Proceedings of ASME Turbo Expo 2024
Turbomachinery Technical Conference and Exposition
GT2024
June 20-28, 2024, London, UK

GT2024-123318

DEVELOPMENT AND APPLICATION OF A HIGH-FIDELITY NUMERICAL TOOL FOR DYNAMIC ANALYSIS OF BLADED DISC SYSTEMS WITH UNDERPLATFORM DAMPERS IN AIRCRAFT ENGINE TURBINES

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ABSTRACT

This study presents VIBRANT: VIbration BehaviouR ANalysis Tool, a numerical tool developed for analysing the dynamic behaviour of complicated mechanical systems. The tool uses Python and Abaqus, employing time-marching algorithms to perform individual time domain simulations under harmonic excitation of certain frequencies until a steady state is reached to predict frequency domain behaviour. Hence, each monoharmonic excitation can be performed independently; the tool can parallelise the computations to shorten the computational time. This software enables the handling of various systems modelled by Abaqus, adeptly addressing numerous nonlinearities and conditions, expanding the capabilities of the software significantly. Properties of interest are measured for each frequency step of the simulation set. To validate and illustrate the capabilities of VIBRANT, two examples are presented. The first example is a beam with a dry friction contact element attached, and the second is a bladed disc system with underplatform dampers in aircraft engine turbines. Parametric studies evaluate the influence of the variation of parameters, such as excitation amplitudes and coefficient of friction, on the system's dynamics. This research extends the scope of computational modelling in mechanical and aerospace engineering and provides a foundational tool that can be implemented to validate future aircraft engine designs.

Keywords: Numerical analysis, time-marching, aeroengine, turbine blade, nonlinear mechanical systems, underplatform damper

NOMENCLATURE

b	Width of the beam cross-section [mm]
F_{ex}	Excitation force [N]
F_N	Normal force in contact [N]
GPa	Modulus of elasticity unit in gigapascals
h	Thickness of the beam cross-section [mm]
HB	Harmonic Balance
K	Temperature unit in Kelvins
kg/m^3	Density unit in kilograms per cubic metre
kg/s	Mass flow rate unit in kilograms per second
L	Length of the beam [mm]
m	Meter
mm	Millimeter
μ	Friction coefficient
rpm	Rotational speed unit in revolutions per minute
s	Time unit in seconds
t	Time [s]
z	Spatial coordinate [mm]
$\dot{z}$	Spatial coordinate's first derivative with respect to time [mm/s]

1. INTRODUCTION

The analysis of nonlinear systems is a difficult challenge in engineering, especially when dealing with complex systems that may be undergoing large deflections, involving contact, and influenced nonlinear material properties [1–5]. The dynamics of aeroengine components/assemblies can be counted among such complicated mechanical systems [6–8]. These sophisticated assemblies are the products of advanced materials science and precision engineering, where each element must function together with others under the most demanding/extreme loads and conditions. Among these components, the bladed disc systems display a remarkable design sophistication. The performance and reliability of these systems are critical, as they are subjected to a variety of forces and environmental loads that can induce nonlinear behaviour. This sensitivity requires comprehensive analytical tools capable of capturing the interactions within the turbine environment to ensure that predictive models accurately reflect the operational behaviour of aeroengines [9].

Building on the foundational works in nonlinear systems analysis, a spectrum of computational tools and methodologies has emerged, each bringing unique strengths to the fore. Among these, NLvib stands out as specialised software for the investiga-

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Documentation for `asmeconf.cls`: Version 1.32, June 19, 2024.

tion of structures with potential discontinuities, such as contacts, which are of particular concern in the field of aeroengine turbines due to their impact on structural integrity and performance [10]. In addition, MANLAB, developed by people in France, provides a user-friendly interface for the continuation of periodic solutions in nonlinear systems [11–13]. It uses the harmonic balance (HB) method combined with the asymptotic numerical method to efficiently navigate the solution space of nonlinear equations, revealing bifurcation points and multiple solution branches with lower computational cost compared to some time domain techniques.

Similarly, Mousai, an open-source HB solver, provides a general-purpose approach to nonlinear vibration problems, further contributing to the range of tools available to researchers [14]. However, Mousai does not have continuation implemented. Instead, it uses forward and backward frequency sweeps. The importance of such tools is emphasised in Lacayo et al., which highlights the need for both frequency and time domain simulations to accurately analyse nonlinear complex systems [15]. This work also suggests that a combination of both domains is essential for a comprehensive understanding and reliable modelling of the complicated dynamics of these systems [15]. Finally, FORSE, an in-house code developed at Imperial College London, is recognised as one of the most powerful numerical tools that can be used to model the frequency domain forced response of nonlinear systems [16–21].

Understanding and modelling the dynamic behaviour of turbine blades has long been a subject of academic and practical interest. The research by Goodman and Klumpp has laid the foundation for quantifying frictional damping by considering both isotropic and anisotropic forces, marking a significant milestone in analytical approach development [22]. Complementing these efforts, studies by Zmitrowicz and Armand et al. have devised mathematical models that accurately model the complex interactions within bladed disc systems [23, 24]. Numerical and experimental investigations on friction damping in bladed disc assemblies were conducted by Charleux et al. [25]. Pesek et al. conducted three-dimensional modelling of bladed discs using the finite element technique and experiments [26]. In order to investigate the energy dissipation of a thin-walled frictional damper intended for turbine blades, Szwedowicz et al. carried out tests and computational simulations [27]. The potential damping properties of a strip damper on a turbine blade were examined by Afzal et al. [28]. Predicting the dissipation of vibrational energy by underplatform dampers is a typical topic that has been the focus of various research articles published in the literature [27, 29–31].

The field has also seen progress in numerical and experimental methods aimed at understanding friction damping in these assemblies. Detailed three-dimensional modelling and finite element analyses have been conducted to complement these studies, providing a more comprehensive picture of the systems in question [26]. Experimental work, combined with computational simulations, revealed the energy dissipation characteristics of various damping mechanisms, including thin-walled dampers and strip dampers, designed to enhance the stability of turbine blades [27, 28]. The critical role of underplatform dampers in dissipating vibrational energy has also been the subject of numerous studies, further emphasising the complexity of these systems [29–31].

Advanced modelling and analysis techniques are essential for understanding the nonlinear vibrations of turbine blade-disc systems with underplatform dampers. The accuracy of contact pressure distribution and the consideration of zero-harmonic terms in multiharmonic expansion have been highlighted as crucial factors in these analyses [32]. Theoretical models integrating experimentally measured contact characteristics have proven effective in predicting the dynamic response and identifying shifts in natural frequency [31]. Furthermore, models that incorporate dry friction dampers into the analysis of turbine blades have utilised quasi-linearisation techniques, facilitating the transformation of nonlinear differential equations into a more manageable algebraic form [33].

The HB method has been applied across a variety of studies focusing on the nonlinear mechanical vibrations of systems influenced by friction. This method's application to turbine blades with friction dampers has provided a means to model the dampers' nonlinear behaviour effectively, which is essential for capturing the complexity of such systems [10, 34–36].

In addition to the HB method, time-marching has also been utilised in capturing the characteristics of nonlinear mechanical vibrations. These methods have proven their efficacy and accuracy through the good agreement of solutions to those obtained from experimental data and their ability to reveal the intricate dynamics present within friction brake systems [37–41].

Exploring the field further, the use of commercial and open-source finite element software has extended the boundaries of research into large amplitude vibrations and nonlinear modal analysis. These software solutions offer valuable tools for investigating the effects of friction and other nonlinearities within various mechanical systems [15, 42–44].

The motivation for this study stems from the industrial need to accurately predict the behaviour of mechanical structures at varying frequencies - a process that is essential for the design and optimisation of components in sectors such as automotive, aerospace and civil engineering. Traditional manual approaches to frequency sweep analysis are fraught with the potential for human error and are inherently inefficient. To address this, the current research introduces a Python-based automation tool that interfaces with Abaqus' robust finite element analysis capabilities. This initiative streamlines the frequency sweep process, ensuring that the dynamic responses of complex structures are captured with greater accuracy and less manual intervention.

This research introduces VIBRANT: VIbration BehaviouR ANalysis Tool, a sophisticated numerical tool designed for the detailed analysis of complex mechanical systems with a primary application in aerospace engineering. VIBRANT is developed using Abaqus and Python and uses time-marching algorithms to conduct high-fidelity time domain simulations, predicting frequency domain behaviour under harmonic excitation. This tool has the ability to parallelise computations, which reduces the computational time spent while being able to handle a wide range of systems modelled in Abaqus. This capability enables accurate simulations of loading conditions, boundary conditions, multi-physical phenomena as well as various nonlinearities, including contacts, impacts, steady state aero-pressure, temperature distributions that affect material elasticity, as well as nonlinear materials. These are often overlooked in conventional analyses as they introduce significant difficulties to the modelling procedures. The proficiency of the tool is demonstrated by mechanically analysing a beam with a localised contact, which is a common case in the literature [45, 46], and a bladed disc system with under-platform dampers in aircraft engine turbines. VIBRANT applies harmonic excitation and monitors the response until a steady state is reached, systematically tracking the critical physical properties such as kinetic and potential energies, vibration damping and amplitude across a selected frequency range relevant to operational conditions. Parametric studies are conducted to investigate the influence of design parameters, such as friction coefficient and excitation levels, on the dynamic behaviour of these systems. VIBRANT can be used to validate future aircraft engine designs, enhancing the design and manufacturing processes of next-generation aircraft engines, with significant implications for efficiency, safety, and reliability. VIBRANT is a standardised yet flexible numerical tool for modelling complex dynamical systems, producing practical, industry-applicable outcomes.

2. SOFTWARE DESCRIPTION

This section presents an overview of VIBRANT, a comprehensive software developed and designed by the author to simplify the frequency sweep analysis of complex mechanical systems. VIBRANT integrates Abaqus' robust capabilities with Python's flexibility. It allows accurate representations of dynamic behaviours of any complicated mechanical system that can be modelled using Abaqus. VIBRANT software is proficient in conducting high-fidelity time domain simulations and producing frequency domain results. It can handle a wide range of non-linearities and multi-physical phenomena, making it useful in accurately assessing the performance of engineering structures. This stresses the importance of using existing powerful tools that work in the time domain to gather realistic frequency domain results. Even though the time-marching approach is known to be computationally very demanding, VIBRANT has the features to mitigate the significant computational load to reduce the computational time. One of its outstanding features is the ability to parallelise computations, significantly reducing the computational time associated with extensive simulations time domain. VIBRANT is designed to be user-friendly and achieve computationally efficient numerical modelling whilst remaining accurate and allowing for systematic exploration of various design parameters and operational scenarios. VIBRANT is an effective and powerful tool for modern mechanical and aerospace engineers, providing a streamlined approach to understanding and optimising complex dynamical systems. It effectively bridges the gap between theoretical modelling and practical application.

2.1 Dependencies

The software is based on Python scripting and Abaqus tools,. Its functionality relies on the following key components:

- **Abaqus and Python**: For model setup, simulation, and data extraction.
- **NumPy**: For numerical operations.
- **mathplotlib**: For visualisation of the extracted data.
- **Standard Python libraries**: `os`, `math`, and `time` for file and system operations.

2.2 Installation and Setup

The installation process is designed for user convenience. To use the Python script, users should place it in a designated directory, usually where the Abaqus files are located, or define a specific path to it. It is crucial to ensure that both Abaqus and Python are not only installed but also correctly configured on the user's system to avoid any compatibility issues.

2.3 Code Architecture

The software automates dynamic analyses within the Abaqus environment, ensuring precise evaluation of mechanical structures under various excitation frequencies. It consists of several modules, each carrying out a specific task of the simulation process.

In the initialisation module, the software loads the necessary libraries and sets up system variables such as frequency range, amplitude, and node labels, which serve as parameters for the software's operation. Additionally, data lists are initiated for orderly data management and later retrieval during post-processing. The program then loads the specified Abaqus model, marking the beginning of the analysis.

After the setup, the model configuration module takes over. This module is responsible for adjusting frequency values within a predetermined range. At each frequency step, the model is updated to reflect the current frequency, and a simulation is prepared. This process ensures a comprehensive frequency sweep is performed, covering all critical points that could reveal resonant frequencies or other significant dynamic characteristics.

Job management is the next component of the software. The simulations, now configured with appropriate settings, are submitted to Abaqus for processing. It automates the submission and tracking of multiple analysis jobs, enhancing efficiency and minimising the risk of human error associated with manual job handling.

The post-processing module is the next stage of the process. After the simulations are completed, this module extracts and computes response parameters and energy dissipation values from the results. The module processes the raw data to extract meaningful results and prepares the results for output, allowing their analysis to be made in the next and the final stage of the software, which is data processing. The processed data reveals the dynamic characteristics of the complicated mechanical system analysed. The results can be used to make design decisions, optimise performance, or gain a better understanding of the system's dynamic behaviour.

The structure of the code is summarised below, providing a high-level overview:

- **Initialisation**:

- Sets system variables such as frequency range, amplitude, and node labels, aligning with the detailed initialisation steps.
- Initialises data lists for storing extracted data with preallocation for efficiency, corresponding to creating empty lists in the minipage.
- Defines the frequency range for the sweep, ensuring it covers the necessary spectrum for analysis.
- Loads the Abaqus model, preparing the simulation environment.

- **Model Configuration**:
  - Systematically changes frequency values and simulation settings in the existing Abaqus model, matching the process of opening the model file, updating the frequency, and saving changes.
  - For each frequency step/increment, the model is updated with the current frequency, and the simulation is run to capture the dynamic response.
- **Job Management**:
  - Manages and submits the configured simulation jobs to Abaqus for analysis, ensuring that each modified model is processed and that the software waits for each job to complete.
- **Post-Processing**:
  - Extracts and computes response parameters and energy dissipations from the completed simulations, which involves opening the output database and processing the relevant data like displacement and stress.
  - Stores the extracted data in lists for subsequent analysis, which allows for a structured and accessible dataset.
- **Data Analysis**:
  - Conducts analysis on the extracted data, writing results to text files or spreadsheets for documentation and review.
  - Optionally, generates plots for visual representation and interpretation of the data, facilitating an understanding of the system's behaviour.

The architecture of VIBRANT ensures that the tool is efficient in processing and analysing data while also being user-friendly. It allows even those with limited coding experience to effectively utilise its full capabilities.

2.4 Model Preparation and VIBRANT Execution

Before using VIBRANT for dynamic analysis, the user is advised to gain a basic understanding of the mechanical system under consideration. This involves performing preliminary analyses to determine the static and dynamic characteristics of the system. For systems subject to static loads, a pre-stressed modal analysis is recommended to accurately capture the influence of these loads on the modal characteristics. In the absence of significant static loads, basic linear modal analysis is usually sufficient to identify the modes of interest, which are important to form the time domain dynamic simulations in the next stages.

The preparation of the model phase continues with the configuration of the model within the finite element tool. Here, the user must accurately define the geometry, material properties, boundary conditions and meshing to reflect the real system as accurately as possible. Next, the dynamic loads and the range of excitation frequencies relevant to the analysis must be specified within the software. Then, the execution of VIBRANT comes in, using the defined model parameters and simulation settings to run the time domain algorithms essential for frequency domain analysis.

By carefully preparing the model and appropriately setting the analysis parameters, users can efficiently utilise VIBRANT to perform comprehensive frequency sweep analyses. This ensures reliable simulation results and helps the user understand the system response and characteristics for better design and optimisation strategies.

2.5 Evaluation of VIBRANT's Capabilities

Table 1 delineates the capabilities of VIBRANT in comparison to other widely-used dynamic analysis methods such as HB. VIBRANT stands out for its comprehensive time-marching analysis and seamless transition to frequency domain modeling, courtesy of its integration with Abaqus. This feature set allows VIBRANT to tackle a broad spectrum of problems, including complex contact scenarios with advanced damping considerations, which are often challenging for other methods.

While HB is renowned for its efficiency in frequency response and nonlinear modal analysis, it lacks the versatility and depth in contact analysis that VIBRANT offers. Numerical Integration techniques, particularly those employing the Newmark integrator, provide unconditional stability but may not capture the actual complex of contact and damping as effectively as VIBRANT.

Furthermore, VIBRANT's capability for parallel computation of independent frequency steps offers significant computational advantages, especially for extensive analyses. This contrasts with the more focused applications of other methods, which might excel in specific scenarios but do not offer the same breadth of application as VIBRANT, especially in simulations demanding high realism in contact mechanics and damping behaviour.

TABLE 1: COMPARISON OF VIBRANT WITH OTHER NONLINEAR DYNAMIC ANALYSIS SOFTWARE

Feature/ Capability	VIBRANT	Other Software (Harmonic Balance, Numerical Integration, etc.)
Analysis Method	Time-marching using Abaqus	Alternating Frequency–Time HB, Newmark integrator, Shooting method
Frequency Domain Modeling	Achieved through time-marching	Directly in frequency domain
Nonlinearities	A very large spectrum of classes depending on Abaqus' capabilities	Local generic nonlinear elements, distributed polynomial stiffness
Types of Analysis	Time domain, broad spectrum, using Abaqus' capabilities	Frequency response, nonlinear modal analysis
Computational Considerations	Expensive but allows for parallelisation	Varies; some methods may offer unconditional stability or require predictor-corrector schemes

3. NUMERICAL EXAMPLES

Two examples are presented in this study. The first example aims to validate VIBRANT, showing a good agreement between the results produced with VIBRANT and NLvib, a validated numerical tool that uses an entirely different approach compared to VIBRANT and is written in MATLAB [47–50]. The second example is there to display the capabilities of VIBRANT through a more complicated system that is part of an aircraft engine's turbine, a bladed disc system with underplatform dampers.

3.1 Validation Example: Beam with Localised Contact

This example focuses on a cantilever Euler-Bernoulli beam, which is excited with a concentrated force at its free end. The beam is represented with a one-dimensional finite element model and is a nonlinear system due to the attachment of a dry (Coulomb) friction element at a specific node close to the fixed end. This means that the friction force depends on the velocity of the point where the frictional contact element is attached to the beam. This localised contact represents the interaction between the beam and an external independent component, a scenario frequently encountered in practical engineering systems such as rotating blades with a damper attached [51].

The properties of the beam being investigated are presented in Table 2.

The beam is discretised into 19 Euler-Bernoulli beam elements, 20 nodes, with the first node clamped, represented with a fixed boundary condition, and the twentieth node is set free. The twentieth node is also where the excitation force F_{ex} is located. The nonlinear dry friction element that uses $\tanh(\dot{z}\, t)$ to represent the frictional behaviour is introduced at the fourth node. Here, z is the spatial coordinate aligned with the transverse deflection of the beam, $\dot{z}$ indicates the first time derivative of the coordinate z, and t denotes time. The system is visualised in Figure 1.

TABLE 2: BEAM DIMENSIONS AND MATERIAL PROPERTIES

Property	Symbol	Value
Length	L	2 m
Height	h	0.1 m
Thickness	b	0.15 m
Young's Modulus	E	185 GPa
Density	ρ	7830 kg/m^3

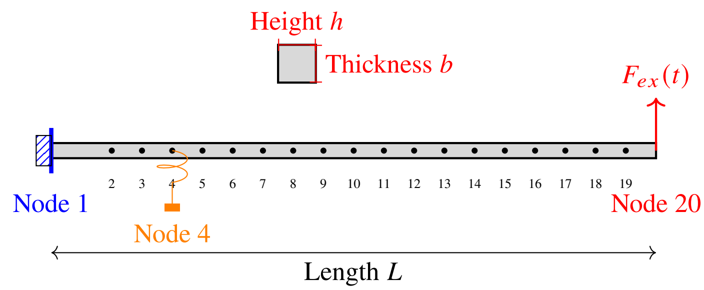


FIGURE 1: SCHEMATIC VISUALISATION OF THE 20-NODE EULER-BERNOULLI BEAM WITH THE CONTACT ELEMENT AT NODE 4 AND APPLIED CONCENTRATED FORCE AT NODE 20.

The system is analysed and solved using two different computational tools that use different approaches. The first is the open-source MATLAB toolbox NLvib, which specialises in the analysis of nonlinear vibrations of beams and discrete/lumped systems. NLvib enables the analysis of the system's response to harmonic excitation using the HB method. This tool and technique allow for the efficient computation of periodic solutions and the direct examination of the influence of nonlinearities on the system's frequency response.

The second tool is VIBRANT, a software designed for the analysis of vibrations in complex mechanical systems through Abaqus. VIBRANT utilises a different numerical strategy to handle nonlinearities, providing a robust platform for modelling complex nonlinear systems. VIBRANT performs time domain simulations and uses a time-marching technique to calculate the frequency response of the system. This tool is validated against the results obtained from NLvib by comparing the frequency response functions generated by both NLvib and VIBRANT.

The frequency range of interest covers only the first bending mode (first peak in the frequency response curves) of the beam in linear cases where there is no contact element, and node 4, where the contact element is attached, is fixed. It is expected that in the presence of the frictional contact element, the first peak in the frequency response should remain in between these linear cases. Also the variation of the maximum friction force $\mu\, F_N$, where μ denoted the friction coefficient and F_N the normal contact force.

3.2 A Large-Scale Example: Bladed Disc System with Underplatform Dampers

After validating the results produced by VIBRANT against a well-known open-source numerical tool, NLvib, this section is designated to describe the large-scale example that shows the capabilities of the tool VIBRANT. This example's finite element model is prepared in ANSYS and exported to Abaqus to execute the time domain dynamic simulations with VIBRANT.

3.2.1 Description of the Model. This example uses a representative geometry for a realistic bladed disc system to represent one stage of an aeroengine turbine, as shown in Figure 2. In the analysis of the bladed disc structure, the geometry, applied loads, and the boundary conditions demonstrate precise symmetry around the turbine's axis of rotation. This indicates that the cyclic symmetry assumption can be employed to reduce the computational load. The use of symmetry permits a reduction in the scope of the finite element model by initially focusing on a single sector. Then, the analysis of this sector is extended to other sectors, leading to results for the whole system. Thus, to build the finite element model and perform the necessary analyses, a turbine sector consisting of two blades, which makes 1/60 of the whole system, was modelled. Using two blades permits achieving a more realistic simulation environment in the time domain, as the application of the cyclic symmetry boundary conditions in the time domain is not as straightforward as in the frequency domain.

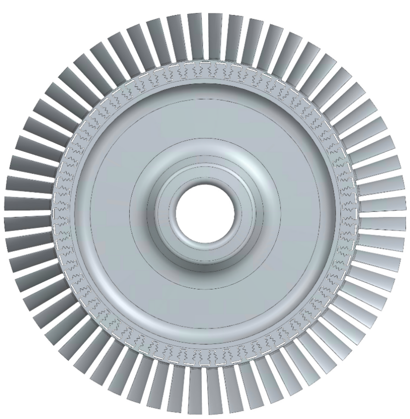


FIGURE 2: VIEW OF THE FULL BLADED DISC SYSTEM WITH UNDERPLATFORM DAMPERS.

The sector that the finite element model considers consists of two blades, underplatform dampers, fir tree joints and disc parts and the contacts between these components as shown in Figure 3. The finite element model comprises 171,523 SOLID185 elements, 85,932 nodes, and 257,796 degrees of freedom. Also, a total of 5,512 contact nodes are assigned to the contact regions, which are linked by a Coulomb friction model. The contact interfaces between the blade and the damper and between the fir tree and the disc are illustrated in Figure 4.

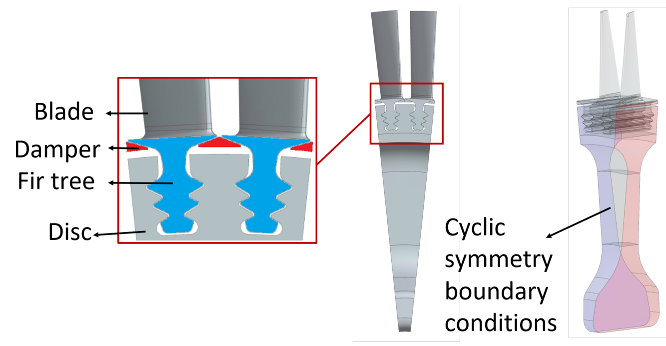


FIGURE 3: THE GEOMETRY USED FOR THE CONSTRUCTION OF THE FINITE ELEMENT MODELS.

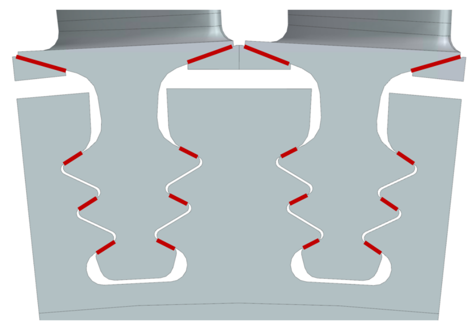


FIGURE 4: THE SURFACES THAT ARE IN CONTACT.

3.2.2 Static Analysis. As explained earlier, the static analysis aims to consider the effects of the static loads and conditions to get more accurate results from the modal analyses and time domain dynamic simulations. In this example, the static model considers cyclic symmetry and takes into account the effects of the steady state temperature distribution on the material properties, the rotational body forces at 5000 rpm, and the static aero-pressure loads acting on the blades and the pressure loads acting on the pressure and suction sides.

It is important to state that this example only contains the case where the system is rotating at 5000 rpm. The changes in the rotational speed affect the centrifugal body forces, the contact conditions as well and the aero-pressure field on the blade, which requires performing more of the computational fluid dynamics simulations. Given the focus on modelling nonlinear mechanical systems, exploring changes in rotational speed is left outside of the scope of this research, due to the substantial computational effort and complexity involved.

In ANSYS, the nonlinear static analysis accommodates the cyclic symmetry of the system by initially solving equilibrium equations for a single sector and then applying cyclic symmetry boundary conditions to derive the complete system's solution.

To obtain pressure and thermal maps on blades, blade geometry is meshed in ANSYS TurboGrid®. The mesh consists of 158,433 linear elements and 17,166 nodes. The mass flow rate is assumed to be 82 kg/s, and the total temperature at the turbine inlet is specified as 1250 K. Rotating speed is taken as

$5000rpm$. The fluid is modelled as ideal air gas, with wall conditions assumed to be smooth, adiabatic, and exhibiting no slip. Reynolds–Averaged Navier–Stokes equation-based shear stress transport turbulence model is used for turbulence modelling, and ANSYS CFX®, which is a coupled solver, is utilised. The simulation setup, where the aero-pressures and temperature fields are calculated, is shown in Figure 5.

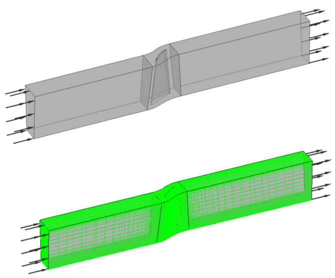


FIGURE 5: COMPUTATIONAL FLUID DYNAMICS SIMULATION SETUP FOR THE BLADES TO COMPUTE THE STEADY STATE PRESSURE LOADS AND TEMPERATURE DISTRIBUTION.

The resultant steady state aero-pressure loads and the temperature field are shown in Figures 7 and 6.

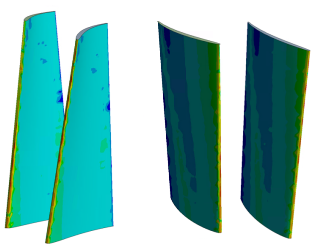


FIGURE 6: STATIC PRESSURE LOADS ACTING ON THE BLADES.

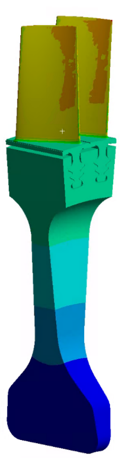


FIGURE 7: TEMPERATURE DISTRIBUTION THROUGHOUT THE BLADED DISC SYSTEM IN STEADY STATE.

3.2.3 Modal Analysis. After the static analysis is complete, the dynamic behaviour of the structure is investigated using modal analysis to determine natural frequencies and mode shapes. This modal analysis takes into account the combined effects of mechanical loading, temperature, and rotational body forces, incorporating the results of the static analysis as pre-stress. The results, obtained primarily by solving the eigenvalue problem, reflect the structure's dynamic characteristics under these conditions. However, these models do not consider the nonlinearities resulting from component contacts, despite including static analysis results. Regarding the contacts, two cases are explored. First, all the contacting surfaces are assumed to be bonded, and their interfaces remain rigidly tied, whilst the second removes the underplatform dampers, and the rest of the contacts are tied as in the first case. This permits the investigation of the influence of the underplatform dampers on the overall dynamics of the system. Consequently, these simulations provide the necessary understanding of the dynamic behaviour of the system so that the frequency range of interest can be determined. Thus, the modal analysis lays the foundation for the calculation of the system's forced response considering the nonlinearities using VIBRANT.

Building upon the baseline modal analysis, two linear cases were simulated within the scope of this study using Abaqus' steady-state dynamics: (1) rigidly tied contact conditions for all contacts, including the underplatform damper, and (2) no underplatform damper with rigid contact conditions among other components. The results were then compared with VIBRANT to provide another step of validation.

3.2.4 Time-Marching and Frequency Sweep. After performing the baseline simulations in ANSYS, which include nonlinear static and pre-stressed modal analyses, a preliminary understanding of the system characteristics is obtained. The ANSYS simulation results are then converted to the Abaqus format for use in VIBRANT. This conversion is essential to facilitate the time domain and frequency-sweep analyses in VIBRANT, which is adept at handling complex dynamic simulations. The frequency range for the sweep is strategically chosen to cover the first in-plane and out-of-plane bending modes of the blades, with the disc mode remaining at 0_{th} nodal diameter. Here, all the contacts are evaluated using the Coulomb friction model. The dynamic excitation is selected as the aerodynamic pressure loads that are assumed to be 10% of the static pressure loads, in magnitude, acting on the blade [52].

In the end, VIBRANT generates a range of output files that contain displacement data in all three coordinate directions. The files are then used to generate plots to understand the behaviour of the system at different frequencies.

4. RESULTS AND DISCUSSIONS OF THE EXAMPLES

4.1 Validation Example: Beam with Localised Contact

The frequency response curves shown in Figure 8 provide a comprehensive visual comparison between the linear (with fixed contact and without any contact) and nonlinear dynamic responses of an Euler-Bernoulli beam subject to harmonic excitation. The computations for all these cases are performed via both NLvib and VIBRANT. The curves in each plot represent the frequency response as calculated by NLvib and VIBRANT for different excitation amplitudes, F_{ex}. The plots demonstrate the system's behaviour as it shifts from a linear dynamic regime to one that is significantly affected by nonlinear frictional contact. The curves are normalised to provide a dimensionless perspective, facilitating comparison regardless of the system's absolute scale.

Normalisation is achieved by normalising the frequency and displacement responses, providing a clearer view of the relative changes in the system's behaviour. The reference values are chosen based on the fixed contact condition under the lowest excitation force (F_{ex}). Specifically, the frequency and displacement values at the peak response of this scenario are selected for normalisation. This approach highlights the influence of nonlinearity in the dynamic response, which is particularly noticeable through the shift in natural frequencies and peak response amplitudes. The impact of the excitation amplitude (F_{ex}) is also highlighted, as larger forces broaden the response curve and increase the frequency shift. This is aligned with the known characteristics of the contact nonlinearity being amplitude-dependent.

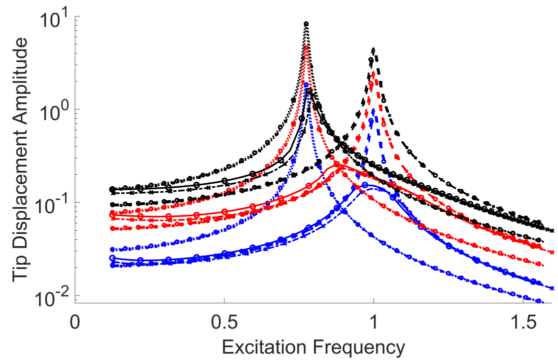
The dynamic response of the Euler-Bernoulli beam, with a localised frictional contact, is significantly influenced by the contact forces and the beam. This interaction is primarily controlled by the coefficient of friction, μ, and the amplitude of excitation, F_{ex}, which dictate the stick, where the contact surfaces move together without relative motion, slip, where the surfaces slide against each other, and stick-slip, where both stick and slip are observed during the motion.

As μ increases, the system initially exhibits a stiffening effect due to the predominance of stick behaviour, which resists motion and increases the apparent stiffness of the structure. This behaviour is complex and highly dependent on the vibration amplitude and frictional contact characteristics.

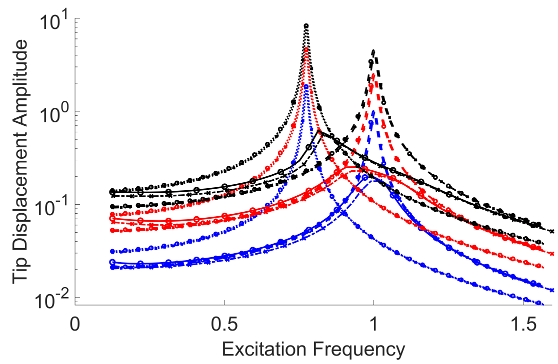
At lower excitation amplitudes, the beam remains largely in the stick phase, suggesting that the stiffness change due to nonlinearity is minimal. However, as the excitation force increases, the system transitions more frequently into the slip phase, particularly at peak vibration velocities. This transition results in energy dissipation through frictional damping, which results in a softening effect characterised by reduced peak amplitudes in the frequency response. The nonlinear response observed does not simply indicate an increase in stiffness. Instead, it reflects the dissipation of energy at the frictional interface.

The nonlinear frictional damping, therefore, plays a crucial role in the system's response to increased excitation forces. It prevents a linear escalation in peak amplitudes, which would be expected in a frictionless system. The resulting frequency response curves, therefore, do not exhibit a proportional relationship between F_{ex} and peak amplitudes. Instead, they reveal a complex dependency on both μ and F_{ex}, which governs the dynamic transition between stick and slip regimes and shapes the observed nonlinear behaviour.

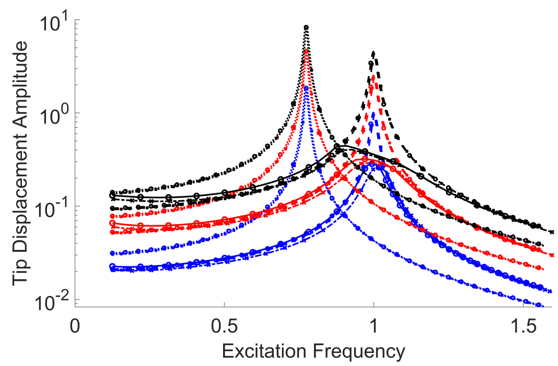
The physics behind these observations is primarily governed by the dynamic equilibrium between inertia, stiffness and damping within the beam structure. The introduction of a frictional contact element at a given node introduces additional forces dependent on displacement and velocity, which are not present in a linear context. These forces alter the equilibrium of the system, with their effects becoming more pronounced as the excitation frequency approaches the natural frequency, causing the observed



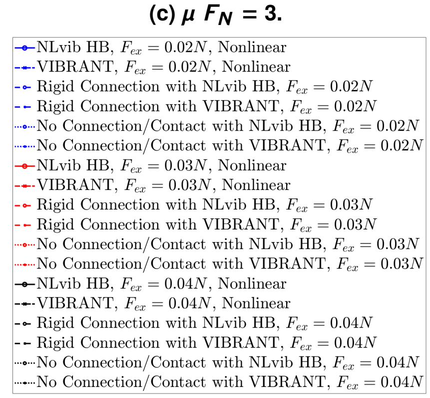
(a) μ F_N = 1.5.



(b) μ F_N = 2.



(c) μ F_N = 3.



(d) Common legend for the plots.

FIGURE 8: FREQUENCY RESPONSE CURVES OF THE BEAM. (MEASUREMENTS ARE TAKEN FROM THE FREE END.)

diversion from linear behaviour.

It is clear that there is a very good agreement between the results produced by NLvib and VIBRANT. Although there is a good agreement between the two sets of curves, the amplitude computed through VIBRANT seems to be consistently smaller than the amplitude calculated with NLvib. This is explained with the artificial/additional damping that Abaqus inherently implements in its element definition, aiming to facilitate numerical convergence and stability in time domain simulations whereas NLvib does not have that. Therefore, the amplitudes calculated with VIBRANT are found to be approximately up to 11% (maximum observed error) smaller than the ones NLvib yield, as can be seen in Table 3.

TABLE 3: MAXIMUM ERROR BETWEEN VIBRANT AND NLVIB FOR DIFFERENT EXCITATION AMPLITUDES.

Nonlinear Cases	Maximum Error (%)
$F_{ex} = 0.02\ N$	8.82
$F_{ex} = 0.03\ N$	9.13
$F_{ex} = 0.04\ N$	11.09
Rigid Contact Linear Cases	**Maximum Error (%)**
$F_{ex} = 0.02\ N$	4.02
$F_{ex} = 0.03\ N$	4.81
$F_{ex} = 0.04\ N$	5.06
No Contact Linear Cases	**Maximum Error (%)**
$F_{ex} = 0.02\ N$	4.34
$F_{ex} = 0.03\ N$	3.78
$F_{ex} = 0.04\ N$	4.46

4.2 Bladed Disc Example

4.2.1 Static Analysis. Static analysis results under pressure, thermal and rotational body force are presented in Figure 9. Here, the stress distribution shows a smooth characteristic, suggesting that the modelling is performed well, although the model consists of significant discontinuities due to complex geometry and loading conditions. The absence of extreme stress concentrations and discontinuities indicates that the modelling and the design of the bladed disc structure are appropriate to represent a realistic bladed disc structure.

4.2.2 Modal Analysis. Figure 10 shows the first six mode shapes for the 0^{th} harmonic index/nodal diameter, providing a clear illustration of how the system's cyclic symmetry is reflected in its vibrational modes.

The first mode is characterised by a flap motion with all blades moving in unison, resulting in maximum deflection at the blade tips. This mode is crucial as it typically has the lowest frequency and can be easily excited during operation. The second mode represents an out-of-phase flap, where adjacent blades move in opposite directions, effectively cancelling out some of the overall system displacement and potentially reducing stress on the disc.

For the third and fourth modes, the presence of a single nodal circle suggests the disc modes are involved, pointing out more complex vibrational behaviour. These blade modes involve a combination of flap and twist. These modes are significant as

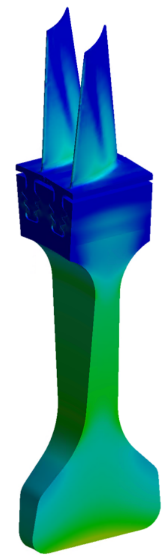


FIGURE 9: STRESS DISTRIBUTION THROUGHOUT THE BLADED DISC SYSTEM UNDER STEADY STATE CONDITIONS.

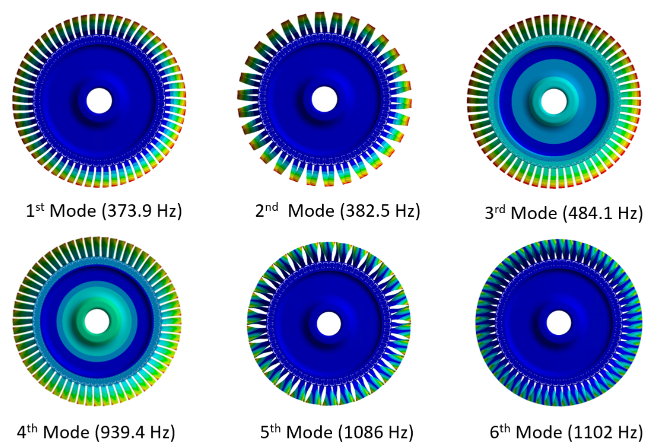


FIGURE 10: THE FIRST SIX MODE SHAPE FOR THE 0^{th} HARMONIC INDEX/NODAL DIAMETER IN THE PRESENCE OF THE UNDERPLATFORM DAMPERS.

they can lead to asymmetric loading on the disc, influencing its fatigue life.

Modes five and six, reverting back to the 0^{th} nodal circle, indicate a proceeding towards more complicated blade motion, coupled blade bending-torsion modes that are out-of-phase and in-phase, respectively.

Figure 11, on the other hand, illustrates the normalised frequencies of the bladed disc system plotted against the number of nodal diameters.

The frequencies of certain modes, namely, modes 1, 2, 4 and 6, remain relatively constant regardless of the number of nodal diameters. This suggests that these modes are less sensitive to the circumferential wave pattern changes around the disc. They represent the flap modes of the blades, which are not significantly affected by the nodal diameters of the disc.

Whereas, mode 3 exhibits a distinct increase in frequency as the nodal diameter grows before plateauing for higher nodal diameters. This mode indicates a significant coupling between the blade and disc, where the addition of a nodal diameter changes the mode's nature from being mainly blade-dominated to one more influenced by the characteristics of the disc. This transition could be due to a shift from a flap-dominated mode to a more torsion-dominated mode or a coupling between blade and disc bending.

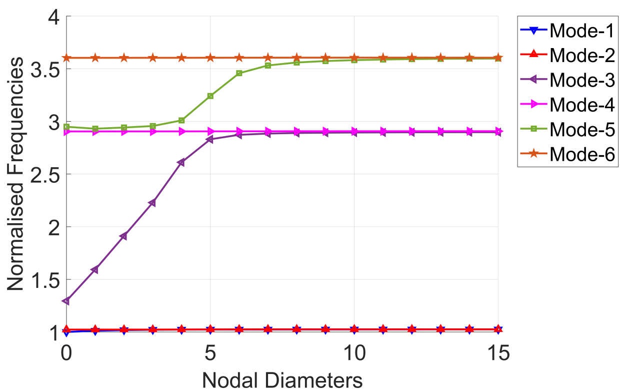


FIGURE 11: THE FIRST SIX NORMALISED NATURAL FREQUENCIES FOR THE NODAL DIAMETERS.

The first and second modes identified in the modal analysis as the target frequency for further nonlinear transient analysis are of particular interest. These modes exist within the lower nodal diameter range and can be greatly affected by the presence of underplatform dampers. Understanding their behaviour is important for making sure the structural integrity and operational efficiency of the bladed disc system remain intact.

4.2.3 Time-Marching and Frequency Sweep. The forced response of the bladed disc system calculated with VIBRANT, as presented in Figure 12 covering the first two modes of the nodal diameter 0, is significantly influenced by the presence of underplatform dampers. Observing the response with and without the underplatform dampers allows for a comparative assessment of their impact on the system's dynamics.

To begin with the evaluation of the linear cases, the linear response with dampers attached, as well as the response without dampers, serves as a foundation to understand the system's behaviour in the absence of nonlinearities. These cases are performed with Abaqus' steady state dynamics simulation approach as well as VIBRANT's time-marching scheme using Abaqus' time domain simulation solver to provide an additional step of validation of VIBRANT. It can be stated that the results are in good agreement. The frequency shift is around $1-2\%$ and the amplitudes are captured well within $6-8\%$.

In the linear scenario, the response of the system with a damper rigidly tied to the blade shows a peak amplitude at a slightly higher frequency compared to the system without a damper. This suggests that the damper's inertia and the added stiffness of the rigid connection have a stiffening effect on the system. When the damper is removed, the system exhibits slightly higher amplitudes, and the resonance frequencies shift to the left, which is slightly lower, indicating a reduction in the system's overall stiffness.

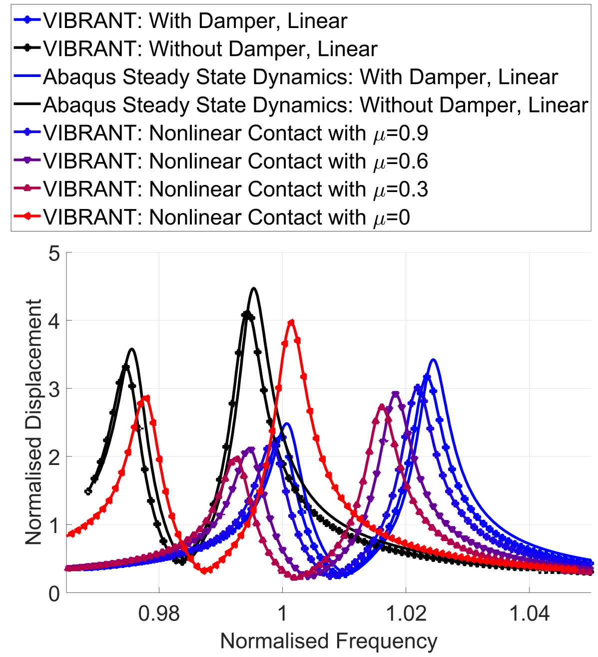


FIGURE 12: FORCED RESPONSE OF THE BLADED DISC SYSTEM MEASURED FROM THE TIP OF A BLADE CONSIDERING ONLY THE NODAL DIAMETER 0.

Moving to the nonlinear cases, the responses calculated with varying friction coefficients μ lie between the two linear cases, leaning closer to the rigidly tied response. This proximity indicates that the normal forces resulting from rotational body forces are substantial, maintaining a predominantly stick regime for this case. Despite this, the presence of frictional damping is still discernible, as evidenced by the dampened peak amplitudes and the frequency shift towards the left, compared to the linear case without a damper. In the presence of contact and the absence of damping where the friction coefficient $\mu = 0$, the results are found to be very close to the linear case without the underplatform damper. The amplitudes are significantly higher than other nonlinear cases as frictional damping is absent from the system.

The second mode, characterised by the out-of-phase blade motion, is notably more heavily damped. The dampers are more effective in this mode due to the relative motion between the blades, which enhances energy dissipation through friction. Consequently, there is a marked frequency shift indicating a weakening of the stick regime, thus reducing the system's stiffness. Furthermore, the amplitude levels drop across the frequency spectrum, signifying that frictional forces are increasingly engaged as the excitation level rises.

Overall, it needs to be stated that the system exhibits a behaviour that is consistent with the findings of previous studies [31, 53, 54].

5. CONCLUSION

This manuscript introduces VIBRANT: VIbration BehaviouR ANalysis Tool, a numerical tool designed to enhance the computational modelling of complex mechanical systems, which can particularly contribute to the field of aerospace engineering. VIBRANT integrates Abaqus' robust capabilities and Python's scripting flexibility, permitting precise/high-fidelity time domain simulations and then transitions into frequency domain analysis. The computational load is significantly reduced by VIBRANT's ability to parallelise computations, enabling efficient handling of various systems and a wide range of nonlinearities.

The numerical examples, including the analysis of an Euler-Bernoulli beam with localised frictional contact and bladed disc systems with underplatform dampers, validate the accuracy and demonstrate the practical application of VIBRANT.

The first example, the beam with a localised contact element, is solved both with NLvib and VIBRANT. NLvib, a validated software that uses a very different approach compared to VIBRANT, is used for comparison against VIBRANT. The results of both software are found in very good agreement. Meanwhile, the second example, which is a very complicated system compared to the first example, shows that VIBRANT can handle any sort of system that can be modelled using Abaqus.

The research mainly contributes to the fields of mechanical and aerospace engineering. VIBRANT can provide a significant understanding of complicated nonlinear systems that could significantly influence the development of future aircraft engines. VIBRANT is a powerful and reliable tool for examining dynamic systems, establishing a foundation for future research and real-world applications. The development of the tool and its application in this study contributes to computational modelling and emphasises the benefits of taking advantage of existing powerful time domain solver/numerical tools to work on complicated systems.

ACKNOWLEDGMENTS

Mertol Tüfekci would like to thank Mr Furkan Sevencan, Mr Ozan Yurdakul and the Composite Materials Laboratory in Faculty of Mechanical Engineering of Istanbul Technical University for their valuable technical support for contributing to this research.

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