. Introduction

Nowadays, as Internet is widely spread, a volume of multimedia traffic is streamed into networks and the number of users is ever increasing. Hence, efficient wideband communication systems which accommodate multiple users are necessary. One of the good candidate for wideband communication systems is subcarrier multiplexing (SCM) based free space optical (FSO)

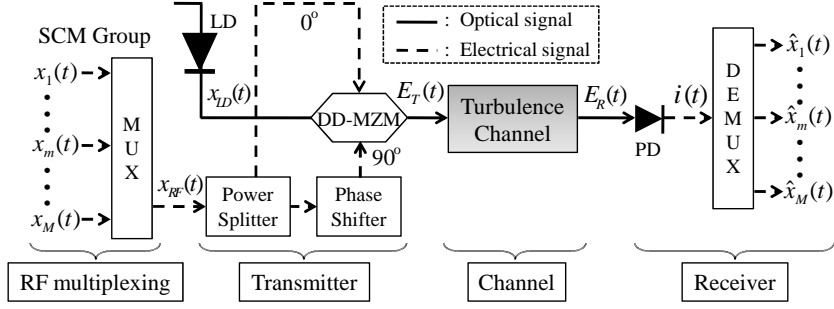


Fig. 1. Overall architecture of the FSO system considering of optical transmitter, turbulence channels, and optical receiver.

systems because SCM schemes have great flexibility in allocating bandwidth, with utilizing a single wavelength, thereby enabling them to accommodate a wide range of users and applications [1] and FSO systems can provide cost-effective broadband bandwidth with high data rates [2].

The primary methods for enhancing the performance of the SCM based FSO systems has been to increase the signal power to ensure a high signal-to-noise ratio (SNR). However, this increase in power also results in problems that degrade system performance, such as an increase of laser relative intensity noise (RIN), and harmonic distortion from the Mach-Zehnder modulator (MZM) and other devices in FSO systems. Among these degradation factors, intermodulation distortion can significantly degrade the system performance because intermodulation power is typically higher than noise power in systems with high input power [3]. Fortunately, a majority of second-order intermodulation (IM2) terms can be eliminated by a symmetrical dual-drive MZM (DD-MZM). However, third-order intermodulation (IM3) terms are more severe as they are too close to the fundamental frequency component, and their power increase is faster [4]. Thus, a more detailed investigation of SCM-based FSO system performance that considers the effects of IM3 is required.

In this paper, we first represent the signal-to-noise-and-distortion ratio (SNDR) that includes the IM3 factor by using a Bessel expansion with a small-signal approximation. We then analyze the average BER by considering the effects of IM3 under atmospheric channels, where the log-normal and exponential distributions describe turbulence-induced fading in a range from severe to moderate. Finally, numerical results are provided to illustrate the degradation of performance according to the input signal power and the scintillation index.

2. FSO system architecture and derivation of SNDR

Figure 1 describes the overall architecture of an SCM-based FSO system. Here, $x_{LD}(t)$ is the optical signal from a laser diode (LD), we assume that the phase noise of LD is negligible, $x_{RF}(t) = x_1(t) + \dots + x_m(t) + \dots + x_M(t)$ is the group of input RF signals, $x_m(t)$ is the tone signal allocated to one user in the SCM group, and PD is a photodetector. As shown in the Fig. 1, the optical signal $x_{LD}(t)$ and the input RF signals $x_{RF}(t)$ can be modulated using DD-MZM and a 90° phase shifter to generate optical single-sideband (OSSB) signals. Then, The input signals are $x_{LD}(t) = A \cdot \exp(j\omega_{LD}t)$, $x_{RF}(t) = \sum_{m=1}^M x_m(t)$, and $x_m(t) = V_{RF} \cdot \cos(\omega_m t)$ where $1 \leq m \leq M$, A and V_{RF} are the optical carrier and RF signal amplitudes, respectively, and ω_{LD} and ω_m are the angular frequencies of the signals. We assume that the frequency difference between users is larger than the bit duration; thus, we use tone for the user signal.

Using a Bessel function, the output signal of DD-MZM can be expressed as [3]

$$\begin{aligned}
E_T(t) &= \frac{L_{att} \cdot A \cdot e^{j\omega_{LD}t}}{2} \left\{ \exp j \left[\frac{\pi \cdot x_{RF}(t)}{\sqrt{2}V_\pi} \right] + \exp j \left[\frac{\pi}{2} + \frac{\pi \cdot \tilde{x}_{RF}(t)}{\sqrt{2}V_\pi} \right] \right\} \\
&\simeq \frac{L_{att} \cdot A \cdot e^{j\omega_{LD}t}}{2} \{B(t) + C(t)\} \\
B(t) &= \sum_{n=-\infty}^{\infty} j^n e^{jn\omega_1 t} J_n(\beta\pi) \times \dots \times \sum_{n=-\infty}^{\infty} j^n e^{jn\omega_m t} J_n(\beta\pi) \times \dots \times \sum_{n=-\infty}^{\infty} j^n e^{jn\omega_M t} J_n(\beta\pi) \\
C(t) &= j \sum_{n=-\infty}^{\infty} e^{jn\omega_1 t} J_n(\beta\pi) \times \dots \times j \sum_{n=-\infty}^{\infty} e^{jn\omega_m t} J_n(\beta\pi) \times \dots \times j \sum_{n=-\infty}^{\infty} e^{jn\omega_M t} J_n(\beta\pi)
\end{aligned} \tag{1}$$

where V_π and $\beta = V_{RF}/(\sqrt{2}V_\pi)$ are the switching voltage and the normalized ac of DD-MZM, respectively. In addition, $\tilde{x}_{RF}(t)$ is the phase-shifted version of $x_{RF}(t)$ and $L_{att} = 10^{-L_{DM}/20}$ is the attenuation of DD-MZM due to insertion loss L_{DM} . From (1), the fundamental and IM3 components will be generated by beating each signal; then, after the transmission of the turbulence channels, the optical signal can be detected by the PD. In this paper, we focus our investigation on the worst performances among the SCM group due to the IM3 components, since the worst performance may limit the whole system. Specifically, we consider IM3 components having the same frequency as the user—at most by $\lfloor (M-1)/2 \rfloor$ times—because when M is odd, the worst user has an $(M-1)/2$ pair, and when M is even the worst user has an $M/2 - 1$ pair. In this case, the photocurrent $i(t)$ after passing PD can be obtained as

$$\begin{aligned}
i(t) &= \Re |E_R(t)|^2 + n(t) \\
&= i_{f_w}(t) + i_{2f_{w+1}-f_{w+2}}(t) + i_{2f_{w+2}-f_{w+4}}(t) + i_{2f_{w+3}-f_{w+6}}(t) + \dots \\
&\quad + i_{2f_{w+m}-f_{w+2m}}(t) + \dots + i_{2f_{w+\lfloor \frac{M-1}{2} \rfloor} - f_{w+2\lfloor \frac{M-1}{2} \rfloor}}(t) + i_{s,o}(t) + n(t)
\end{aligned} \tag{2}$$

where \Re is the responsivity, $E_R(t)$ is the received optical signal at PD, $i_{f_w}(t)$ is the fundamental frequency component of the worst user, $i_{2f_{w+1}-f_{w+2}}(t) + \dots + i_{2f_{w+\lfloor (M-1)/2 \rfloor} - f_{w+2\lfloor (M-1)/2 \rfloor}}(t)$ are the IM3 components happening at the same frequency as the worst user by $\lfloor (M-1)/2 \rfloor$ times, $i_{s,o}(t)$ are other spurious terms such as IM2, and $n(t)$ are additive noises such as thermal and shot noises. Note, however, that IM2 terms are ignored in this paper since they are easily removed by utilizing an appropriate filter; hence, we concentrate on the fundamental frequency and IM3 components. Additionally, we assume that high-order components of the Bessel function are negligible since the value of $\beta\pi$ in a Bessel function is very small due to the fact that $V_\pi \gg V_{RF}$ in general. As such, using power $P = |i(t)|^2$ and $J_n(\beta\pi) \approx (\beta\pi)^n / 2^n n!$ for $\beta\pi \ll 1$ [3], the power of a fundamental frequency component of the worst user P_{f_w} can be expressed as

$$\begin{aligned}
P_{f_w} &= (\delta L_{att}^2 A^2 \Re \cdot J_0^{2M-1}(\beta\pi) \cdot J_1(\beta\pi))^2 \\
&\simeq \left(\frac{1}{2} \beta\pi \delta L_{att}^2 A^2 \Re \right)^2, \quad \beta\pi \ll 1
\end{aligned} \tag{3}$$

where δ is the turbulence channel coefficient. Similarly, the power of IM3 $P_{2f_{w+m}-f_{w+2m}}$ can be obtained as

$$\begin{aligned}
P_{2f_{w+m}-f_{w+2m}} &= 2 \left[\frac{M-1}{2} \right] (\delta L_{att}^2 A^2 \Re)^2 (D_{IM}^2 + E_{IM}^2) \\
&\simeq 2 \left[\frac{M-1}{2} \right] \left(\frac{1}{16} (\beta\pi)^3 \delta L_{att}^2 A^2 \Re \right)^2 \Psi, \quad \beta\pi \ll 1
\end{aligned} \tag{4}$$

$$D_{IM} = \frac{1}{\sqrt{2}} \left\{ J_0^M(\beta\pi) J_1(\beta\pi) J_2(\beta\pi) [\cos(\phi_{2f_{w+m}} - \phi_{f_{w+2m}} + \pi/4) + \sin(\phi_{2f_{w+m}-f_{w+2m}} - \pi/4)] \right. \\ \left. - J_0(\beta\pi)^{M-1} J_1^3(\beta\pi) \sin(\phi_{f_{w+2m}-f_{w+m}} - \phi_{f_{w+m}} + \pi/4) \right\} \quad (5)$$

$$E_{IM} = \frac{1}{\sqrt{2}} \left\{ J_0^M(\beta\pi) J_1(\beta\pi) J_2(\beta\pi) [\cos(\phi_{2f_{w+m}} - \phi_{f_{w+2m}} - \pi/4) - \sin(\phi_{2f_{w+m}-f_{w+2m}} + \pi/4)] \right. \\ \left. - J_0(\beta\pi)^{M-1} J_1^3(\beta\pi) \sin(\phi_{f_{w+2m}-f_{w+m}} - \phi_{f_{w+m}} - \pi/4) \right\} \quad (6)$$

$$\Psi = 3 - \cos(\phi_{2f_{w+m}-f_{w+2m}} - \phi_{2f_{w+m}} + \phi_{f_{w+2m}}) + 2 \cos(\phi_{2f_{w+m}-f_{w+2m}} \\ + \phi_{f_{w+2m}-f_{w+m}} - \phi_{f_{w+m}}) - 2 \cos(\phi_{2f_{w+m}} - \phi_{f_{w+2m}} + \phi_{f_{w+2m}-f_{w+m}} - \phi_{f_{w+m}}) \quad (7)$$

where the frequency is changed to $\phi_{f_m} = \pi L D \lambda^2 f_m^2 / c$ due to dispersion [6] in the turbulence channel, λ is the LD wavelength, L is the communication distance, D is the dispersion parameter, and c is the speed of light. Since the frequencies (f_w, f_m) of signals are much higher than the difference ($|f_w - f_m|$) [7], Ψ can be approximated as

$$\Psi \simeq 5 - 3 \cos\left(\frac{2\pi L D \lambda^2 f_w^2}{c}\right). \quad (8)$$

Next, the probability density function (pdf) of the turbulence channel coefficient δ can be modeled as [5]

$$f_\delta(\delta) = \frac{1}{\sqrt{2\pi\sigma_k^2}\delta} \exp\left(-\frac{(\ln(\delta) - \mu_k)^2}{2\sigma_k^2}\right), \quad \delta > 0 \quad (9)$$

where $\delta = e^K$, μ_k and σ_k denote the mean and standard deviation of K , respectively. Here, the scintillation index is defined as $SI = \frac{E[\delta^2]}{E[\delta]^2} - 1$, and the log-normal channel is characterized by a scintillation index less than 0.75 (i.e., weak turbulence) [5]. If the scintillation index is close to 1 and/or the propagation length is long (i.e., strong turbulence), the pdf of δ becomes the exponential distribution [5], such that

$$f_\delta(\delta) = \frac{1}{\bar{\delta}} \exp\left(-\frac{\delta}{\bar{\delta}}\right), \quad \delta > 0 \quad (10)$$

where, $\bar{\delta}$ is $E[\delta]$.

3. Average BER analysis of FSO systems

In this section, we derive a closed form of the average BER by considering the turbulence channels and IM3. First, we define the SNDR as follows:

$$SNDR = \frac{P_{f_w}}{P_{2f_{w+m}-f_{w+2m}} + P_{th} + P_{shot}} \quad (11) \\ \simeq \frac{\delta^2 G P_{RF}}{\delta^2 H P_{RF}^3 + 4kTB + q\mathfrak{R}A^2\delta B} \quad \text{for } \beta\pi \ll 1$$

where

$$G = \left(\frac{\pi L_{att}^2 A^2 \mathfrak{R}}{2V\pi}\right)^2, \quad H = \frac{1}{128} \left[\frac{M-1}{2}\right] \left(\frac{\pi^3 L_{att}^2 A^2 \mathfrak{R}}{V\pi^3}\right)^2 \cdot \Psi, \quad (12)$$

$k = 1.38 \times 10^{-23} J/K$ is the Boltzmann constant, $q = 1.6 \times 10^{-19} C$ is the electron charge, P_{th} is the thermal noise power, P_{shot} is the shot noise power, $T = 300K$ is the absolute temperature, B is the effective noise bandwidth, and $P_{RF} = V_{RF}^2/2$ is the input signal power. Using the above SNDR and turbulence channels, we then derive the average BER (P_b) as

$$P_b = \int_0^\infty Q(\sqrt{SNDR}) f_\delta(\delta) d\delta. \quad (13)$$

Using the change of variable $x = (\ln\delta - \mu_k)/\sqrt{2}\sigma_k$ and the Gauss-Hermite quadrature formula [8], (13) can then be simplified as

$$P_b = \frac{1}{\sqrt{\pi}} \sum_{i=1}^N w_i Q \left(\sqrt{\frac{GP_{RF} e^{2(\sqrt{2}\sigma_k x_i + \mu_k)}}{HP_{RF}^3 e^{2(\sqrt{2}\sigma_k x_i + \mu_k)} + 4kTB + q\mathfrak{R}A^2 B e^{(\sqrt{2}\sigma_k x_i + \mu_k)}}} \right) \quad (14)$$

where N is the order of approximation, $x_i, i = 1, \dots, N$ are the zeros of the N th-order Hermite polynomial and $w_i, i = 1, \dots, N$ are weight factors for the N th-order approximation; $N = 10$ is used for the analysis [5]. Finally, by employing the same process as for the log-normal channel, the average BER for the exponential channel can be given as

$$P_b = \sum_{i=1}^N w_i |x_i| Q \left(\sqrt{\frac{\bar{\delta}^2 GP_{RF} x_i^4}{\bar{\delta}^2 HP_{RF}^3 x_i^4 + 4kTB + q\mathfrak{R}A^2 B \bar{\delta} x_i^2}} \right). \quad (15)$$

4. Numerical results

Figure 2 illustrates the results of the average BER in both the log-normal and the exponential channels. The system parameters used for the analysis include the following: the wavelength (λ) is 1550 nm, the switch voltage (V_π) is 2.5 V, the DD-MZM insertion loss (L_{DM}) is 6 dB, the responsivity (\mathfrak{R}) is 0.8 A/W, the communication distance (L) is 2 km, and the worst RF frequency (f_w) is 25 GHz. Since SNDR is sensitive to the input signal power and the total number of users (M), as shown in (14), in Fig. 2(a) we present the average BER as a function of the input signal power according to the total number of users under the log-normal and the exponential channels. In Fig. 2(b), we show the relationship between the scintillation index and the average BER.

In Fig. 2(a), the numerical results also show that when the user number doubles, the input signal power decreases by almost 2 dBm under both turbulence channels at a given average BER. Additionally, for a given the input signal power of 20 dBm, it is shown that the difference of average BER corresponding to 8 users and 32 users under weak turbulence is almost 2.5 times larger than that under strong turbulence. Then, Fig. 2(b) shows that as the scintillation index increases from 0.25 to 0.75, the average BER decreases almost 4.1832 dB for 8 users at a signal power of 20 dBm; the numerical results for these conditions show that the difference of the average BER is almost 2.74 times larger than for 32 users at the signal power of 18 dBm under the log-normal channel.

5. Conclusions

In this paper, we investigated SNDR using a Bessel expansion by considering the effect of IM3 components. Also, we derived a closed-form average BER performance under atmospheric turbulence channels using log-normal and the exponential distributions based on the Gauss-Hermite quadrature formula. As a result, we could more easily predict average BER performance without requiring complicated calculations. In practical terms, when establishing FSO

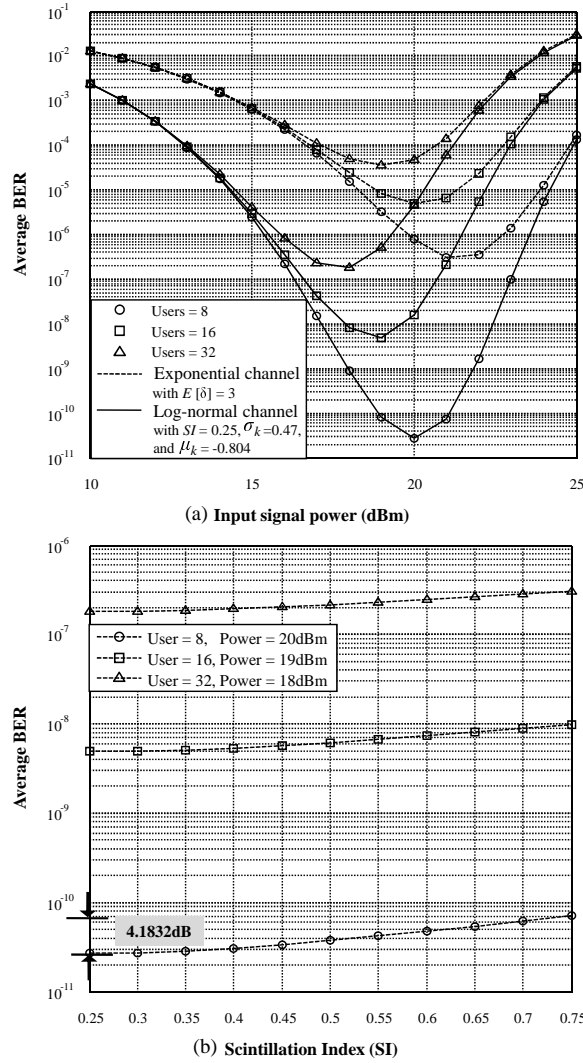


Fig. 2. Average BER: (a) as a function of the input signal power under the log-normal channel (with $SI = 0.25$) and the exponential channel ($E[\delta] = 3$) according to three sets of users (e.g., 8, 16, 32), and (b) as a function of the scintillation index under the log-normal channel according to three sets of users with an optimal input signal power.

systems, we could construct an engineering table using this derived average BER formula according to the input signal power and the number of users. It is noteworthy that all the derived equations are reasonable only $\beta\pi \ll 1$. Thus, when we establish FSO systems practically, we should carefully consider the desired environment.

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