Proceedings Letters

Letters Section Update

Please note that technical letters are no longer being considered for publication in the Proceedings of the IEEE. As announced in earlier issues, the Letters Section will be discontinued in 1988. Letters already under review will be published upon acceptance as usual.

Pages previously occupied by letters will gradually be used for papers to expand coverage. To that end, readers are encouraged to submit proposals for high-quality, general-interest papers, especially in areas less well covered in the past. For further information, see "Notice to Readers" in the July 1987 issue, page 868, and the boxed text on the second page of each issue of the Proceedings.

The Electromagnetic Field in Rotating Coordinates

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The electromagnetic field equations in rotating coordinates obtained by various authors, are based on incorrect expressions for the divergence and curl operators and the electromagnetic spatial vectors. This is a serious error in this most important aspect of electromagnetic theory. Maxwell's equations are shown to be valid in rotating coordinates.

The problem of the electromagnetic field equations in rotating coordinates has been discussed in the literature by a number of authors. In particular, Schiff [1], and Atwater [2] obtained field equations which are extremely complicated and distinctly unlike Maxwell's equations. This is a serious problem in electromagnetic theory, because it challenges the validity of Maxwell's equations in rotating coordinates. We shall show that these treatments are faulty because they are based on incorrect expressions for the electromagnetic 3-vectors and the 3-current and charge densities, as well as the divergence and curl operators.

Shiozawa, (see remarks by T. Shiozawa in [2]), seems to have obtained Maxwell-like equations in rotating coordinates, but his treatment is also based on incorrect expressions for the various quantities. The main source of error in all these treatments, is the failure to take account of the fact that the geometry of physical 3-space in rotating coordinates is non-Euclidean.

A treatment of Maxwell's equations in general coordinates was given by Møller [3]. We have also shown that these equations are valid in all coordinate systems in flat and curved spaces [4]. Here we shall deal specifically with rotating coordinates.

We shall use the convention in which Greek indices take the values 1, 2, 3, 4 for the spacetime coordinates with $x^4 = t$, Roman indices take the values 1, 2, 3 for the space coordinates, the signature of the spacetime metric tensor $g_{\mu\nu}$ is +2, a comma denotes partial differentiation and we shall use MKS units.

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The electromagnetic field equations in vacuum electrodynamics expressed in tensor form are

$$F_{\mu\nu,\lambda} + F_{\nu\lambda,\mu} + F_{\lambda\mu,\nu} = 0$$

$$\frac{1}{\mu_0 \sqrt{-g}} (\sqrt{-g}\, F^{\mu\nu})_{,\nu} = J^\mu \tag{1}$$

where $F_{\mu\nu}$ and $F^{\mu\nu} = g^{\mu\alpha} g^{\nu\beta} F_{\alpha\beta}$ are the covariant and contravariant electromagnetic tensors, J^μ is the electric 4-current, μ_0 is the magnetic permeability of the vacuum, and $g = |g_{\mu\nu}|$. We shall assume that (1) are valid in all coordinate systems in both curved and flat spacetimes. The controversy concerns Maxwell's equations, which are the electromagnetic field equations expressed in terms of the spatial vectors **E**, **H**, **D**, **B**, **j** and scalar ρ. Here, **E** and **H** are the electric and magnetic intensities, **D** and **B** the corresponding inductions, **j** and ρ the 3-current and charge densities.

It may be shown [3], [4] that in any coordinate system in curved or flat spacetimes, (1) may be reduced to Maxwell's equations

$$\text{curl } \mathbf{E} = -\frac{1}{\sqrt{\gamma}} \frac{\partial}{\partial t} (\sqrt{\gamma}\, \mathbf{B}), \qquad \text{div } \mathbf{B} = 0$$

$$\text{curl } \mathbf{H} - \frac{1}{\sqrt{\gamma}} \frac{\partial}{\partial t} (\sqrt{\gamma}\, \mathbf{D}) = \mathbf{j}, \qquad \text{div } \mathbf{D} = \rho \tag{2}$$

where

$$\mathbf{E} = E_i = F_{i4} \qquad \mathbf{D} = D^i = -\frac{1}{\mu_0 c} \sqrt{-g_{44}}\, F^{i4}$$

$$\mathbf{H} = H_i = \frac{1}{2\mu_0 c}\, e_{ikp} \sqrt{-g_{44}}\, F^{kp} \qquad \mathbf{B} = B^i = \frac{1}{2}\, e^{ikp} F_{kp}$$

$$\mathbf{j} = j^i = \frac{1}{c} \sqrt{-g_{44}}\, J^i \qquad \rho = \frac{1}{c} \sqrt{-g_{44}}\, J^4 \tag{3}$$

with c the vacuum speed of light. Furthermore, $\gamma = |\gamma_{ij}|$ where $\gamma_{ij} = g_{ij} + \gamma_i \gamma_j$, with $\gamma_i = g_{i4}/\sqrt{-g_{44}}$, is the spatial metric tensor, and $e_{ikp} = \sqrt{\gamma}\epsilon_{ikp}$, $e^{ikp} = \epsilon_{ikp}/\sqrt{\gamma}$ are the completely antisymmetric permutation tensors, ϵ_{ikp} being the Levi-Civita symbol whose values are +1, −1, or 0 according as to whether ikp is an even permuation, an odd permaution, or no permutation of 123. Note that the magnitude of the spatial vector γ_i will not be used here, so no confusion should arise between this magnitude and γ. The divergence and contravariant curl of a spatial vector ***a*** and the contravariant vector product of two such vectors ***a*** and ***b*** are defined in a coordinateindependent manner by

$$\text{div}\,\mathbf{a} = \frac{1}{\sqrt{\gamma}}(\sqrt{\gamma}\,a^k)_{,k} \qquad \text{curl}\,\mathbf{a} = e^{ikp}a_{p,k} \qquad (\mathbf{a}\times\mathbf{b})^i = e^{ikp}a_k b_p. \tag{4}$$

We note that the indices of spatial vectors are lowered and raised using the tensors γ_{ij} and γ^{ij} respectively, and that the forms (covariant or contravariant) of the various quantities in (2), are those indicated by (3) and (4). We may further deduce after some lengthy algebra by expressing the $F^{\mu\nu}$ in terms of the $F_{\mu\nu}$ and using (3) and the third equation of (4), that the constitutive relations are

$$\mathbf{D} = \frac{\epsilon_0 c}{\sqrt{-g_{44}}}(\mathbf{E} + \mu_0 c\mathbf{H}\times\gamma) \qquad \mathbf{B} = \frac{\mu_0 c}{\sqrt{-g_{44}}}(\mathbf{H} - \epsilon_0 c\mathbf{E}\times\gamma) \tag{5}$$

where ϵ_0 is the electric permittivity of the vacuum and $\epsilon_0\mu_0 = 1/c^2$.

In the case of the coordinates $x^\mu = (x\,y\,z\,t)$ rotating with respect to the Lorentz coordinates $X^\mu = (X\,Y\,Z\,T)$ in the manner described in [1], [2], we have the transformation equations

$$\begin{aligned} X &= x\cos\omega t - y\sin\omega t \qquad Y = x\sin\omega t + y\cos\omega t \\ Z &= z \qquad T = t \end{aligned} \tag{6}$$

where $\omega = (0\,0\,\omega)$ is the (constant) angular velocity. The line element in the rotating coordinates x^μ is given by

$$ds^2 = dx^2 + dy^2 + dz^2 + 2\omega x\,dydt - 2\omega y\,dxdt - \frac{c^2}{\Gamma^2}dt^2 \tag{7}$$

where $\Gamma = (1 - \omega^2 r^2/c^2)^{-1/2}$ with $r^2 = x^2 + y^2$. We thus obtain for the nonzero elements of the symmetric tensors $g_{\mu\nu}$ and $g^{\mu\nu}$

$$\begin{aligned} &g_{11} = 1 \qquad g_{14} = -\omega y \qquad g_{22} = 1 \qquad g_{24} = \omega x \\ &g_{33} = 1 \qquad g_{44} = -\frac{c^2}{\Gamma^2} \end{aligned} \tag{8}$$

$$\begin{aligned} &g^{11} = 1 - \frac{\omega^2 y^2}{c^2} \qquad g^{12} = \frac{\omega^2 xy}{c^2} \qquad g^{14} = -\frac{\omega y}{c^2} \\ &g^{22} = 1 - \frac{\omega^2 x^2}{c^2} \qquad g^{24} = \frac{\omega x}{c^2} \qquad g^{33} = 1 \qquad g^{44} = -\frac{1}{c^2} \end{aligned} \tag{9}$$

with $g = -c^2$. The spatial vector γ_i is $\gamma_i = \Gamma(-\omega y/c\;\omega x/c\;0)$ and so we find for the nonzero elements of the symmetric tensors γ_{ij} and γ^{ij}

$$\begin{aligned} &\gamma_{11} = 1 + \frac{\Gamma^2\omega^2 y^2}{c^2} \qquad \gamma_{12} = -\frac{\Gamma^2\omega^2 xy}{c^2} \\ &\gamma_{22} = 1 + \frac{\Gamma^2\omega^2 x^2}{c^2} \qquad \gamma_{33} = 1 \end{aligned} \tag{10}$$

$$\gamma^{11} = 1 - \frac{\omega^2 y^2}{c^2} \qquad \gamma^{12} = \frac{\omega^2 xy}{c^2} \qquad \gamma^{22} = 1 - \frac{\omega^2 x^2}{c^2} \qquad \gamma^{33} = 1 \tag{11}$$

with $\gamma = \Gamma^2$. It is important to emphasize that the geometry of physical 3-space in rotating coordinates with spatial metric tensor γ_{ij} whose elements are given by (10) is non-Euclidean [3].

Using (3) we obtain

$$\begin{aligned} &\boldsymbol{E} = E_i = F_{i4} \qquad \mathbf{D} = D^i = -\frac{1}{\mu_0\Gamma}F^{i4} \qquad \mathbf{H} = H_i = \frac{1}{\mu_0}(F^{23}\,F^{31}\,F^{12}) \\ &\mathbf{B} = B^i = \frac{1}{\Gamma}(F_{23}\,F_{31}\,F_{12}) \qquad \mathbf{j} = j^i = \frac{1}{\Gamma}J^i \qquad \rho = \frac{1}{\Gamma}J^4. \end{aligned} \tag{12}$$

With these and the expressions in (4) for the divergence and curl operators in curvilinear coordinates, the electromagnetic field equations (1) reduce to Maxwell's equations (2). After a somewhat lengthy algebra involving the use of (9) for the elements of $g^{\mu\nu}$ to obtain the $F^{\mu\nu}$ in terms of the $F_{\mu\nu}$, and the use of (3) and the third equation of (4), we may show that the constitutive relations (5) are also satisfied.

Schiff [1] and Atwater [2] do not see the need to use all four of the electromagnetic spatial vectors. Furthermore, they treat the geometry of physical 3-space in rotating coordinates as if it were Euclidean, although it is clearly not so. Consequently, they wrongly use Cartesian forms for the divergence and curl operators. These are the reasons for the complicated form of their electromagnetic field equations in rotating coordinates.

Shiozawa (see [2]) uses all four of the electromagnetic spatial vectors and obtains Maxwell-like equations. His expressions for $\boldsymbol{B}$ in terms of $F_{\mu\nu}$, of $\boldsymbol{D}$ in terms of $F^{\mu\nu}$ and of $\boldsymbol{j}$ and ρ in terms of J^μ are incorrect, however. Furthermore, he wrongly treats the spatial geometry as Euclidean and so his divergence and curl operators have Cartesian forms. The combination of these errors give Maxwell-like equations.

We have pointed out the errors made by Schiff [1], Atwater [2], and Shiozawa (see [2]), and have shown that Maxwell's equations are valid in rotating coordinates. According to the general principle of relativity, accelerated systems of coordinates are equivalent to inertial systems for the description of nature, but we must be prepared to abandon Euclidean geometry in these cases. (See p. 255 in [3].)

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A Novel Interpretation of Prony's Method

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This letter presents a novel interpretation of Prony's method. The key steps in this method are shown to be: i) the generation of a nonhomogeneous, constant coefficient, linear difference equation, whose nonhomogeneous part is minimized in the least squares sense; and ii) the approximation of the general solution to the difference equation by the homogeneous solution that minimizes the particular solution.

I. Introduction

The purpose of this letter is to present a new interpretation of Prony's method [1]. This interpretation, which originally appeared in [2], provides new insights into a technique which is almost two hundred years old.

The key step in the derivation of Prony's technique, as is usually presented [3]-[5], is to recognize that the exponential approximation is the homogeneous solution of a constant coefficient, *homogeneous*, linear difference equation on the approximation to the measured samples. The key step in our interpretation is to recognize that the exponential approximation can be viewed as the homogeneous solution of a *nonhomogeneous*, linear difference equation on the measured samples.

Once this fact is recognized, the extended Prony method can be set up in a more straightforward manner than is usually presented. In addition, it provides a new interpretation of the procedure in terms of two minimization problems with an intermediate polynomial rooting. It is shown that the technique first minimizes the Euclidean norm of the vector whose components are the values of the discrete forcing function of the nonhomogeneous difference equation. The second step can also be viewed as the minimization of the Euclidean norm of the vector whose components are the values of the particular solution of the nonhomogeneous difference equation.

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