# The Distributive Role of Managerial Incentives in a Mixed Duopoly

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# Abstract

We study a mixed oligopoly where a partially public firm competes with a private firm. When the private firm offers managerial incentives, there is a redistribution of profit and output from the private to the public firm, but the aggregate output and social welfare may remain unchanged. When the private firm is foreign owned, the extent of privatization is less while managerial incentives are milder.

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#### 1. Introduction

The mixed oligopoly literature traditionally focuses on the aggressive behavior of a public firm against its private rivals, and its impact on social welfare (De Fraja and Delbono, 1989; Matsumura, 1998). This aggression primarily comes not from the government's apathy or rivalry to private firms, but from its equal treatment of consumer surplus and profit, which favors redistribution of gains from firms to consumers. What is less emphasized is that if profit becomes relatively more important to the government than consumer surplus, it will care not only about the public firm's profit but also the private firms' profit. Once private firms realizes this, they can afford to be aggressive by adopting profit enhancing strategies such as managerial incentives.

In the context of private oligopoly firms indeed offer sales-oriented incentives to their managers to enhance their individual output and profit; in doing so collectively they might over-produce ending up with less profit but generating higher social welfare than in the no-incentive case (Sklivas, 1985; Vickers, 1985; Fershtman and Judd, 1987). We show that such aggression might pay off against a rival public firm because the government in its concern for the private firms' profit will try to accommodate their aggression by optimally reducing its share-holding in the public firm and cutting back its production. In some cases the government will cut back the public firm's output by exactly the same amount that the private firms seek to increase their output by resorting to managerial incentives, giving rise to a scenario where managerial incentives merely redistribute profit and output in the private firms' favor without affecting the aggregate output and social welfare. This situation arises when marginal cost is constant and equal in all firms, so that the distribution of output among firms does not matter for social welfare.

We also show that if the government values the rival firm's profit less than the public firm's, the outcome will be quite different. A special case of this asymmetric treatment arises when the rival firm is foreign-owned. The government will then not care for the rival firm's profit and will be prepared to counteract the rival firm's aggression. Realizing this the foreign firm will be less aggressive in its choice of managerial incentives and the government will respond with less privatization. The aggregate output will be greater than that in the no-incentive case as well as in the case of domestic competition. Thus, foreign competition benefits the consumers more than domestic competition does, though there is no technological superiority associated with foreign firms. The literatures on mixed oligopoly and managerial incentives are fairly well-established, but attempts to bring them together have been very few. Only Chang (2007), Nakamura and Inoue (2007) and Heywood and Ye (2008) considered mixed duopoly allowing for managerial incentives, but while Chang's work focussed exclusively on trade policy, the latter two papers did not consider partial privatization.

#### 2. The Model

We consider a two-stage game between a partially public firm (indexed 0) and a private firm (indexed 1). In the first stage, the government decides on the share of public ownership in firm 0, while firm 1 decides on managerial incentives. In the second stage the two firms engage in Cournot competition.

The market demand is linear:  $p = a - b(q_0 + q_1), b > 0$ . Marginal costs of the two firms are symmetric and constant at c. We assume c < a < 3c. The first restriction (c < a) is natural. The second restriction (a < 3c) is needed to ensure that managerial incentives are effective. As managerial incentives work by altering the effective cost of production from the manager's perspective, incentives will be useful only if the marginal cost is not too small.

The public firm is jointly owned by the government and a private partner, and the choice of its output is made by the firm's board of management consisting of a government representative and the private partner. The public firm's objective function is a weighted average of social welfare and profit:  $z = \theta SW + (1 - \theta)\pi_0$ , where SW is given by the sum of consumer surplus and profit of the two firms, and  $\theta$  represents the government's share-holding.

The public firm's output choice is preceded by a decision of how much to divest or privatize, and this decision lies at a higher level of government, whose concern is to maximize social welfare. The government is also concerned with the solvency of the firms which makes it 'profit oriented'. In recent times governments around the world have been concerned about financial health of firms in general, and public sector enterprises in particular, for reasons of employment if not anything else. In emerging economies such profit-concerns may be necessary to encourage greater investment in both private and public sectors. We capture this 'profit orientation' through a modified social welfare function

$$V = CS + \beta_0 \pi_0 + \beta_1 \pi_1. \tag{1}$$

Here  $\beta_i \geq 1$  i.e. the government values profit more than consumer surplus. We first consider the case  $\beta_0 = \beta_1 = \beta$ . One can then rewrite V as  $V = SW + (\beta - 1)(\pi_0 + \pi_1)$ . Note that the government's objective function (V) differs from the objective of the government representative (SW) in the public firm's management board.

The private firm hires a manager and offers her incentives. Following the strategic delegation literature, we assume a linear incentive scheme  $M = \rho \pi_1 + (1 - \rho)pq_1$ , which rewards sales when  $\rho < 1$ . The manager is instructed to choose  $q_1$  to maximize M.

We begin with stage 2 of the game. The public firm's output reaction function is  $q_0 = \frac{(a-c)-bq_1}{b(2-\theta)}$ , and the private firm's output reaction function is  $q_1 = \frac{(a-\rho c)-bq_0}{2b}$ . The equilibrium outputs are obtained as

$$q_{0} = \frac{2(a-c) - (a-\rho c)}{b(3-2\theta)}, q_{1} = \frac{(a-\rho c)(2-\theta) - (a-c)}{b(3-2\theta)}, q_{1} = \frac{(a-c) + (a-\rho c)(1-\theta)}{b(3-2\theta)}.$$
(2)

 $q_0 > 0$  if  $a > c(2 - \rho)$ , and  $q_1 > 0$  if  $a(1 - \theta) > c[\rho(2 - \theta) - 1]$ . The corresponding expressions for profits are

$$\pi_0 = \frac{(1-\theta)[a+c(\rho-2)]^2}{b(3-2\theta)^2},$$
  
$$\pi_1 = \frac{(1-\theta)[a+c(\rho-2)][(a-\rho c)(2-\theta)-(a-c)]}{b(3-2\theta)^2}.$$

We now move to the first stage of the game. The government and the private firm determine their respective choice variables, *viz.*  $\theta$  and  $\rho$ , simultaneously. The private firm's owner chooses  $\rho$ , its incentive reaction function, by maximizing  $\pi_1$  as follows

$$\rho(\theta) = \frac{c(5-2\theta)-a}{2c(2-\theta)} = 1 - \frac{a-c}{2c} \left[\frac{1}{2-\theta}\right].$$

The 'incentive reaction function' is downward sloping in  $\theta$ , which means if the government increases its ownership, the private firm will make its managerial incentive stronger by reducing  $\rho$ . This reflects the fact that the private firm is aware of the government's concern for its rival firm's profit. In effect, greater nationalization induces the private firm to become more aggressive.

The government's privatization reaction function  $(\theta)$  is derived by maximizing

$$V = CS + \beta(\pi_0 + \pi_1) = b\frac{q^2}{2} + \beta [a - bq - c] q.$$

The first order condition is

$$\frac{\partial V}{\partial \theta} = \left[ bq + \beta (a - 2bq - c) \right] \frac{\partial q}{\partial \theta} = 0.$$

Since  $\frac{\partial q}{\partial \theta} > 0$  as is evident from (2) we must have  $[bq + \beta(a - 2bq - c)] = 0$ . That is given any  $\rho$ , the government's best response  $\theta$  must be such that the aggregate output q remains unchanged at

$$q = \frac{(a-c)}{b} \left[ \frac{\beta}{2\beta - 1} \right].$$
(3)

Substituting the expression of q from (2) in this relation we derive the government's reaction function as

$$\theta = \frac{\beta[c(2\rho - 1) - a] + 2a - c(1 + \rho)}{2\beta c(\rho - 1) + a - \rho c},$$

Several points are noteworthy. First, if  $\beta = 1$ , optimal  $\theta$  is 1 regardless of  $\rho$ . Second, given  $\beta > 1$ ,  $\theta \in (0, 1)$ . Privatization must be partial if profit is valued more than consumer surplus. This is true even if  $\rho = 0$ . This is because the government wishes to ensure positive profit for both firms. Third,  $\theta'(\rho) > 0$ ; from the government's point of view nationalization and (milder) managerial incentives are strategic complements. This is exactly opposite of the perspective the private firm has. This is where a mixed duopoly is crucially different from a pure duopoly. As the government cares about the industry profit more than consumer surplus, it internalizes some of the negative effects that would follow from aggressive output mobilization by both firms. So when the private firm is expected to increase its output incentives and managerial aggression (by reducing  $\rho$ ), the public firm divests its ownership in order to accommodate this.

Equilibrium  $\theta$  and  $\rho$  are

$$\theta^* = 3 - 2\beta, \rho^* = \frac{c(4\beta - 1) - a}{2c(2\beta - 1)}.$$

It can be verified that given our assumption c < a < 3c, we have  $0 < \theta^* < 1$ and  $0 < \rho^* < 1$  for  $\beta \ge 1$ . The equilibrium  $q_0$  and  $q_1$  are also strictly positive at  $(\theta^*, \rho^*)$ . Clearly, in equilibrium privatization is partial, and managerial incentives are sales-oriented. This shows that managerial incentives can be an effective strategy to counter the public firm's nationalization. The government's optimal privatization is such that the industry output remains unchanged at a value given by (3) regardless of whether the private firm offers managerial incentive or not. If there were no incentives  $(\rho = 1)$ , optimal privatization would be far less at  $\theta = 2 - \beta$ . Though this would give rise to the same q as in (3), the private firm would earn much lower profit and produce far less. But managerial incentives induce the public firm to cut back its production and transfer profit to the private firm. Thus, managerial incentives become purely redistributive. It is also of some interest to note that aggregate output can fall below or exceed the pure duopoly (without managerial incentives) level, depending on how much profit oriented the government is. In the pure duopoly case without managerial incentives, total output is  $q = \frac{2(a-c)}{3b}$ . This will be less than  $q^*$ , the mixed duopoly equilibrium output, if  $\beta < 2$ .

**Proposition 1** Given  $\beta > 1$ , equilibrium  $(\theta, \rho)$  will both lie strictly between 0 and 1. Compared to the 'no managerial incentives' case,  $\theta$  will be smaller causing  $q_0$  and  $\pi_0$  also to be smaller, but  $q_1$  and  $\pi_1$  will be greater, while q and SW will be unchanged. Thus, managerial incentives become merely redistributive having no efficiency effect.

#### 3. Foreign competition

We now consider a special case where the government does not value the rival firm's profit at all, though it continues to value the public firm's profit more than consumer surplus. This case arises when the rival firm is foreign firm. We will see that the private firm's incentive response will be quite different. Here SW consists of consumer surplus and the public firm's profit while the government's objective is obtained by setting  $\beta_0 = \beta$  and  $\beta_1 = 0$  in (1) i.e.  $V = CS + \beta \pi_0$ . Solving the game as usual by backward induction we get the following output reaction functions:  $q_0 = \frac{(a-c)-b(1-\theta)q_1}{b(2-\theta)}$  and  $q_1 = \frac{(a-\rho c)-bq_0}{2b}$ .

The equilibrium outputs are obtained as

$$q_{0} = \frac{2(a-c) - (a-\rho c)(1-\theta)}{b(3-\theta)}, q_{1} = \frac{(a-\rho c)(2-\theta) - (a-c)}{b(3-\theta)}, q_{1} = q_{0} + q_{1} = \frac{2a-c(1+\rho)}{b(3-\theta)}.$$
(4)

As before the aggregate output is increasing in  $\theta$  and decreasing in  $\rho$ . We now determine the simultaneous (first stage) choice of  $\rho$  and  $\theta$ . From the private firm's profit maximization we get

$$\rho(\theta) = 1 - \frac{a-c}{2c} \left[ \frac{1-\theta}{2-\theta} \right].$$

To solve for the public firm's response, we maximize the following with respect to  $\theta$ 

$$V = CS + \beta \pi_0 = b \frac{q^2}{2} + \beta \left[ a - bq - c \right] q_0.$$
(5)

Substituting the expressions of  $q_0$  and q from (4) and carrying out the maximization we derive the government's privatization reaction function as

$$\theta(\rho) = \frac{\beta[a - c(2 - \rho)] + [2a - c(1 + \rho)]}{\beta[3a - c(2 + \rho)]}.$$

It can be checked that  $\theta'(\rho) > 0$  and  $d\theta/d\beta < 0$  as before. Further,  $\rho(0) = \frac{5c-a}{4c} > 0$ ,  $\rho(1) = 1$  and  $\rho'(\theta) > 0$ . On the other hand  $\theta(0) = \frac{a(2+\beta)-c(1+2\beta)}{\beta(3a-2c)} > 0$  and  $\theta(1) = \frac{2+\beta}{3\beta} < 1$ . Thus the intersection of  $\rho(\theta)$  and  $\theta(\rho)$  must occur at some  $(\theta, \rho)$ , such that  $0 < \theta < 1$ and  $0 < \rho < 1$ . Let this solution be denoted as  $(\tilde{\theta}, \tilde{\rho})$ . So as before privatization is partial and managerial incentive is sales oriented.

But there are some interesting differences. The incentive reaction function is now upward sloping, while it was previously downward sloping. As the government chooses greater nationalization, the foreign firm makes its incentives milder (as opposed to making it stronger earlier). Realizing that the government will not care about its profit, the foreign firm cannot afford to increase aggression. This reversal in the foreign firm's reaction and the hardening of the government's stance against the rival firm are shown in Figure 1. The foreign firm's reaction function is given by the solid curve  $\rho_F(\theta)$  and the government's reaction function by the solid curve  $\theta_F(\rho)$ , where the subscript F denotes the case of foreign competition. As argued above, they must cross at an interior point like  $(\tilde{\theta}, \tilde{\rho})$ . The case of domestic competition has been shown by the two dotted reaction curves. There the private firm's reaction function  $\rho(\theta)$  was declining, as shown by the curve  $\rho_D(\theta)$  (D denotes domestic competition). So it must be the case that the equilibrium  $\rho$  in the previous case was smaller than  $\frac{5c-a}{4c}$  and in the present case it is greater than  $\frac{5c-a}{4c}$ . That is, the private firm offers much milder incentive when it is foreign-owned, than when it is domestically owned.

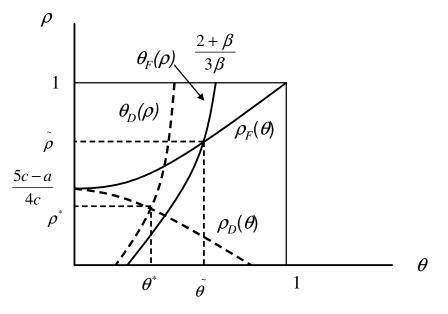


Figure 1: Equilibrium  $\theta$  and  $\rho$ 

As for privatization,  $\theta^*$  will be smaller than  $\tilde{\theta}$  if  $\theta_D(\rho)$  lies to the left of  $\theta_F(\rho)$ . This, however, cannot be directly ascertained, though it is clear that  $\theta_D(\rho = 1) < \theta_F(\rho = 1)$ . But it can be proved (what we do in the following proposition) that the aggregate output will now be greater. From (4) we know that the aggregate output cannot rise if  $\rho$  has risen, and if  $\theta$  has also fallen. Therefore, an increase in q must imply a rise in  $\theta$ . Thus we must have  $\tilde{\theta} > \theta^*$  as depicted in Figure 1.

Greater  $\theta$  also implies that the output of the public firm will rise and so will its market share. It is also apparent from (5) that social welfare (which can be obtained by setting  $\beta = 1$ ) depends not only on the aggregate output, but also on the output of the public firm (unlike in the previous case). The government's optimal choice of  $\theta$  dictates that both the aggregate output and the public firm's output change in response to the foreign firm's managerial incentives. Therefore, social welfare will not remain unchanged from the no-managerial incentive case, though it is unclear whether it will rise or fall.

Nevertheless, it can be ascertained that q under managerial incentives will be greater. This can be checked by visualizing that the iso-output line (obtained from equation (4))  $\rho = \frac{2a - \bar{q}b(3-\theta)}{c} - 1$ , when passing through the point  $(\tilde{\theta}, \tilde{\rho})$  must correspond to the equilibrium q. If a similar iso-output line passes through the 'no-incentive equilibrium' point ( $\theta = \frac{\beta+2}{3\beta}, \rho = 1$ ), it must correspond to a lower q; for greater  $\rho$  at a given  $\theta$ , qmust be smaller. Therefore, when the rival firm is foreign-owned, managerial incentives are not merely redistributive; they clearly affect social welfare.

**Proposition 2** As compared to the case of domestic competition, the equilibrium aggregate output will be greater under foreign competition, and the associated privatization will be smaller and managerial incentive milder; i.e.  $\tilde{\theta} > \theta^*$ ,  $\tilde{\rho} > \rho^*$ . Given foreign competition, the case of no-managerial incentive produces a smaller aggregate output and smaller privatization than the case of managerial incentives. Social welfare will not remain unchanged between these two cases.

*Proof:* We need to prove that q under foreign competition is greater. Recall from the previous section that q under domestic competition was obtained as  $q^* = \frac{a-c}{b} \left[ \frac{\beta}{2\beta-1} \right]$ . Now rewrite V from (5) as

$$V = b\frac{q^{2}}{2} + \beta [a - bq - c] q - \beta [a - bq - c] q_{1}.$$

Maximizing V with respect to  $\theta$  we get as the first order condition

$$\frac{\partial V}{\partial \theta} = \left[ bq + \beta (a - 2bq - c) \right] \frac{\partial q}{\partial \theta} - \beta \left[ a - bq - c \right] \frac{\partial q_1}{\partial \theta} + \beta bq_1 \frac{\partial q}{\partial \theta} = 0.$$

Since  $\frac{\partial q_1}{\partial \theta} < 0$  and  $\frac{\partial q}{\partial \theta} > 0$ , the last two terms must be positive, and therefore, we must have  $[bq + \beta(a - 2bq - c)] < 0$ . This implies  $q > \frac{a-c}{b} \left[\frac{\beta}{2\beta-1}\right] = q^*$ . The rest of the proposition follows, as explained in the discussion above.

### 4. Conclusion

In this paper, we have introduced managerial incentives in a mixed duopoly with constant marginal cost. We show that if the government is profit oriented, it will accommodate the private firm's aggression (via managerial incentives) and cut back its own production through partial privatization. This accommodation does not occur if the private firm is foreign-owned. Realizing this the foreign firm will offer milder managerial incentives and the public firm will witness less privatization. Consumers benefit the most under foreign competition. This difference in government's attitude to private firms (depending on it being domestic or foreign) and its implications for privatization and social welfare could be studied in other contexts as well, such as increasing marginal cost, differentiated products and Stackelberg duopoly.

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