The Frequency of Low-Mass Exoplanets. III. Toward $\eta_\oplus$ at Short Periods

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ABSTRACT

Determining the occurrence rate of terrestrial-mass planets ($m_p < 10 M_\oplus$) is a critically important step on the path towards determining the frequency of Earth-like planets ($\eta_\oplus$), and hence the uniqueness of our Solar system. Current radial-velocity surveys, achieving precisions of $1 \, \mathrm{m \, s^{-1}}$, are able to detect terrestrial-mass planets and provide meaningful estimates of their occurrence rate. We present an analysis of 67 solar-type stars from the Anglo-Australian Planet Search specifically targeted for very high-precision observations. When corrected for incompleteness, we find that the planet occurrence rate increases sharply with decreasing planetary mass. Our results are consistent with those from other surveys: in periods shorter than 50 days, we find that 1.5% of stars host a giant ($m_p > 100 M_\oplus$) planet, and that 18.5% of stars host a planet with $m_p < 10 M_\oplus$. The preponderance of low-mass planets in short-period orbits is in conflict with formation simulations in which the majority of terrestrial-mass planets reside at larger orbital distances. This work gives a hint as to the size of $\eta_\oplus$, but to make meaningful predictions on the frequency of terrestrial planets in longer, potentially habitable orbits, low-mass terrestrial planet searches at periods of 100-200 days must be made an urgent priority for ground-based Doppler planet searches in the years ahead.

Subject headings: planetary systems – techniques: radial velocities

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1. Introduction

To date, 20 extrasolar planets are known[1] with minimum masses (m sin i) less than $10M_\oplus$. Hundreds more planet candidates with sizes smaller than a few Earth radii, and therefore potentially terrestrial in nature, have been identified by the *Kepler* spacecraft (Borucki et al. 2011a,b). It is clear that terrestrial-mass planets exist, but what is not yet clear is the percentage of stars that form such planets, and how often those planets survive post-formation dynamical interactions in order to be observed today. The frequency of Earth-mass planets in the habitable zone, often referred to as $\eta_\oplus$, is a key science driver for the *Kepler* and *CoRoT* missions. Within this decade, these space missions are anticipated to provide an estimate of $\eta_\oplus$ with unparalleled accuracy and precision. However, the bottleneck for large transit surveys has always been the radial-velocity follow-up to obtain mass estimates for planet candidates. Until the multifarious candidates identified by these spacecraft have mass determinations, radial-velocity surveys capable of $1\,\mathrm{m\,s^{-1}}$ precision will make a critical contribution to constraining $\eta_\oplus$ and the planetary mass function. This work is prompted by the recent results of Howard et al. (2010), who presented estimates for the occurrence rate of planets in short periods ($P < 50$ days) from the NASA-UC Eta-Earth survey. Our aim is to verify those results by using their methods on our own independent data set.

The Anglo-Australian Planet Search (AAPS) has undertaken two long, continous observing campaigns (48 and 47 nights) with the aim of detecting low-mass planets in periods shorter than 50 days (O'Toole et al. 2009c). These two "Rocky Planet Search" campaigns have targeted a total of 54 bright, stable stars with spectral types between G0 and K5. This strategy of observing through a dark lunation facilitates the detection of planets in that period regime by suppressing the window function near one lunar month (29 days). Previous AAPS planet discoveries arising from these observing campaigns include HD 16417b (O'Toole et al. 2009b), 61 Vir b,c,d (Vogt et al. 2010), and HD 102365b (Tinney et al. 2011). In addition, we have chosen a subset of the AAPS main program stars for observation at high precision by requiring a signal-to-noise (S/N) of at least 300 per epoch. Since the aim is a single-epoch radial-velocity precision of $1\,\mathrm{m\,s^{-1}}$ (in the absence of stellar jitter), we designate these as "One Meter Per Second" (OMPS) stars. There are 67 OMPS stars in the AAPS target list, of which the 54 Rocky Planet Search targets form a subset. All of these stars receive at least 20 minutes of integration time per epoch, in order to average over the stellar p-mode oscillations (O'Toole et al. 2008).

We have previously presented detailed simulations of planet detectability based on data

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[1]http://www.exoplanets.org

from the 24 stars observed in the first Rocky Planet Search campaign in 2007: exploring the frequency of planets with periods less than ∼20 days in O'Toole et al. (2009c), and investigating the nature of the "period valley" in Wittenmyer et al. (2010). In this work, we consider the entirety of the OMPS and Rocky Planet Search targets, applying our simulation algorithms to the 67 stars in the AAPS sample which have data with the highest velocity precisions. In particular, we seek to compute the completeness of this sample and to estimate the occurrence rate of low-mass planets with periods $P < 50$ days. We choose this period bin to match the primary focus of Howard et al. (2010) and compare our results with theirs. In Section 2, we present the input data sample and the methods used to obtain detection limits. In Section 3, we compute the completeness of the sample and determine the occurrence rate of planets in four mass bins, before drawing conclusions in Section 4.

2. Data Properties and Analysis Methods

In this work, we focus on the 67 "OMPS" stars in the AAPS program which have the highest radial-velocity precision. The OMPS target stars were selected on the basis of being apparently inactive (log $R'_{HK} < -4.7$) and lacking a massive (i.e. $>10M_{\rm Jup}$) companion, and bright enough to obtain S/N>300 in no more than 30 minutes of integration time. This represents essentially all of the AAPS target stars down to $V = 6.50$, with a few additional stars being added between $V = 6.5 - 7.0$ to fill in observing gaps in right ascension.

We fit for and removed velocity trends due to stellar companions, as well as the orbits of known planets. The data are summarized in Table 1, and a histogram of the number of observations is shown in Figure 1. The parameters of all known planets in this sample are given in Table 2.

We determined the detectability of planets in these data by adding simulated Keplerian signals to the velocity data, then increasing the velocity amplitude (K) of the artificial planet until 100% of signals at that period were recovered. For a given K at a given orbital period P, we use a grid of 30 values of periastron passage T_0. A signal was considered recovered if its period in a standard Lomb-Scargle periodogram (Lomb 1976; Scargle 1982) had a false-alarm probability (FAP) of less than 0.1%. Trials were also performed at recovery rates ranging from 10% to 90%. The simulated planets had periods between 2 and 1000 days, with 100 trial periods evenly spaced in log P. As in Howard et al. (2010), the simulated planets had zero eccentricity. This method is identical to that used in our previous work (e.g. Wittenmyer et al. 2006, 2009, 2010, 2011).

3.   Results and Discussion

Figure 2 shows the mass limits (m sin i) averaged over all 67 stars at four recovery rates: 100%, 70%, 40% and 10%. Detected planets in the sample are represented by large filled circles. Throughout the discussion on the results of these simulations, we use "mass" to refer to the projected planetary mass m sin i obtained from radial-velocity measurements. Since the inclination of the system is generally unknown, the planetary mass (m sin i) is a minimum value.

3.1.   Completeness Correction

The 67 stars considered here host 18 currently known planets orbiting 12 stars. However, to determine the *underlying* frequency of planets in the sample, we need to use the detectabilities we obtain from the simulations to correct these detections for our survey's varying completeness as a function of planet period and planet mass. Moreover, because these detectabilities vary from star to star, we need to make this completeness correction on a star-by-star, rather than on a whole-of-survey basis.

We therefore estimate how many planets have been "missed" from our survey as a whole, by calculating the "missed planet" contribution for each detected planet using

$$N_{missed} = \left[\frac{1}{N_{stars}} \sum_{j=1}^{N_{stars}} f_{R,j}(P_i, M_i) \right]^{-1} - 1, \tag{1}$$

where $f_{R,j}(P_i, M_i)$ is the recovery fraction as a function of mass M_i at period P_i (for the ith detected planet), and N_{stars} is the total number of stars in the sample ($N = 67$). There are i detected planets in the sample, and j stars in total. For a detected planet with period P_i and mass M_i, each star contributes a detectability $f_R(P_i, M_i)$ between 0 and 1 to the sum in Equation (1). The quantity $f_R(P_i, M_i)$ is the fraction of simulated planets with period P_i and mass M_i which were recovered. In this way, we compute the detectability averaged over the whole sample for each detected planet at the specific (P_i, M_i) of that planet. This approach, also employed in Wittenmyer et al. (2011), thus accounts for the non-uniformity of detectability across the sample. We show the results of these calculations in the column labeled "Method 1" of Table 3.

This method for estimating the number of "missed planets" (i.e. the correction for survey completeness) is nearly identical to that used by Howard et al. (2010), except that they defined "completeness" as the fraction of *stars* for which a planet of mass M at period

P was recovered in 100% of trials. That is, each star contributes either 0 or 1 to the sum in Equation (1). Since that work considered only the 100% recovery level, a star whose detection limit falls just short of the mass for a given planet would be counted as *never* able to detect that planet, whereas the true detectability may still be significant (i.e. >90%). We have used our simulation results (at 100% recovery) to estimate the number of "missed planets" using this method, and this is given in Table 3 as "Method 2." The completeness computations of Howard et al. (2010) excluded stars with detected planets; our results for Method 2 thus excluded the 12 planet hosts, leaving 55 stars. The last column of Table 3 ("Method 3") gives the results obtained using this method when we include the 12 planet-host stars in the calculations.

We see a pronounced difference in the results obtained by Methods 1 and 2/3 for the lowest-mass planets: when the completeness is a binary function (either a planet is detected 100% of the time or it is never detected), the number of missed planets is poorly sampled. Indeed, this can lead to nonsensical results, e.g. an infinite number of missed planets. This unphysical result occurs when considering the detection of planets with properties that match those of the super-Earth HD 115617b (=61 Vir b; Vogt et al. 2010). At the mass and period of HD 115617b, none of the data sets for the 55 non-planet-hosting stars in our sample enabled the detection of the simulated planet in 100% of trials. This resulted in $f_R(P, M) = 0$, and hence an infinite number of missed planets by Equation (1). HD 115617 is an unusual target in that we have a large number of observations ($N = 139$) and it is a very stable star, with a residual velocity rms (to the three-planet fit) of only $2.3\,\mathrm{m\,s}^{-1}$. This extreme example highlights two important points to consider in the estimation of the frequency of extremely low-mass planets: first, that meaningless results are obtained when the completeness approaches zero, and second, inhomogeneities in planet-search data sets require a detailed, star-by-star approach to best determine the true underlying frequency of low-mass planets. Great caution is therefore required when interpreting results from survey-completeness simulations such as these, especially when considering terrestrial-mass planets, where current radial-velocity surveys are heavily affected by incompleteness.

For the AAPS data considered here, the missed-planet correction used by Howard et al. (2010) (given in Table 3 as Method 2), is clearly not useful. Even when planet-hosting stars are included (Method 3), the correction for missed planets gives a result that is unjustifiably overestimated. This is due to the uneven data density for our sample, as the detection limits achievable depend heavily on the number of observations (Wittenmyer et al. 2011). The AAPS sample has a mean $N_{obs} = 88\pm45$, whereas the Keck sample has a mean $N_{obs} = 40\pm22$.

Nonetheless, the high-N_{obs} tail seen in Figure 1 reflects a reality for all radial-velocity programs: stars with candidate low-mass planets (e.g. 61 Vir) are prioritised and receive a

larger number of observations. This directly leads to the situation described above, where a very low-mass planet *only* has a high detectability for that one star, and can have detectabilities of, e.g. only 10% for the remaining targets in the sample. Because the Keck sample of Howard et al. (2010) has a somewhat more uniform distribution in N_{obs}, their data are less prone to this feature, and their method is therefore less problematic. It is, however, inappropriate for our data and we adopt Method 1 for this work and all subsequent discussion.

3.2. Close-in Planets ($P < 50$ days)

To directly compare our results with those of Howard et al. (2010), we now focus on periods shorter than 50 days. While Howard et al. (2010) used mass bins of width $\log_{10}(\Delta M_{Earth}) = 0.5$, our sample has fewer detected planets, so we use four mass bins of width $\log_{10}(\Delta M_{Earth}) = 1.0$.

We estimate the frequency of planets in each bin using binomial statistics, after Howard et al. (2010). That is, we compute the binomial probability of detecting exactly k planets in a sample of n stars, with the underlying probability p of hosting a planet. We compute this over all p to find the most probable value (Figure 3). In this way, we estimate the planet frequency and its 1-sigma uncertainty (68.3% confidence interval) for each of the four mass bins. The planet frequencies obtained are then adjusted for incompleteness by multiplying each bin's frequency and its uncertainty by a factor $(N_{detected} + N_{missed})/N_{detected}$. The results are shown in Table 4. The uncertainties on our measured planet frequency are large, owing to the small number of detections (7 planets with $P <$50 days) compared to the Keck survey (16 planets). As previous studies have shown (e.g. Howard et al. 2010, Wittenmyer et al. 2010, O'Toole et al. 2009c), planet frequency increases as planet mass decreases; more low-mass planets are found despite the fact that they are much more difficult to detect. Figure 4 plots the derived planet frequencies from this work and those of Howard et al. (2010) for direct comparison. Our results are consistent with those of the NASA-UC Eta-Earth survey: we find that $18.5^{+12.9}_{-18.5}\%$ of stars host terrestrial-mass planets ($M_p < 10 M_\oplus$) at periods of less than 50 days.

4. Conclusions

Our data are consistent with the estimation of Mayor et al. (2009) that $30\pm10\%$ of solar-type stars host a planet with m sin $i \lesssim 30 M_\oplus$ and $P < 50$ days. These results, and those

of other radial-velocity planet search teams, support the idea that close-in terrestrial-mass planets (or "super-Earths" with $m_p < 10 M_\oplus$) are quite common in orbital periods less than 50 days. These observational data are, at present, in disagreement with planet-formation simulations (e.g. Mordasini et al 2009, Ida & Lin 2005, 2008, Kornet & Wolf 2006) which predict an under-abundance of such planets orbiting inside of ~ 1 AU. Ida & Lin (2008) instead predict a large number of super-Earths to accumulate near the ice line, beyond 2 AU. Such objects are completely undetectable by current radial-velocity surveys, but the observational data in hand suggest that the planet population synthesis models require significant revision in order to reproduce the high abundance of close-in super-Earths for which there is now a growing body of evidence.

The frequency of habitable Earth-like planets ($\eta_\oplus$) is a key quantity to measure as we seek to understand the frequency of habitable environments in the Universe. However, it is important to note that while these results (and those of Howard et al. 2010) provide hints on the size of $\eta_\oplus$, they do not determine $\eta_\oplus$ directly. Given their orbital periods ($P < 50$d), and therefore semi-major axes (0.24–0.30 AU), none of the terrestrial-mass planets probed by these studies are actually habitable – they are all far too hot.

A key next step for this research will be extending searches for the lowest-mass planets to larger orbital periods (and so semi-major axes). If we can at least understand the trends in the frequency with which planet formation makes planets as a function of period, at periods from 50 to 100 and even 150d, then we will be in a much better position to make *robust* predictions as to the frequency with which habitable terrestrial planets (i.e. planets in 200-400d orbits) are formed around solar-type stars. Low-mass terrestrial planet searches at 100-200d must be made an urgent priority for ground-based Doppler planet searches in the years ahead (Guedes et al. 2008; Endl et al. 2009).

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Table 1.   Summary of Radial-Velocity Data

Star	N	RMS $(\mathrm{m\,s^{-1}})$	$<\sigma>$ $(\mathrm{m\,s^{-1}})$
HD 142	74	10.97	3.19
HD 1581	97	3.59	1.26
HD 2151	175	4.28	0.84
HD 3823	70	5.82	1.75
HD 4308	107	4.51	1.36
HD 7570	43	6.34	1.53
HD 10360	61	4.48	1.33
HD 10361	60	4.56	1.23
HD 10700	231	3.68	1.09
HD 13445	60	4.87	1.93
HD 16417	113	3.99	2.53
HD 20794	134	3.45	1.07
HD 20807	89	4.48	1.50
HD 23249	79	3.47	0.62
HD 26965	94	4.79	0.83
HD 27442	87	7.11	0.87
HD 28255A	61	7.23	1.69
HD 38382	36	4.83	1.69
HD 39091	59	5.57	2.23
HD 43834	123	4.98	1.15
HD 44120	32	3.55	1.71
HD 45701	30	5.86	2.00
HD 53705	125	4.55	1.60
HD 53706	38	3.02	1.47
HD 65907A	58	6.20	1.75
HD 72673	55	3.66	1.25
HD 73121	38	5.86	1.92
HD 73524	78	5.25	1.63
HD 75289	41	5.80	1.77
HD 84117	123	5.45	1.70
HD 100623	75	5.01	1.09
HD 102365	153	2.76	1.11
HD 102438	47	4.59	1.71
HD 108309	55	3.54	1.25
HD 114613	198	5.68	0.98
HD 115617	139	2.32	1.96
HD 122862	93	4.22	1.68
HD 125072	68	5.28	1.19
HD 128620	99	4.06	0.93
HD 128621	134	3.58	0.70
HD 134060	86	5.65	1.44
HD 134987	67	2.96	1.30
HD 136352	146	4.74	1.27
HD 140901	102	10.36	1.26
HD 146233	62	5.25	1.16

Table 1—Continued

Star	N	RMS $(\mathrm{m\,s^{-1}})$	$<\sigma>$ $(\mathrm{m\,s^{-1}})$
HD 156274B	92	4.83	1.29
HD 160691	167	2.25	0.89
HD 168871	62	4.91	1.92
HD 172051	49	3.37	1.13
HD 177565	90	3.98	1.15
HD 189567	79	5.55	1.63
HD 190248	208	4.05	0.96
HD 191408	168	4.20	1.17
HD 192310	146	3.93	1.15
HD 193307	76	4.27	1.79
HD 194640	70	4.83	1.46
HD 196761	38	4.78	1.01
HD 199288	68	5.48	2.23
HD 207129	114	4.95	1.22
HD 210918	65	5.32	1.28
HD 211998	40	14.69	3.02
HD 212168	42	5.59	1.67
HD 214953	76	4.98	1.73
HD 216435	74	7.05	2.08
HD 216437	49	4.92	1.74
HD 219077	60	3.89	1.36
HD 221420	70	4.77	1.51

Table 2.   Planets From This Sample

| Planet	Period (days)	M sin i $(\mathrm{M_{\oplus}})$	a (AU)	Discovery Ref.
HD 142 b	350.4±1.5	419.5±38.9	1.05±0.02	Tinney et al. (2002)
HD 4308 b	15.609±0.007	13.0±1.4	0.118±0.009	Udry et al. (2006)
HD 13445 b	15.7656±0.0005	1280.8±69.9	0.114±0.002	Queloz et al. (2000)
HD 16417 b	17.24±0.01	22.1±2.0	0.14±0.01	O'Toole et al. (2009b)
HD 27442 b	430.8±0.8	508.5±29.5	1.27±0.02	Butler et al. (2001)
HD 75289 b	3.50918±0.00003	146.3±6.8	0.048±0.001	Udry et al. (2000)
HD 102365 b	122.1±0.3	16.0±2.6	0.46±0.04	Tinney et al. (2011)
HD 115617 b	4.2150±0.0006	5.1±0.5	0.050201±0.000005	Vogt et al. (2010)
HD 115617 c	38.021±0.034	18.2±1.1	0.2175±0.0001	Vogt et al. (2010)
HD 115617 d	123.01±0.55	22.9±2.6	0.476±0.001	Vogt et al. (2010)
HD 134987 b	258.19±0.07	505.3±6.4	0.81±0.02	Vogt et al. (2000)
HD 134987 c	5000±400	260.6±9.5	5.8±0.5	Jones et al. (2010)
HD 160691 b	644.9±0.6	534.4±18.8	1.53±0.02	Butler et al. (2001)
HD 160691 c	4060±49	641.0±32.3	5.2±0.1	McCarthy et al. (2004)
HD 160691 d	9.641±0.002	9.1±1.0	0.093±0.001	Santos et al. (2004)
HD 160691 e	308.7±0.7	156.4±12.4	0.94±0.01	Pepe et al. (2007)
HD 216435 b	1332±14	405.6±33.4	2.59±0.05	Jones et al. (2003)
HD 216437 b	1354±6	714.5±34.9	2.54±0.04	Jones et al. (2002)
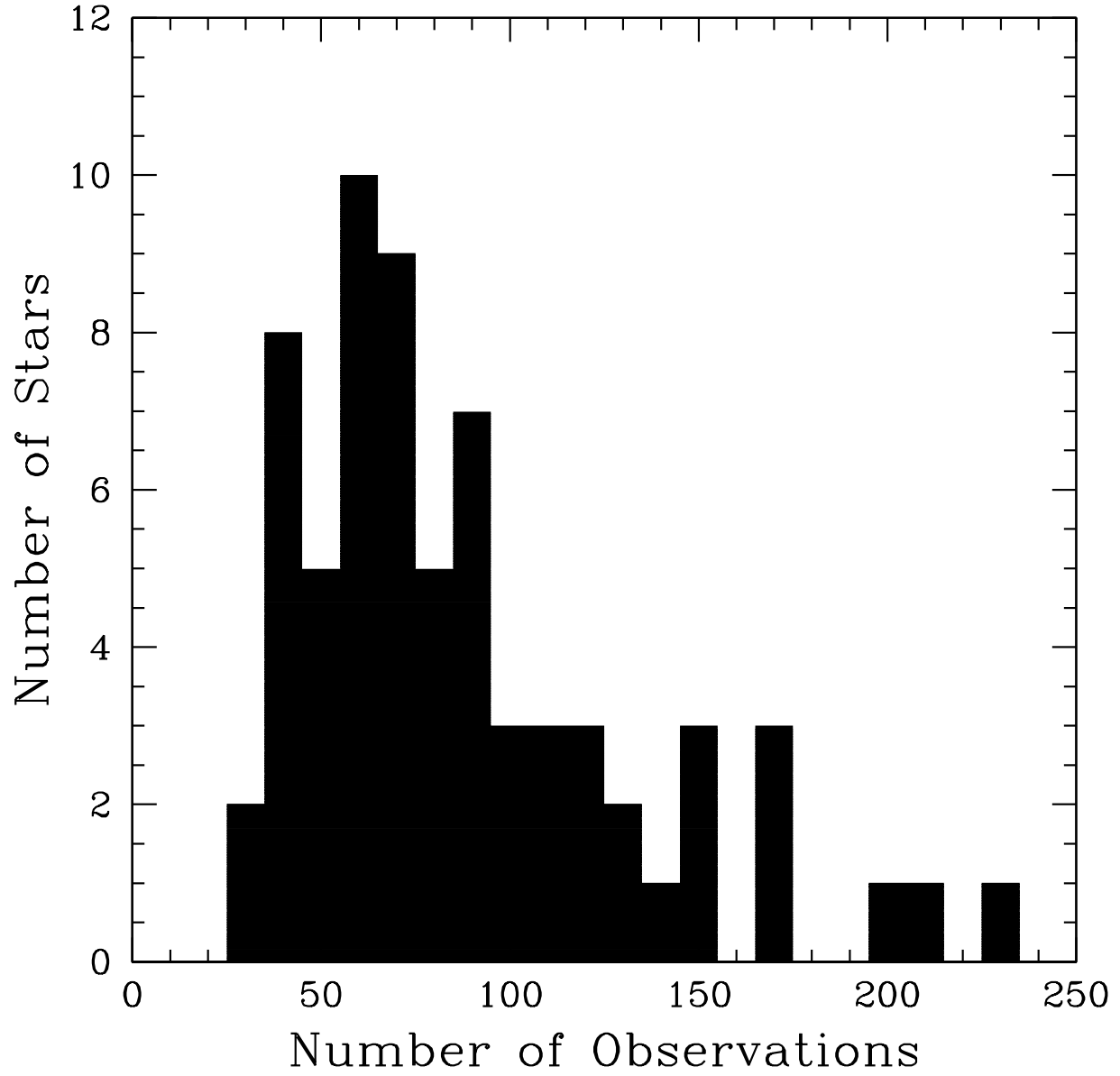
Fig. 1.— Histogram of the number of observations for the 67 AAPS stars considered here. Stars found to host low-mass planets contribute to the high-N tail.
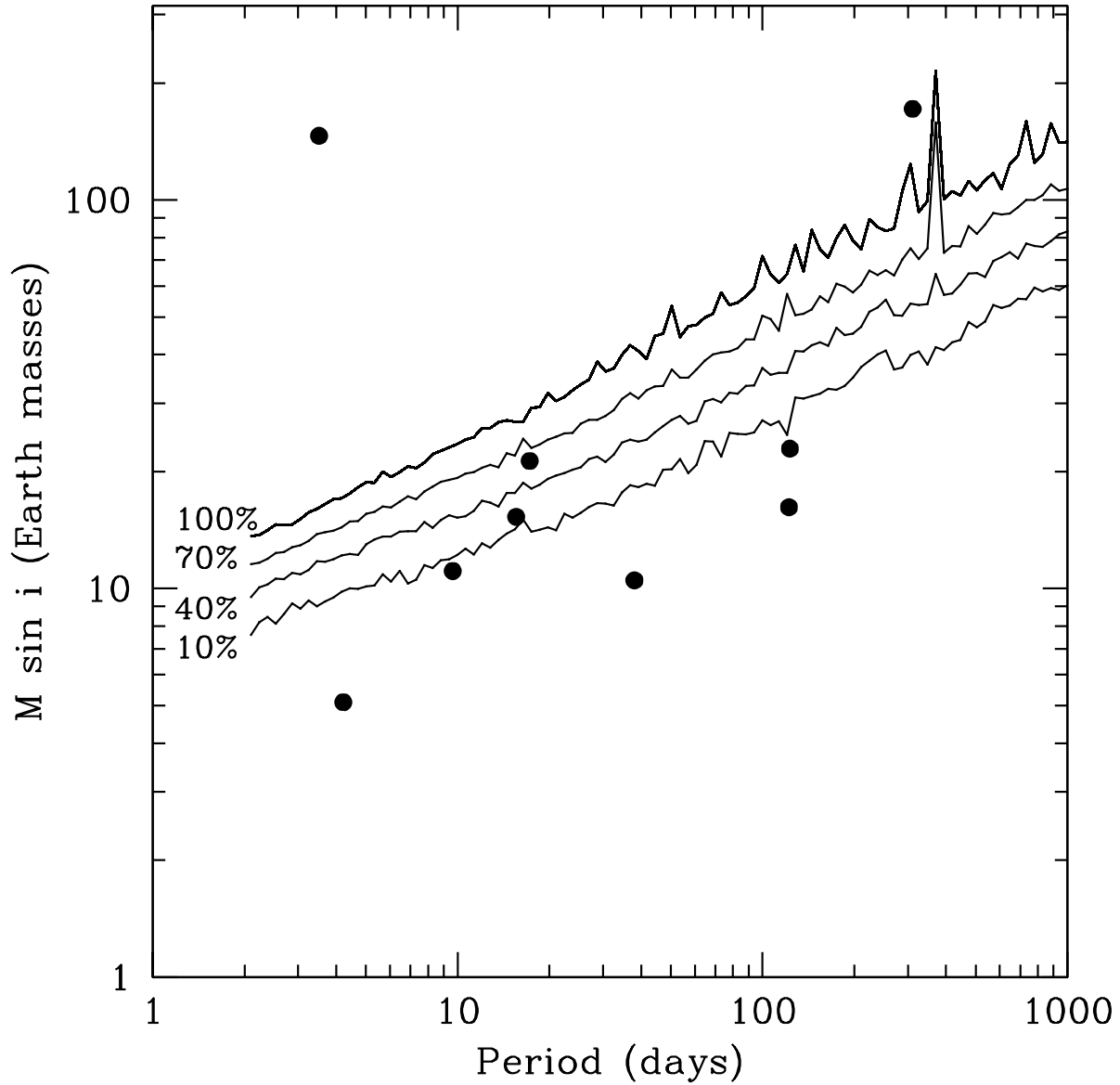
Fig. 2.— Detection limits for planets in circular orbits, averaged over the 67 stars considerd here. The contours indicate the fraction of injected planets which were recovered. Filled circles represent detected planets in the sample; a further 9 planets with large masses or long periods are off the scale.

Table 3.   Missed Planets in the Sample

Planet	Method 1	Method 2[a]	Method 3[b]
HD 142 b	0.0	0.1	0.1
HD 4308 b	1.4	6.9	5.1
HD 13445 b	0.0	0.0	0.0
HD 16417 b	0.3	1.0	1.0
HD 27442 b	0.0	0.0	0.0
HD 75289 b	0.0	0.0	0.0
HD 102365 b	4.7	54.0	21.3
HD 115617 b	7.0	Inf	21.3
HD 115617 c	1.2	12.8	7.4
HD 115617 d	2.1	54.0	21.3
HD 134987 b	0.0	0.1	0.0
HD 134987 c	⋯ [c]	⋯	⋯
HD 160691 b	0.0	0.1	0.0
HD 160691 c	⋯	⋯	⋯
HD 160691 d	3.3	17.3	10.2
HD 160691 e	0.0	0.1	0.1
HD 216435 b	⋯	⋯	⋯
HD 216437 b	⋯	⋯	⋯

[a]After Howard et al. (2010)

[b]Same as Howard et al. (2010) but including detectabilities from the planet hosts also.

[c]The simulations here considered only periods shorter than 1000 days, so detectability information is not available for these four long-period planets.

Table 4.   Short-Period Planet Frequencies

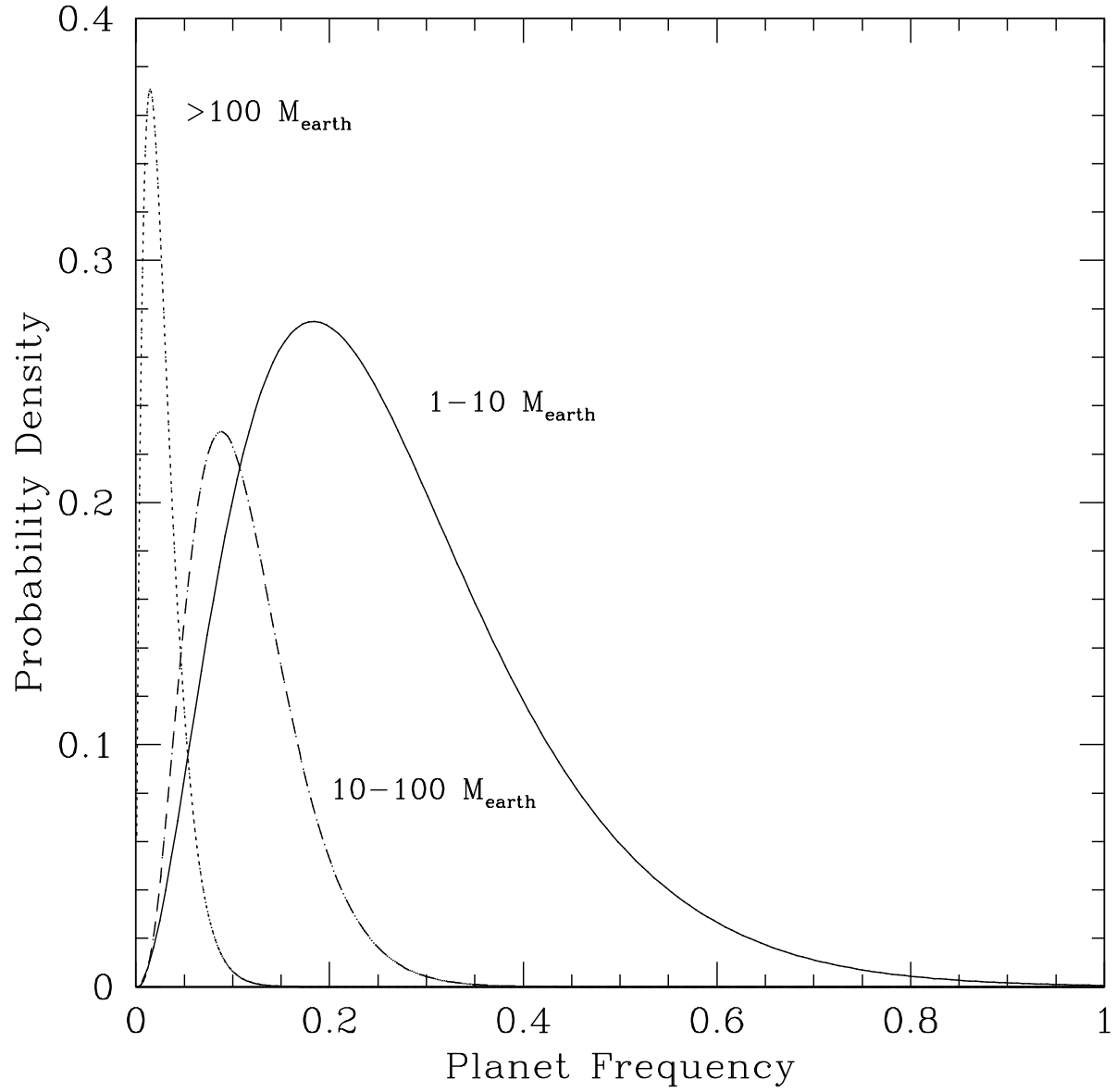| Mass Bin	Detections	N_{missed}	Frequency
1-10$M_\oplus$	2	10.3	$18.5^{+12.9}_{-18.5}\%$
10-100$M_\oplus$	3	2.9	$8.9^{+5.1}_{-6.1}\%$
100-1000$M_\oplus$	1	0.0	$1.5^{+1.6}_{-1.5}\%$
1000-10000$M_\oplus$	1	0.0	$1.5^{+1.6}_{-1.5}\%$

Fig. 3.— Binomial probability density functions computed for three mass bins and periods less than 50 days. Less-massive planets are clearly more prevalent.
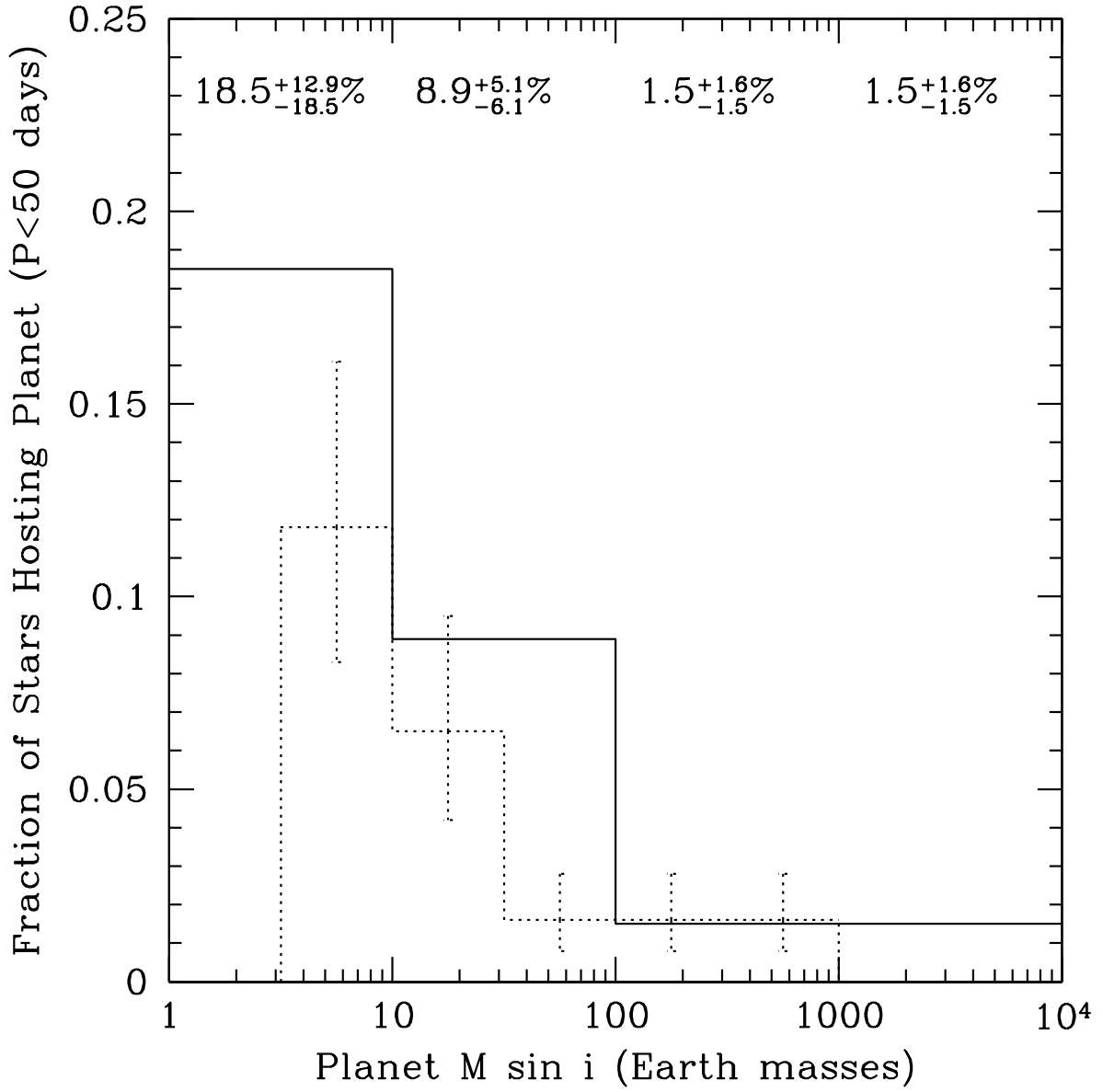
Fig. 4.— Planet frequency as a function of mass, from this work (solid histogram) and compared with Howard et al. (2010) (dashed histogram). The two sets of results are consistent within their uncertainties.