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Was Proxima captured by Alpha Centauri A and B?

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ABSTRACT

The nearest stellar system consists of the stars Proxima, Alpha Centauri A and B and at least one planet Proxima b. The habitability of Proxima b and any other planets are likely to be significantly influenced by the orbital evolution of the system. To study the dynamical evolution of the system, we simulate the motions of Proxima and Alpha Centauri A and B due to the perturbations from the Galactic tide and stellar encounters in a Monte Carlo fashion. From 100 clones, we find that 74 per cent orbits of Proxima Centauri are bound to Alpha Centauri A and B while 17 per cent and 9 per cent orbits become unbound in the simulations over the past and future 5 Gyr. If the system migrated outward in the Milky Way to its current location, more than 50 per cent of clones could become unstable in backward simulations. The ratio of unstable clones increases with the simulation time-scale and encounter rate. This provides some evidence for a capture scenario for the formation of the current triple system. Despite large uncertainties, the metallicity difference between Proxima and Alpha Centauri A and B is also suggestive of their different origin. None the less, further improvements in the available data and models will be necessary for a reliable assessment of the history of the Proxima–Alpha Centauri system and its impact on the habitability of Proxima b.

Key words: catalogues – binaries: general – stars: individual: Alpha Centauri – stars: kinematics and dynamics – Galaxy: kinematics and dynamics – solar neighbourhood.

1 INTRODUCTION

Since the discovery of Proxima Centauri b, a planet with a mass of $1.3 M_{\oplus}$ orbiting Proxima (Innes 1915) with a period of 11.2 d (Anglada-Escudé et al. 2016), intensive studies have been performed to investigate its habitability. Various scenarios for the formation of Proxima b have been proposed (Alibert & Benz 2017; Coleman et al. 2017) and its habitability has been comprehensively studied (Barnes et al. 2016; Ribas et al. 2016). The association between Proxima and Alpha Centauri A and B have been frequently discussed in previous studies (Voûte 1917; Wertheimer & Laughlin 2006; Matvienko & Orlov 2014; Pourbaix & Boffin 2016). Proxima, a late-type M dwarf with a mass of $0.1221 \pm 0.022 M_{\odot}$ (Mann et al. 2015), has been confirmed to be bound to Alpha Centauri A ($1.1055 \pm 0.0039 M_{\odot}$; Kervella et al. 2016b) and B ($0.9373 \pm 0.0033 M_{\odot}$; Kervella et al. 2016b) by the recent use of the HARPS radial velocity data to constrain its orbit (Kervella, Thévenin & Lovis 2017). This triple system is composed of a close binary and a wide companion, which is a typical configuration for triples (Tokovinin et al. 2006). Proxima has a semimajor axis of $8.7_{-0.4}^{+0.7}$ kau, eccentricity of $0.50_{-0.09}^{+0.08}$, orbital period of 547_{-40}^{+66} kyr, and inclination of $107.6_{-2.0}^{+1.8}$ deg with respect to Alpha Centauri A and B (Kervella et al. 2017).

According to Barnes et al. (2016), the tidal force from Alpha Centauri A and B is important for our understanding of the evolution and habitability of Proxima b. However, they did not account for the anisotropy in the velocity space of stellar encounters with respect to the primary star (Feng & Bailer-Jones 2014). They probably overestimate the radial migration of the Sun and thus assume the formation of Alpha Centauri A and B at a Galactocentric distance from 1.5 to 4.5 kpc, which is inconsistent with recent studies of the solar motions in the Galaxy accounting for the uncertainties of the Sun’s motion (e.g. Feng & Bailer-Jones 2013; Martínez-Barbosa, Brown & Portegies Zwart 2015). In addition, they did not use the new data from Kervella et al. (2017) and thus adopt arbitrary initial conditions for the Proxima–Alpha Centauri system.

Although Kervella et al. (2017) find a >10 Gyr stability of the Proxima and Alpha Centauri system, the tidal radius (Jiang & Tremaine 2010) they have adopted is not reliable due to an underestimation of stellar density and unrealistic assumption of isotropic encounter velocities (Feng & Bailer-Jones 2014). Considering these limitations, we assess the stability of the Proxima–Alpha Centauri system by simulating the motions of this system under the perturbations from the Galactic tide and stellar encounters. We will derive the initial conditions of Alpha Centauri A and B and Proxima from the data in Kervella et al. (2017) and adopt a comprehensive encounter model according to Feng & Bailer-Jones (2014).

The paper is structured as follows. In Section 2, we describe the numerical method and the models for encounters and the Galactic

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tide. We report the results in Section 3 and conclude and discuss in Section 4.

2 METHOD

We derive the initial conditions of Proxima and Alpha Centauri in the Galactocentric reference frame from the data in Kervella et al. (2017). The initial conditions of the Sun and the model of the Galactic potential are the same as in Feng & Bailer-Jones (2014). We simulate the motions of clones under perturbations from the Galactic tide and stellar encounters using the Bulirsch–Stoer method (Bulirsch & Stoer 1964) implemented in the `PRACMA` package of `R`. According to our tests, this integrator conserves the orbital energy and angular momentum with a relative numerical error down to 10^{-8} over 1 Gyr with a time-step of 0.01 Myr. We also define a clone of the Proxima–Alpha Centauri system as unstable if the eccentricity e of Proxima with respect to Alpha Centauri A and B is larger than one. Since some clones that become eccentric may be perturbed back to a bound state, a clone is considered as unstable only if it never comes back on to a stable orbit (i.e. $e < 1$) within the simulation time span.

Since the orbital period of Proxima around Alpha Centauri is around 0.55 Myr which is much longer than the interaction time-scale of about 0.02 Myr between encounters and the binary, assuming an encounter velocity of 50 km s^{-1} according to Feng & Bailer-Jones (2014) and periapsis of 1 pc. Thus, we can apply the so-called ‘impulse approximation’ (Rickman 1976) to calculate the perturbation of velocity from an encounter, which is

$$\Delta \mathbf{v}_* = \frac{2GM_{\text{enc}}}{v_{\text{enc}}d_{\text{enc}}}\mathbf{e}_{\text{enc}}, \quad (1)$$

where \mathbf{v}_* is the impulse gained by the target star, G is the gravitational constant, M_{enc} is the encounter mass, d_{enc} is the impact parameter or periapsis, and \mathbf{e}_{enc} is the direction of periapsis. For every 1 Myr, we randomly generate stellar encounters according to the model in Feng & Bailer-Jones (2014) and calculate the velocity kick for Alpha Centauri A and B and Proxima.

The main uncertain parameters in the above encounter model is the encounter rate. In Feng, Jones & Tanvir (2017), we find that the encounter rate for the Solar system is larger than 15 per Myr for encounters with perihelia less than 1 pc, which is consistent with the value of 20 per Myr given by Bailer-Jones (2017) based on studies of TGAS encounter candidates. However, this value may be underestimated because GL 710 and the Scholtz’s star are found to pass the solar system at 0.06 and 0.25 pc about 1.35 Myr from the present and 0.07 Myr ago, respectively (Mamajek et al. 2015; Berski & Dyczyński 2016). This is roughly equivalent to one encounter passing the Sun within 0.1 pc Myr^{-1} , although the sample of encounters are still incomplete (Feng 2016; Feng et al. 2017). In other words, there are probably around 100 encounters passing the Sun within 1 pc every Myr.

We confirm this estimation using the formula of encounter rate which is

$$F = n\sigma\bar{v}_{\text{enc}}, \quad (2)$$

where σ , \bar{v}_{enc} , n are the cross-section, velocity of the encounter in the heliostatic frame and local stellar number density. According to Binney & Tremaine (2008), the cross-section σ is

$$\sigma = \pi D^2 \left(1 + \frac{2G(M_{\text{enc}} + M_{\odot})}{Dv_{\text{enc}}^2} \right), \quad (3)$$

where D is the maximum perihelion or impact parameter of encounters, M_{\odot} and M_{enc} are the masses of the Sun and the encounters, respectively, and G is the gravitational constant. If D is set to 1 pc, the second term in the bracket is far less than one. Thus, the above cross-section is approximately πD^2 .

The local number density is 0.098 pc^{-3} based on the sample of 211 stars within 8 pc (excluding brown dwarfs) from the Sun collected by Kirkpatrick et al. (2012). However, this number density is a lower limit of the real stellar number density due to the incompleteness of dwarf stars and other faint stars whose astrometry is unknown or poorly known. This is evident from the new discoveries of M, L, T and Y dwarfs (e.g. Finch et al. 2014; Finch & Zacharias 2016; Kirkpatrick et al. 2016). The stellar density of local mid-plane is about $0.1 M_{\odot} \text{ pc}^{-3}$ according to Pham (1997), Holmberg & Flynn (2000), Korchagin et al. (2003) and Soubiran, Bienaymé & Siebert (2003). The mean stellar mass is about $0.4 M_{\odot}$ based on the mass of 129 stars within 12 light years (or 6.44 pc) from the Sun.¹ Thus, the stellar number density local to the Sun would be 0.25 pc^{-3} . Assuming $\bar{v}_{\text{enc}} = 75 \text{ km s}^{-1}$ according to Feng et al. (2017), the flux of encounters with $d_{\text{enc}} < 1 \text{ pc}$ is about 60 per Myr.

If we include brown dwarfs and other low mass objects into our encounter model, the encounter rate could be higher. Compared with Kirkpatrick et al. (2012)’s sample, there are around six new brown dwarfs discovered within a heliocentric distance of 6.5 pc (Bihain & Scholz 2016). In addition, the non-uniform distribution of nearby brown dwarfs indicates a bias in the observations of brown dwarfs (Bihain & Scholz 2016). This anisotropy may disappear due to new discoveries which would increase the brown-dwarf-to-star ratio and not change the local stellar number density much if the ratio is around 1 (Chabrier 2003). Moreover, given ongoing new discoveries the current list of the nearest M dwarfs is probably not complete (e.g. Scholz 2014). Considering all these factors, we set a default encounter rate of 80 per Myr and use the encounter model in Rickman et al. (2008) to generate encounters. We will also investigate the importance of the encounter rate for a sensitivity test in the next section.

Therefore, we will simulate 320 encounters of Alpha Centauri A and B with impact parameter d_{enc} less than 2 pc every Myr, assuming that the local number density of Alpha Centauri A and B is the same as the Solar system. The number of encounters per Myr is proportional to the local stellar density and thus is modulated during the migration of Alpha Centauri A and B in the Galaxy. As the encounter velocity is calculated with respect to Alpha Centauri and thus would not be isotropic, the mean directions of encounters would also be modulated due to a change of the peculiar motion of Alpha Centauri A and B.

We also follow Feng & Bailer-Jones (2015) to predict the strength of an encounter using

$$g = \frac{M_{\text{enc}}}{d_{\text{enc}}^2 v_{\text{enc}}}, \quad (4)$$

where M_{enc} is the mass of encounter. In this work, the impact parameter d_{enc} is calculated in the frame centred at the barycentre of Proxima and Alpha Centauri.

¹ <http://www.johnstonsarchive.net/astro/nearstar.html>

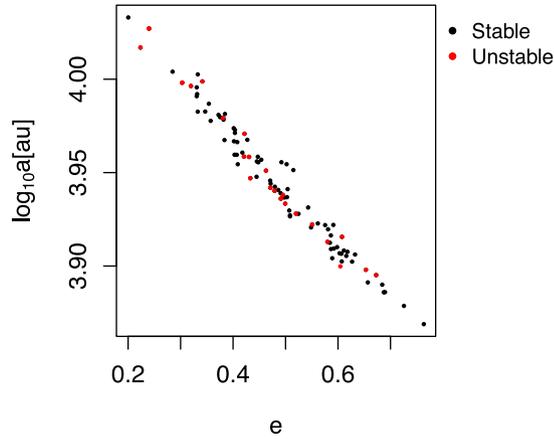


Figure 1. Distribution of initial eccentricity and semimajor axis of 100 clones of Proxima. The colours encode the final stage of their evolution in the simulations from -5 to $+5$ Gyr. The current encounter rate is set to be 80 encounters with $d_{\text{enc}} < 1$ pc every Myr.

3 RESULTS

We generate 100 clones of Proxima and Alpha Centauri according to the data uncertainty given by Kervella et al. (2017). We simulate the orbits of these clones under the perturbations from the Galactic tide and stellar encounters from -5 to $+5$ Gyr appropriate to the age for Alpha Centauri A and B (e.g. 4.7–5.2 Gyr; Bazot et al. 2016). The same sample of encounters are applied to different clones. We then show the distribution of initial eccentricity e and semimajor axis a of these clones with respect to the barycentre of Alpha Centauri A and B in Fig. 1. We find 17 per cent and 9 per cent clones of Proxima were/will be ejected from the system, respectively, while the other clones experience strong orbital variation. Since the age of Proxima is probably older than 5 Gyr (Eggenberger et al. 2004), we simulate another 100 clones of Proxima from -7 to 7 Gyr and find 23 per cent and 15 per cent clones of Proxima were/will be ejected from the system. Since the encounter sample is the same for all clones and some clones are ejected while others are not, the final status of clones are sensitive to their initial conditions. In other words, some clones have trajectories that are close to strong encounters while other clones are not. We also vary the encounter rate F (see equation 2) and find an ejection ratio of 11 per cent, 31 per cent and 54 per cent for simulations with the rate of encounters with $d_{\text{enc}} < 1$ pc in units of Myr^{-1} , $F = 40$, 60 and 100 from -7 to 7 Gyr, respectively. Based on a linear fit of the ejection ratio for these simulations, we find that the percentage of unstable clones r is $\sim 0.5F$. Thus, the stability of the Centauri system is sensitive to the encounter rate.

In Fig. 1, we see that the clones that are ejected seem to be randomly scattered in the $e - a$ distribution. This distribution indicates a high correlation between a and e , which approximately follows $a \propto 10^{-e}$. This is probably due to the fact that eccentricity or angular momentum is less sensitive to astrometric uncertainties than semimajor axis or orbital energy. The ejected clones follows a similar distribution to the stable ones, indicating that the ejection ratio is more sensitive to eccentricity than to semimajor axis. Thus, a more precise measurements of orbital eccentricity is essential to improve the reconstruction of Proxima’s dynamical history.

To see the effect of encounters, we select two stable and two unstable clones and show the evolution of their orbital elements under the perturbations of the Galactic tide alone and of both the Galactic tide and encounters in Figs 2 and 3, respectively. Without stellar encounters, the semimajor axes or orbital energy of clones

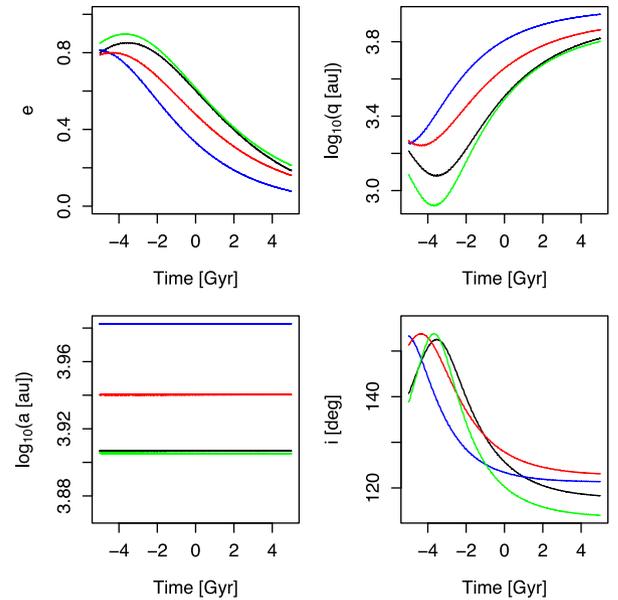


Figure 2. Variation of eccentricity e , periastron q , semimajor axis a and inclination i , of Proxima under the perturbations from the Galactic tide based on simulations from -5 to $+5$ Gyr. The black and blue lines denote the orbital elements of clones which were/will be ejected through combined perturbations of stellar encounters and the Galactic tide. The red and green lines denote the orbits of stable clones.

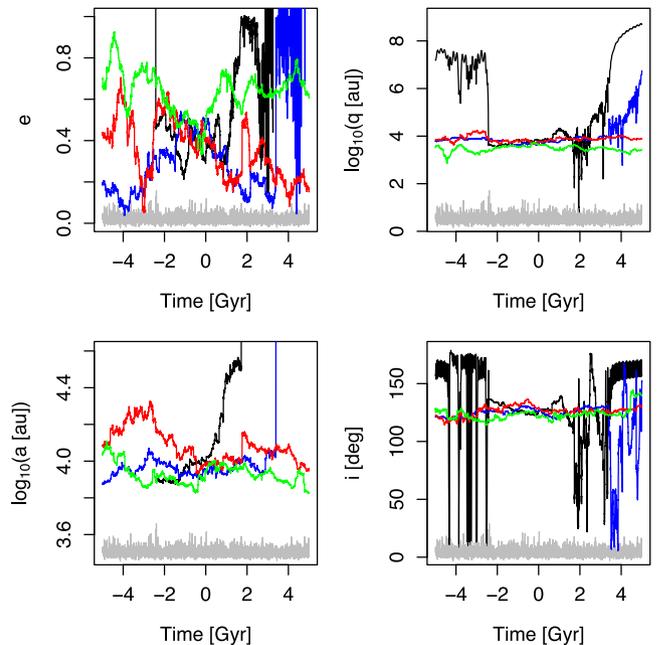


Figure 3. Similar to Fig. 2, but for orbital elements of Proxima under the combined perturbations from the Galactic tide and stellar encounters. The grey lines at the bottom shows the strength of encounters characterized by g (see equation 4), which is smoothed with a 1 Myr time bin.

are almost constant over time. The eccentricity, periastron and inclination vary gradually and continuously in the whole simulations, and the evolution of orbital elements is sensitive to the argument of periastron (Veras & Evans 2013). However, if encounters are included in the simulations, all orbital elements vary significantly and stochastically due to the random impulses from encounters. Apart from the orbital elements shown in Fig. 3, we also find significant

variation in the argument of periastron ω and relatively weaker variation in the longitude of ascending node Ω , indicating a more sensitive response of ω to encounter perturbations than Ω .

In Fig. 3, we also see a strong connection between the jump of the orbital elements and the peaks in the bottom grey lines. That means the strength of the perturbation imposed by an encounter is well characterized by the proxy of g defined in equation (4). This is consistent with the conclusion based on the study of the Oort cloud by Feng & Bailer-Jones (2015) and the analytical expression of encounter-induced eccentricity change derived by Heggie & Rasio (1996).

The orbits of clones are especially sensitive to strong encounters, which are slow, massive and extremely close to the clones. This is evident from the abrupt change of semimajor axis caused by strong encounters. However, strong encounters only play a role to trigger the escape of clones from the system while weak encounters cumulatively randomize the clones' orbits and thus pave the way for escape. In other words, the orbital variation is more like a stochastic process than a deterministic process. Hence, an analytical escape radius/zone based on the occurrence rate of strong encounters (Veras et al. 2014; Portegies Zwart & Jílková 2015) is probably not reliable for stability analysis for multiples, especially for wide multiples like the Proxima–Alpha Centauri system. But a statistical analogue to the escape radius can be derived based on simulations of a large sample of wide multiples covering larger parameter space, which is beyond the scope of this work.

4 DISCUSSION AND CONCLUSION

While Proxima Centauri has been of particular recent interest due to the discovery of Proxima b, the Alpha Centauri system has long been studied for many reasons by a variety of different techniques. Of particular interest to us is the extent to which these studies support a common origin for all three stars. Given the benchmark status of the system, there are a number of diagnostics available.

In this work, we investigate the capture scenario by simulating the Proxima–Alpha Centauri system over the past and future 5 and 7 Gyr and find a considerable percentage of Proxima's clones become unstable in backward simulations. Although most N -body simulations are not strictly time reversible due to cumulative numerical errors (Rein & Tamayo 2017), the simulations in this work are time reversible in a statistical sense. The fact that some clones of Proxima become unstable by looking backward is equivalent to them becoming stabilized by looking forward. Thus, our simulations not only provide evidence against the *in situ* scenario but also for the capture scenario. In addition, many disc stars are likely to have migrated from near the Galactic Centre to their current locations (Sellwood & Binney 2002). If the Proxima–Alpha Centauri system formed near the Galactic Centre, it would experience more intensive perturbations from the Galactic tide and encounters and more Proxima clones would become unbound in backward simulations, and thus the capture scenario would be more favoured. For example, according to Barnes et al. (2016), the Proxima–Alpha Centauri system has migrated outwards at least 3.5 kpc, corresponding to a stellar density five times higher at the formation region than at the Sun's current location. Thus, the encounter rate would also be at least five times higher during the early evolution of the system, leading to more than 50 per cent ejection percentage assuming $r \sim 0.5 F$ and a linear radial migration.

Moreover, the capture of a field star is not rare if the rate of encounters with $d_{\text{enc}} < 1$ pc is 80 every Myr, equivalent to about one encounter with $d_{\text{enc}} < 0.1$ pc every Myr. There are more than 0.1 per cent encounters moving extremely slowly with respect to

the reference star, with a relative velocity less than 1 km s^{-1} (Feng et al. 2017). Hence, a Sun-like system would encounter a few extremely close and slow encounters after the dissolution of its birth cluster. These encounters could be captured by the system and become stabilized through the perturbations from the Galactic tide and stellar encounters, as seen in the simulations of Proxima clones. Moreover, the eccentricity evolution in tide-only simulations shown in Fig. 2 indicates that Proxima was on a highly eccentric orbit about 5 Gyr ago, probably corresponding to an unstable stage after capture. This capture scenario provides another channel for the formation of multiples, in addition to the dynamical unfolding scenario (Reipurth & Mikkola 2012). If the capture scenario is plausible for Proxima, it would be more plausible for the formation of wider multiples. A comparative study of the capture and *in situ* scenarios (e.g. Kouwenhoven et al. 2010; Moeckel & Clarke 2011) would help to infer the formation channels from the orbital characteristics of the current sample of wide binaries (e.g. Caballero 2007; Price-Whelan, Oh & Spergel 2017).

The dynamical evolution of the Proxima–Alpha Centauri triple is sensitive to its encounter history. However, the current sample of encounters of Alpha Centauri A and B is too small for a realistic reconstruction of the encounter history even within 10 Myr (Feng et al. 2017). With the upcoming *Gaia* data releases, we expect to identify more encounters within the past 1 Gyr (Feng 2016). This would provide a realistic sample for the reconstruction of the dynamical history of Proxima and thus for the assessment of the dynamical habitability of Proxima b and other potential planets in the Proxima–Alpha Centauri system.

Considering that many clones are ejected in the 5-Gyr simulations, the dissolution time of >10 Gyr introduced by Jiang & Tremaine (2010) is probably not a reliable metric to estimate the long-term stability of wide binaries. This is probably due to their underestimation of the encounter rate based on a stellar density of 0.05 pc^{-3} . In addition, they assume isotropic encounter velocities or equivalently that the Sun is static with respect to the local standard of rest, which is unrealistic (Feng & Bailer-Jones 2014). A revised version of tidal radius based on realistic encounter model is needed to estimate the stability of wide binaries.

The tidal force caused by the quadratic potential of Alpha Centauri A and B is not accounted for in our simulations. This force becomes significant when the periastron of Proxima is less than 2000 au (Barnes et al. 2016). In our simulations, few stable orbits of clones have periastrons below this limit while the unstable orbits tend to have lower periastron before being ejected. Despite this most unstable clones would become unbound at their apses due to stronger and longer perturbations from encounters and the Galactic tide. In other words, most unstable clones will be kicked out of the system before arriving at their periapses. Thus, the quadratic potential of A and B is not important for the estimation of the ejection ratio. But, it is probably essential for a comprehensive study of the dynamical habitability of Proxima b and other potential planets in the Alpha Centauri A and B since the members in a hierarchical multiple system might influence the secular evolution of each other according to studies of hierarchical multiples (Hamers & Portegies Zwart 2016).

In addition to dynamical studies, co-evality of the components in the Proxima–Alpha Centauri system can be judged from the consistency of their evolutionary history as determined by different isochrone fits to their luminosity and mass and the similarity of their metallicity as judged by spectral synthesis. For Alpha Cen A and B, there is a long history of studies which converge to find them as having a significantly metal-rich composition and solar-type age. For example, Jofré et al. (2015) use eight different

methods to determine abundances to find $[\text{Fe}/\text{H}] = 0.26 \pm 0.08$ and 0.22 ± 0.10 , respectively, and do not find any particular peculiarities relative to a differential analysis with respect to the Sun other than for Mn which is expected due to its sensitivity to microturbulence. In terms of age, Bazot et al. (2016) consider the range of available asteroseismic data for Alpha Cen A with applicable models in the range of 4.7–5.2 Gyr.

Derivations of age and metallicity for Proxima are substantially more uncertain since it is a late-type M dwarf which is relatively less well studied. The question of the evolutionary history of the Alpha Centauri system and particularly the metallicity for Proxima has been recently addressed by Beech, McCowan & Peltier (2017). They find the best match for Proxima to have a metallicity of $[\text{Fe}/\text{H}] = -0.5$ or by constraining a simultaneous fit with the values of mass and radius used by Kervella et al. (2017), a metallicity of $[\text{Fe}/\text{H}] = -0.3$. However, there are a number of uncertainties involved and a metallicity as high as solar is allowed within their 1σ error bars when considering a mass–radius relationship based on absolute K magnitude. There are a number of recent derivations of spectroscopic metallicities for Proxima showing a spread around a solar-like value, e.g. -0.07 ± 0.14 (Passegger, Wende-von Berg & Reiners 2016) and 0.05 ± 0.20 (Kervella et al. 2016a). For hotter stars, ages maybe reliably determined by asteroseismic and activity measurements. However, late-type M dwarfs like Proxima lie in a rather poorly calibrated regime. Proxima presents both a high flare rate (e.g. Davenport et al. 2016) and slow period (e.g. Collins, Jones & Barnes 2017). The data from Sloan Digital Sky Survey indicate that late-type M5 and M6 dwarfs have activity lifetimes of 7 ± 0.5 Gyr (West et al. 2008). So while the metallicity of Proxima (and of all late-type M dwarfs) is relatively poorly constrained, recent literature results are suggestive of a metallicity difference between Proxima and Alpha Centauri A and B which would be consistent with different formation histories.

As concluded in Barnes et al. (2016), Alpha Centauri A and B could be close enough to destabilize the orbit of Proxima b. However, if Proxima was captured by Alpha Centauri, Proxima b may only be influenced by Alpha Centauri A and B for a relatively short period of time, and thus was probably orbiting in the current habitable zone for a long time. In summary, a comprehensive study of the dynamical history and composition of Proxima and Alpha Centauri A and B is crucial for a complete understanding of their formation and evolution and the habitability of their planets.

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