Transmit Power Minimization for MIMO Systems of Exponential Average BER with Fixed Outage Probability

Dian-Wu Yue and Yichuang Sun

Abstract

This paper is concerned with a wireless multiple-antenna system operating in multiple-input multiple-output (MIMO) fading channels with channel state information being known at both transmitter and receiver. By *spatiotemporal* subchannel selection and power control, it aims to minimize the average transmit power (ATP) of the MIMO system while achieving an exponential type of average bit error rate (BER) for each data stream. Under the constraints on each subchannel that individual outage probability (OP) and average BER are given, based on a traditional upper bound and a dynamic upper bound of Q function, two closed-form ATP expressions are derived, respectively, which can result in two different power allocation schemes. Numerical results are provided to validate the theoretical analysis, and show that the power allocation scheme with the dynamic upper bound can achieve more power savings than the one with the traditional upper bound.

Index Terms

Multiple antennas, multiple-input multiple-output (MIMO), power control, channel selection, multi-beam, fading channel, multiplexing, diversity.

This work was supported in part by the Scheme of Research Exchanges with China and India, the Royal Academy of Engineering of UK, the Specialized Research Fund for the Doctoral Program of Higher Education under Grant 20132125110006, and the open research fund of Zhejiang Provincial Key Lab of Data Storage and Transmission Technology, Hangzhou Dianzi University, under Grant 201401.

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I. Introduction

Multiple-input multiple-output (MIMO) wireless systems are those that have multiple antenna elements at both the transmitter and receiver [1]. MIMO technology has attracted much interest in the past couple of decades due the resultant improvement in capacity and reliability, and has already become an integral part of wireless standards such as IEEE 802.11 and 4G LTE. So far many people have studied intensively from various aspects of wireless MIMO systems, especially from the important aspects of system efficiency and reliability [2], [3]. In particular, some fundamental tradeoff results between the system efficiency and reliability have also been given [4]- [6].

Adaptive transmission can utilize the resources efficiently and obtain improvements in terms of the system efficiency and reliability by exploiting the channel knowledge available at transmitter. Therefore, adaptive transmission techniques including adaptive power control are always of great interest in the field of wireless communications. Dating back to early 1968, the author in [7] investigated the adaptive power control problem for a single-input single-output (SISO) system, and presented an optimal power strategy, which can minimize the bit error rate (BER) subject to an average power constraint. On the other hand, from the information theoretic point of view, the authors in [8] showed that the water-filling power control policy can maximize the channel capacity. Moreover, with the help of optimization theory and random matrix theory, several novel optimal power control policies for MIMO systems have been already proposed in [9]- [13].

It is well known that reliability performance of wireless communications can be characterized by the diversity order or diversity gain in the high signal-to-noise ratio (SNR) regime. For an additive white Gaussian noise (AWGN) channel, a SISO wireless system with coherent signalling schemes [14] can achieve an infinite diversity order . This implies that the SISO system has a BER exponentially decreasing with SNR. When the same system operates in a Rayleigh fading channel, however, its average BER decreases only inversely with the SNR. The degradation can be partially mitigated if we replace the SISO system with MIMO. In spite of various efforts, nearly all existing MIMO system schemes can only achieve a finite diversity order, even with spatial power control [15], [16].

For the first time, however, [17] and [18] showed that by adaptive power control in time a SISO system can obtain a BER performance with exponential diversity order in fading environments. Subsequently, the good result was extended to a MIMO system. In particular, [19] and [20] showed that by a combined temporal and spatial power control policy, a MIMO system can also obtain an exponential diversity order. The aforementioned results were given under the total average power constraint. When the more realistic scenario of peak to average power ratio (PAPR) constraint was satisfied, papers [21] and [22] considered such an optimal power control problem for MISO channels and also obtained the minimized BER of exponential diversity. It should be noticed that all of the aforementioned discuss were limited in Rayleigh fading. [23] further showed that a MIMO system in "all" fading channels can achieve the exponential diversity order. After that, for wireless multihop systems, [24] presented two different power allocation strategies of achievable exponential diversity order. In fact, the aforementioned results require perfect channel-state information (CSI) at both the transmitter and receiver. Papers [25] and [26] further showed that even in the practical case with imperfect CSI at the transmitter, the exponential diversity order can also be obtained by appropriate spatiotemporal power allocation.

In the existing literature involving the achievement of exponential diversity order for a MIMO system, the underlying system is limited to transmit only a single information stream along one of its eigen beams. It should be pointed out that the MIMO system based on orthogonal space-time block coding (OSTBC) discussed in [20] is indeed equivalent to a single beamforming system [27]. In order to utilize efficiently the degree of freedom provided by multiple antennas, different from the mentioned-above works, in this paper, we will adopt multi-channel beamforming to transmit ([3], [9]) . Under individual average BER and outage probability (OP) constraints for each data stream, we will pursue such a optimal power control strategy that minimizes the total average transmit power. Our optimal strategy is obviously different from the one adopted in [17]- [26] that can minimize the system BER under the total average transmit power constraint. Of course, our strategy is consistent with the current efforts of green communications [28], [29]. And the average BER will be expressed as an exponential function of SNR, which implies that the underlying MIMO system has exponential diversity order.

The rest of the paper is organized as follows. In Section II, we describe the system model and present the optimization problems. In Section III, with the help of an order statistical result of eigenvalues of complex central Wishart matrices, we derive a closed-form ATP expression based on the traditional upper bound of Q function. In Section IV, we present a dynamic upper bound of Q function, and based on it derive further another closed-form ATP expression. In Section V, we consider further the scenario under frequency selective fading. After that, in Section VI we provide some numerical results to validate the theoretical analysis and make comparisons between the two different power allocation schemes. Finally, in Section VII we conclude the paper.

II. System Model And Optimization Formulation

A. *System model*

Under Raleigh flat fading environments, we consider a single-user MIMO system having $n_T \geq 1$ transmit antennas and $n_R \geq 1$ receive antennas. We assume that both of the transmitter and the receiver can know the perfect CSI. Let $\mathbf{H}$ denote the channel gain matrix whose i, j-th entry is h_{ij}. h_{ij} represents the channel gain between i-th receive antenna and j-th transmit antenna, and is independent and identically distributed (i.i.d.). Thus, $\mathbf{H}$ follows the joint complex Gaussian distribution with zero mean matrix and covariance matrix $\mathbf{I}_{n_R} \otimes \mathbf{I}_{n_T}$ [30], i.e.,

$$\mathbf{H} \sim \mathbb{CN}(0, \mathbf{I}_{n_R} \otimes \mathbf{I}_{n_T}). \tag{1}$$

For a transmission through the MIMO channel with $\mathbf{H}$, the $n_R \times 1$ received vector can be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{2}$$

where $\mathbf{x}$ is the $n_T \times 1$ transmitted vector and $\mathbf{n}$ is the $n_R \times 1$ additive noise vector following complex Gaussian distribution of zero-mean vector and covariance matrix $\mathbf{I}_{n_R}$, i.e., $\mathbf{n} \sim \mathbb{CN}(0, \mathbf{I}_{n_R})$.

Now let $m = \min\{n_T, n_R\}$ and $n = \max\{n_T, n_R\}$. Define

$$\mathbf{\Omega} = \begin{cases} \mathbf{H}^\dagger \mathbf{H}, & \text{for } m = n_T; \\ \mathbf{H}\mathbf{H}^\dagger, & \text{for } n = n_T. \end{cases} \tag{3}$$

From Chapter 3 of [30], it follows that the matrix $\mathbf{\Omega}$ follows Wishart distribution, i.e., $\mathbf{\Omega} \sim \mathbb{CW}(n, \mathbf{I}_m)$. Following the conventional spatial multiplexing method based on singular value decomposition(SVD) [31], [32], the channel matrix can be written as

$$\mathbf{H} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^\dagger \tag{4}$$

where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices, and

$$\mathbf{\Lambda} = \text{diag}(\sqrt{\lambda_1}, \sqrt{\lambda_2}, \ldots, \sqrt{\lambda_m}) \tag{5}$$

with $\{\lambda_i : i = 1, 2, \ldots, m\}$ being the eigenvalues of $\mathbf{\Omega}$ sorted in descending order, i.e.,

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m. \tag{6}$$

Thus we can transmit $r \leq m$ data symbols at one time. Since $\mathbf{H}$ is known perfectly at the transmitter, we can set the transmitted vector as

$$\mathbf{x} = \mathbf{V}_1^r \mathbf{P}\mathbf{s} \tag{7}$$

where $\mathbf{s}$ is the $r \times 1$ modulated data vector with covariance matrix $\mathbf{I}_r$, $\mathbf{V}_1^r$ is the precoding matrix formed with the first r columns of $\mathbf{V}$ associated with the first r largest eigenvalues of $\mathbf{\Omega}$, and $\mathbf{P}$ is a diagonal matrix as follows:

$$\mathbf{P} = \text{diag}(\sqrt{p_1}, \sqrt{p_2}, \ldots, \sqrt{p_r}) \tag{8}$$

where $\{p_i : i = 1, 2, \ldots, r\}$ are the powers allocated to the r established data streams. Due to the assumption that CSI is available at the receiver, the symbols transmitted through the receive filter are recovered from the received vector $\mathbf{y}$ with matrix $\mathbf{U}_1^r$, defined similarly to $\mathbf{V}_1^r$, as

$$\begin{aligned} \hat{\mathbf{s}} &= (\mathbf{U}_1^r)^\dagger(\mathbf{H}\mathbf{x} + \mathbf{n}) \\ &= \mathbf{\Lambda}_1^r \mathbf{P}\mathbf{s} + (\mathbf{U}_1^r)^\dagger \mathbf{n} \end{aligned} \tag{9}$$

where $\mathbf{\Lambda}_1^r$ is a diagonal submatrix of $\mathbf{\Lambda}$ that contains the r largest eigenvalues in descending order, and the filter-processed noise $\eta = (\mathbf{U}_1^r)^\dagger \mathbf{n}$ has the same statistical properties as $\mathbf{n}$, possibly with a reduced dimension. Each data stream then experiences an instantaneous SNR given by

$$\text{SNR}_i = \lambda_i p_i, \quad i = 1, 2, \ldots, r. \tag{10}$$

And the corresponding short term BER is expressed as

$$P_b^{(i)} = \xi_i Q(\sqrt{\beta_i \text{SNR}_i}) \tag{11}$$

where $Q(\cdot)$ is the Gaussian Q function, and the parameters ξ_i and β_i are constants, depending on the used modulation type [33].

B. *Optimization formulation*

In addition to the average symbol error probability, the OP is another often used performance indicator for wireless communications in fading environments. It is well-known that the OP is defined as the probability when the instantaneous SNR falls below a certain threshold [34]. At this time when the i-th subchannel is in bad condition, in order to save transmit power, the subchannel should have a transmit outage temporarily. For this reason, in order to analyze conveniently, here we set the SNR threshold as $p_i\overline{\lambda}_{\text{out}}(i)$ for the i-th subchannel. So we will introduce a transmit outage when $\lambda_i < \overline{\lambda}_{\text{out}}(i)$. Accordingly, the individual OP is expressed as

$$P_{\text{out}}^{(i)} = \int_0^{\overline{\lambda}_{\text{out}}(i)} f_i(\lambda_i) d\lambda_i \tag{12}$$

where $f_i(\lambda_i)$ is the p.d.f. of eigenvalue λ_i.

Once the OP is given, we can carry on adaptive transmission. In particular, based on channel eigenvalues, we can select those MIMO subchannels satisfying the OP constraint condition to transmit data streams, and let each of them transmit a data stream. In order to utilize efficiently MIMO subchannels, we should employ all those satisfactory subchannels to communicate. Note that if any subchannel does not satisfy the constraint, then this implies that the subchannel cannot transmit a data stream, and thereby we force the channel into the state of channel outage; and if none of the MIMO subchannels satisfies the constraint, then this will result in a system outage.

The above-mentioned adaptive transmission involves not only channel selection but also power control, both of which are conducted based on the status of eigenvalues of channel matrix. As already mentioned before, our adaptive power allocation strategy aims at minimizing the total ATP while each data stream achieves an exponential average BER. For this reason, under the constraint that both the individual OP $P_{\text{out}}^{(i)}$ and the individual average BER $\overline{P}_b(i)$ are given, this optimal problem can be formulated as

$$\begin{cases} \underset{\{p_i:1\le i\le m\}}{\text{Minimize}} & \rho = \mathbb{E}\{\sum_{i=1}^m p_i\}; \\ \text{Subject to} & \frac{\mathbb{E}P_b^{(i)}}{\overline{P}_o(i)} \le \overline{P}_b(i), \;\; 1\le i\le m \end{cases} \tag{13}$$

where

$\overline{P}_o(i)$ denotes the transmit probability, and thus is written as

$$\overline{P}_o(i) = 1 - P_{\text{out}}^{(i)}. \tag{14}$$

In addition,

$$\mathbb{E}P_b^{(i)} = \int_{\overline{\lambda}_{\text{out}}(i)}^{\infty} P_b^{(i)} f_i(\lambda_i)d\lambda_i = \int_{\overline{\lambda}_{\text{out}}(i)}^{\infty} \xi_i Q(\sqrt{\beta_i\lambda_i p_i}) f_i(\lambda_i)d\lambda_i \tag{15}$$

where $\mathbb{E}(\cdot)$ stands for the expectation operator.

On the other hand, the required BER $\overline{P}_b(i)$ can be expressed as an exponential function of SNR:

$$\overline{P}_b(i) = \frac{\xi_i}{2} e^{-\beta_i \widehat{\text{SNR}}(i)/2}. \tag{16}$$

It should be pointed out that the SNR $\widehat{\text{SNR}}(i)$ can be designed beforehand.

Obviously, this optimization problem can be translated into m individual optimization problems, and each corresponds to an ordered subchannel:

$$\begin{cases} \underset{\{p_i\}}{\text{Minimize}} & \mathbb{E}\{p_i\}; \\ \text{Subject to} & \mathbb{E}P_b^{(i)} \le \overline{P}_o(i)\overline{P}_b(i). \end{cases} \tag{17}$$

Applying Lagrange Multiplier Method to each of the above sub-optimization problems, we get the following family of unconstrained optimization problems parameterized by multipliers $\omega_i > 0, \;\; 1 \le i \le m$:

$$\underset{\{p_i\}}{\text{Min}} \int_{\overline{\lambda}_{\text{out}}(i)}^{\infty} p_i f_i(\lambda_i)d\lambda_i + \omega_i \int_{\overline{\lambda}_{\text{out}}(i)}^{\infty} P_b^{(i)} f_i(\lambda_i)d\lambda_i - \omega_i \overline{P}_o(i)\overline{P}_b(i) \tag{18}$$

or

$$\underset{\{p_i\}}{\text{Min}} \int_{\overline{\lambda}_{\text{out}}(i)}^{\infty} [p_i + \omega_i(P_b^{(i)} - \overline{P}_b(i))] f_i(\lambda_i)d\lambda_i. \tag{19}$$

If we make use of the exact expression of $P_b^{(i)} = \xi_i Q(\sqrt{\beta_i \text{SNR}_i})$ to solve the problems, then due to the relatively complicated Q function, we can only have an unclosed-form expression based on the Lambert W function [35]. Similar to [20] and [24], we also employ the common upper bound $Q(x) \leq \frac{1}{2}e^{-x^2/2}$ to replace the exact expression and obtain easily a suboptimum solution as follows:

$$p_i = \begin{cases} \frac{2}{\beta_i \lambda_i} \ln(\frac{\lambda_i}{\lambda_0^{(i)}}) & \text{for } \lambda_i > \lambda_0(i); \\ 0 & \text{for } \lambda_i \leq \lambda_0(i). \end{cases} \tag{20}$$

where $\lambda_0(i) = \max\{\lambda_0^{(i)}, \overline{\lambda}_{\text{out}}(i)\}$ and $\lambda_0^{(i)}$ can be found by solving

$$\int_{\lambda_0(i)}^{\infty} \frac{\xi_i}{2} e^{-\beta_i p_i \lambda_i/2} f_i(\lambda_i) d\lambda_i = \overline{P}_o(i)\overline{P}_b(i). \tag{21}$$

This suboptimum solution will provide convenience for us to produce theoretical and numerical results.

III. Minimum Average Transmit Power and A Power Allocation Scheme

A. Individual outage probability

It follows from [36] that the marginal p.d.f. of the i-th largest eigenvalue $\lambda_i,\ i = 1, 2, \ldots, m$, can be expressed as a sum of terms $\lambda_i^a e^{-b\lambda_i}$, which is very friendly for further analysis. By the expression, we can easily get the following expression of individual outage probability.

Lemma 1: The individual OP for the i data stream can be given by

$$P_{\text{out}}^{(i)} = \sum_{k=i}^{m} (-1)^{k-i} \binom{k-1}{i-1} \binom{m}{k} \overline{F}_{\text{min:}k}^{(\text{out})}(\overline{\lambda}_{\text{out}}(i)) \tag{22}$$

where $\overline{F}_{\text{min:}k}^{(\text{out})}(\overline{\lambda}_{\text{out}}(i))$ denotes the distribution function of the smallest random variable considered in a subset of k random variables over the random variable set of all eigenvalues $\{\lambda_i,\ i = 1, 2, \ldots, m\}$, and is given by [36]

$$\overline{F}_{\text{min:}k}^{(\text{out})} = \frac{kC}{m!} \sum_{\alpha} \sum_{\mu} \text{sgn}(\alpha)\text{sgn}(\mu) A_k(\alpha, \mu) \times \sum\nolimits_{\tau} \frac{\gamma(\theta + \alpha_k + \mu_k - 1 + \sum_{\iota=1}^{k-1} \tau_\iota, k\overline{\lambda}_{\text{out}}(i))}{k^{\theta + \alpha_k + \mu_k - 1 + \sum_{\iota=1}^{k-1} \tau_\iota}} \prod_{\iota=1}^{k-1} \frac{(\theta + \alpha_\iota + \mu_\iota - 2)!}{\tau_\iota!} \tag{23}$$

where $\theta = n - m$, C is a constant standing for

$$C = \frac{1}{\prod_{j=1}^{m}(m-j)! \prod_{j=1}^{m}(n-j)!}, \tag{24}$$

$\text{sgn}(\alpha)$ denotes the sign of permutation $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_m)$ for integers $\{1, 2, \ldots, m\}$, $A_k(\alpha, \mu)$ is defined as

$$A_k(\alpha, \mu) = \prod_{\iota=k+1}^{m} (\theta + \alpha_\iota + \mu_\iota - 2)!, \tag{25}$$

and $\sum_\tau$ denotes

$$\sum\nolimits_{\tau} = \sum_{\tau_1=0}^{\theta+\alpha_1+\mu_1-2} \sum_{\tau_2=0}^{\theta+\alpha_2+\mu_2-2} \cdots \sum_{\tau_{k-1}=0}^{\theta+\alpha_{k-1}+\mu_{k-1}-2}. \tag{26}$$

Moreover, $\gamma(q,x)$ is just the incomplete gamma function (See Page 454 of [37]).

On the other hand, the global outage probability for the whole system is written as

$$\begin{aligned} P_{\text{out}} &= \text{Prob}(\lambda_i < \overline{\lambda}_{\text{out}}(i),\ \ 1 \le i \le m) \\ &\le P_{\text{out}}^{(1)}. \end{aligned} \tag{27}$$

When $\overline{\lambda}_{\text{out}}(i) = \lambda_{\text{out}}$ for $1 \le i \le m$, we can have

$$P_{\text{out}} = P_{\text{out}}^{(1)}. \tag{28}$$

B. Another BER constraint condition

In order to provide convenience for the system design, we hope that $\lambda_0(i) = \overline{\lambda}_{\text{out}}(i)$. For that, in this subsection we will derive out such a BER constraint condition that can let $\lambda_0(i) = \overline{\lambda}_{\text{out}}(i)$ hold. So we revisit the derivation process of optimum solution in Subsection II.B, and rewrite the expression (20) as

$$p_i = \begin{cases} \frac{\widehat{\text{SNR}}(i)}{\lambda_i} + \frac{2}{\beta_i\lambda_i}\ln(\frac{\lambda_i\Delta(i)}{\overline{\lambda}_{\text{out}}(i)}), & \lambda_i > \overline{\lambda}_{\text{out}}(i) \\ 0, & \lambda_i \le \overline{\lambda}_{\text{out}}(i) \end{cases} \tag{29}$$

where the unknown optimization parameter $\Delta(i)$ should meet the following BER constraint condition:

$$\int_{\overline{\lambda}_{\text{out}}(i)}^{\infty} \frac{\xi_i}{2} e^{-\beta_i p_i \lambda_i/2} f_i(\lambda_i) d\lambda_i = \overline{P}_o(i)\overline{P}_b(i). \tag{30}$$

Substituting (29) and (16) into (30), we have after a simplifying process

$$\overline{P}_b(i) \int_{\overline{\lambda}_{\text{out}}(i)}^{\infty} \frac{\overline{\lambda}_{\text{out}}(i)}{\lambda_i\Delta(i)} f_i(\lambda_i) d\lambda_i = \overline{P}_o(i)\overline{P}_b(i). \tag{31}$$

(31) can be simplified further to

$$\Delta(i) = \overline{\lambda}_{\text{out}}(i) \int_{\overline{\lambda}_{\text{out}}(i)}^{\infty} \frac{1}{\lambda_i} f_i(\lambda_i) d\lambda_i / \overline{P}_o(i). \tag{32}$$

From the theorem of integral mean value, there is a constant g satisfying

$$\int_{\overline{\lambda}_{\text{out}}(i)}^{\infty} \frac{1}{\lambda_i} f_i(\lambda_i) d\lambda_i = g\overline{P}_o(i). \tag{33}$$

With g, we can define a new function of $\overline{\lambda}_{\text{out}}(i)$ as follows:

$$\overline{\lambda}_{\text{mea}}(i) = \frac{1}{g} = \frac{\overline{P}_o(i)}{\int_{\overline{\lambda}_{\text{out}}(i)}^{\infty} \frac{1}{\lambda_i} f_i(\lambda_i) d\lambda_i}. \tag{34}$$

Furthermore, it can follow from the theorem of integral mean value that

$$\overline{\lambda}_{\text{mea}}(i) \ge \overline{\lambda}_{\text{out}}(i). \tag{35}$$

Then $\Delta(i)$ can be rewritten as

$$\Delta(i) = \frac{\overline{\lambda}_{\text{out}}(i)}{\overline{\lambda}_{\text{mea}}(i)}. \tag{36}$$

Taking account of the requirement of $p_i \geq 0$, from (29) $\Delta(i)$ should also meet another BER constraint condition:

$$\frac{\widehat{\text{SNR}}(i)}{\lambda_i} + \frac{2}{\beta_i \lambda_i} \ln(\frac{\lambda_i \Delta(i)}{\overline{\lambda}_{\text{out}}(i)}) \geq 0 \tag{37}$$

or

$$\Delta(i) \geq \frac{\overline{\lambda}_{\text{out}}(i)}{\lambda_i} e^{-\beta_i \widehat{\text{SNR}}(i)/2} \tag{38}$$

Due to the fact of $\lambda_i \geq \overline{\lambda}_{\text{out}}(i)$, the constraint condition becomes under the help of (16)

$$\Delta(i) \geq \frac{2}{\xi_i} \overline{P}_b(i). \tag{39}$$

By (36), the constraint condition can be rewritten as

$$\overline{P}_b(i) \leq \frac{\xi_i}{2} \frac{\overline{\lambda}_{\text{out}}(i)}{\overline{\lambda}_{\text{mea}}(i)}. \tag{40}$$

So we have the following lemma finally.

Lemma 2: If $\overline{P}_b(i) \leq \frac{\xi_i}{2} \frac{\overline{\lambda}_{\text{out}}(i)}{\overline{\lambda}_{\text{mea}}(i)}$, then,

$$\lambda_0(i) = \overline{\lambda}_{\text{out}}(i) \tag{41}$$

and the optimum solution of power allocation is (29).

C. Minimum average transmit power

Under the condition that the mentioned-above inequality (40) holds, we derive the minimum average transmit power and obtain the following result.

Proposition 1: Suppose that $\overline{P}_b(i) \leq \frac{\xi_i}{2} \frac{\overline{\lambda}_{\text{out}}(i)}{\overline{\lambda}_{\text{mea}}(i)}$. Let $\hat{\rho}^{(i)}(\overline{P}_b(i), P_{\text{out}}^{(i)})$ denote the average needed transmit power for i-th data stream achieving the BER given by (16) under the condition that the OP $P_{\text{out}}^{(i)}$ is given. Then

$$\begin{aligned} \hat{\rho}^{(i)} &= \rho_s(\overline{P}_b(i), P_{\text{out}}^{(i)}) + \rho_\Delta(P_{\text{out}}^{(i)}) \\ &= \rho_s(\widehat{\text{SNR}}(i), \overline{\lambda}_{\text{out}}(i)) + \rho_\Delta(\overline{\lambda}_{\text{out}}(i)) \end{aligned} \tag{42}$$

where

$$\rho_s(\widehat{\text{SNR}}(i), \overline{\lambda}_{\text{out}}(i)) = \sum_{k=i}^{m} (-1)^{k-i} \binom{k-1}{i-1} \binom{m}{k} \overline{F}_{\text{min}:k}^{(\text{pow})}(\widehat{\text{SNR}}(i), \overline{\lambda}_{\text{out}}(i)) \tag{43}$$

and

$$\rho_\Delta = \sum_{k=i}^{m} (-1)^{k-i} \binom{k-1}{i-1} \binom{m}{k} \overline{F}_{\text{min}:k}^{(\text{del})}(\overline{\lambda}_{\text{out}}(i)) \tag{44}$$

with

$$\overline{F}^{(\text{pow})}_{\text{min}:k} = \widehat{\text{SNR}}(i) \cdot \frac{kC}{m!} \sum_{\alpha} \sum_{\mu} \text{sgn}(\alpha)\text{sgn}(\mu) A_k(\alpha, \mu) \times$$

$$\sum_{\tau} \frac{\Gamma(\theta + \alpha_k + \mu_k - 2 + \sum_{\iota=1}^{k-1} \tau_\iota, k\overline{\lambda}_{\text{out}}(i))}{k^{\theta+\alpha_k+\mu_k-2+\sum_{\iota=1}^{k-1}\tau_\iota}} \prod_{\iota=1}^{k-1} \frac{(\theta + \alpha_\iota + \mu_\iota - 2)!}{\tau_\iota!} \tag{45}$$

$$\overline{F}^{(\text{del})}_{\text{min}:k} = \frac{kC}{m!} \sum_{\alpha} \sum_{\mu} \text{sgn}(\alpha)\text{sgn}(\mu) A_k(\alpha, \mu) \times$$

$$\sum_{\tau} \frac{2 \cdot \jmath(N_k(\boldsymbol{\tau}), k\overline{\lambda}_{\text{out}}(i), \Delta(i))}{\beta_i k^{N_k(\boldsymbol{\tau})}} \prod_{\iota=1}^{k-1} \frac{(\theta + \alpha_\iota + \mu_\iota - 2)!}{\tau_\iota!} \tag{46}$$

$$\jmath(N_k(\boldsymbol{\tau}), k\overline{\lambda}_{\text{out}}(i), \Delta(i)) =$$

$$(N_k(\boldsymbol{\tau}) - 1)! \sum_{\iota=0}^{N_k(\boldsymbol{\tau})-1} \frac{\Gamma(\iota, k\overline{\lambda}_{\text{out}}(i))}{\iota!} + \Gamma(N_k(\boldsymbol{\tau}), k\overline{\lambda}_{\text{out}}(i)) \ln \Delta(i) \tag{47}$$

and

$$N_k(\boldsymbol{\tau}) = \theta + \alpha_k + \mu_k - 2 + \sum_{\iota=1}^{k-1} \tau_\iota. \tag{48}$$

The derivation of Proposition 1 is given in Appendix. Moreover, it should be noticed that the function $\overline{F}^{(\text{pow})}_{\text{min}:k}(\widehat{\text{SNR}}(i), \overline{\lambda}_{\text{out}}(i))$ represents the cumulative distribution function of the smallest random variable considered in a subset of k random variables over the random variable set of all eigenvalues, which corresponds to $\overline{F}^{(\text{out})}_{\text{min}:k}(\overline{\lambda}_{\text{out}}(i))$ while $\overline{F}^{(\text{del})}_{\text{min}:k}(\overline{\lambda}_{\text{out}}(i))$ corresponds to $\overline{F}^{(\text{pow})}_{\text{min}:k}(\widehat{\text{SNR}}(i), \overline{\lambda}_{\text{out}}(i))$. Moreover, $\Gamma(q, x)$ in the above equations stands for the complementary incomplete gamma function (See Page 454 of [37]).

IV. Modified Power Allocation Scheme

A. *A dynamic upper bound of Q function*

Fig.1 makes comparison between Q function $Q(\sqrt{2\text{SNR}})$ and its traditional upper bound $\frac{1}{2}e^{-\text{SNR}/2}$. As seen in Fig.1, at the important BER region their SNR deviation is relatively large and slowly becomes smaller as SNR increases. For example, when $P_b = 10^{-6}$, the SNR deviation is 0.65 dB. Therefore, in order to improve the system performance, we consider to find a new upper bound of Q function replacing the old one and with it give a modified power allocation scheme. In order to continue to employ the analysis method given in Section III, we now need to study the following type of exponential upper bounds of Q function:

$$Q(\sqrt{2\text{SNR}}) \leq \frac{1}{c(\text{SNR})} e^{-\text{SNR}/2} \tag{49}$$

Note that different from the old upper bound, here we allow the designated parameter c(SNR) to be dynamically variable.

For any given BER $= Q(\sqrt{2\text{SNR}})$, we easily find a $c(\text{SNR})$ which makes the new upper bound to approximate appropriately to the given BER. At the BER region from 10^{-3} to 10^{-8} Fig.1 also plots the new upper bound by using some appropriate values of $c(\text{SNR})$. As seen in Fig.1, the dynamic upper bound approximates well to the exact value of Q function. The computed results are also presented in Table I. From the table, it can be observed that the optimized value of parameter c increases as the SNR increases. Therefore, we have the following property of Q function:

Lemma 3: If $Q(\sqrt{2\text{SNR}}) \leq \frac{1}{c(\text{SNR})}e^{-\text{SNR}/2}$ for a given SNR, then

$$Q(\sqrt{2(\text{SNR}+\Lambda)}) \leq \frac{1}{c(\text{SNR})}e^{-(\text{SNR}+\Lambda)/2},\ \Lambda > 0. \tag{50}$$

B. Modified minimum ATP

For any given $\overline{P}_b(i)$, we can find appropriate $\widetilde{\text{SNR}}(i)$ and $c(\widetilde{\text{SNR}}(i))$ satisfying

$$\overline{P}_b(i) \approx \frac{\xi_i}{c(\widetilde{\text{SNR}}(i))}e^{-\beta_i(i)\widetilde{\text{SNR}}(i)/2}. \tag{51}$$

With the help of Lemma 3, thus the power allocation scheme can be modified as

$$p_i = \begin{cases} \frac{\widetilde{\text{SNR}}(i)}{\lambda_i} + \frac{2}{\beta_i\lambda_i}\ln(\frac{\lambda_i\Delta(i)}{\overline{\lambda}_{\text{out}}(i)}), & \lambda_i > \overline{\lambda}_{\text{mea}}(i) \\ 0, & \lambda_i \leq \overline{\lambda}_{\text{out}}(i) \\ \frac{\widehat{\text{SNR}}(i)}{\lambda_i} + \frac{2}{\beta_i\lambda_i}\ln(\frac{\lambda_i\Delta(i)}{\overline{\lambda}_{\text{out}}(i)}), & \text{Otherwise.} \end{cases} \tag{52}$$

For the modified power allocation scheme, the BER constraint condition in (13) can be still met since

$$\begin{aligned} \mathbb{E}P_b^{(i)} &= \int_{\overline{\lambda}_{\text{out}}(i)}^{\infty} \xi_i Q(\sqrt{\beta_i\text{SNR}_i}) f_i(\lambda_i)d\lambda_i \\ &= \int_{\overline{\lambda}_{\text{out}}(i)}^{\infty} \xi_i Q(\sqrt{\beta_i p_i\lambda_i}) f_i(\lambda_i)d\lambda_i \\ &= \int_{\overline{\lambda}_{\text{mea}}(i)}^{\infty} \xi_i Q(\sqrt{\beta_i p_i\lambda_i}) f_i(\lambda_i)d\lambda_i \\ &\quad + \int_{\overline{\lambda}_{\text{out}}(i)}^{\overline{\lambda}_{\text{mea}}(i)} \xi_i Q(\sqrt{\beta_i p_i\lambda_i}) f_i(\lambda_i)d\lambda_i \\ &\leq \int_{\overline{\lambda}_{\text{mea}}(i)}^{\infty} \frac{\xi_i}{c(\widetilde{\text{SNR}}(i))}e^{-\beta_i p_i\lambda_i/2} f_i(\lambda_i)d\lambda_i \\ &\quad + \int_{\overline{\lambda}_{\text{out}}(i)}^{\overline{\lambda}_{\text{mea}}(i)} \frac{\xi_i}{2}e^{-\beta_i p_i\lambda_i/2} f_i(\lambda_i)d\lambda_i \\ &\approx \overline{P}_b(i)\int_{\overline{\lambda}_{\text{mea}}(i)}^{\infty} \frac{\overline{\lambda}_{\text{out}}(i)}{\lambda_i\Delta(i)} f_i(\lambda_i)d\lambda_i \\ &\quad + \overline{P}_b(i)\int_{\overline{\lambda}_{\text{out}}(i)}^{\overline{\lambda}_{\text{mea}}(i)} \frac{\overline{\lambda}_{\text{out}}(i)}{\lambda_i\Delta(i)} f_i(\lambda_i)d\lambda_i \\ &= \overline{P}_b(i)\overline{P}_o(i). \end{aligned} \tag{53}$$

Moreover, we can verify the fact that if (40) holds, the optimal solution (52) exits.

Accordingly, the ATP for i-th subchannel is derived again and Proposition 1 is modified as follows:

Proposition 2: Suppose that $\overline{P}_b(i) \leq \frac{\xi_i}{2}\frac{\overline{\lambda}_{\text{out}}(i)}{\overline{\lambda}_{\text{mea}}(i)}$. Let $\tilde{\rho}^{(i)}(\overline{P}_b(i), P_{\text{out}}^{(i)})$ denote the average needed transmit power for i-th data stream achieving the BER given by (51) under the condition that the OP $P_{\text{out}}^{(i)}$ is given. Then

$$\begin{aligned}\tilde{\rho}^{(i)} &= \hat{\rho}^{(i)}(\overline{P}_b(i), P_{\text{out}}^{(i)}) - \rho_s(\widehat{\text{SNR}}(i) - \widetilde{\text{SNR}}(i), \overline{\lambda}_{\text{mea}}(i)) \\ &= \rho_s(\widehat{\text{SNR}}(i), \overline{\lambda}_{\text{out}}(i)) + \rho_\Delta(\overline{\lambda}_{\text{out}}(i)) - \rho_s(\widehat{\text{SNR}}(i) - \widetilde{\text{SNR}}(i), \overline{\lambda}_{\text{mea}}(i)). \end{aligned} \tag{54}$$

By comparing (54) in Proposition 2 with (42) in Proposition 1, we clearly see that the amount of power savings is just equal to $\rho_s(\widehat{\text{SNR}}(i) - \widetilde{\text{SNR}}(i), \overline{\lambda}_{\text{mea}}(i))$.

V. Extension to Frequency Selective Fading Channels

Now we consider a broadband OFDM-MIMO system operating over a frequency-selective channel with K subcarriers, n_T transmit and n_R receive antennas. Furthermore, we assume ideal OFDM transmission with proper cyclic prefix extension. Then the frequency selective MIMO channel can be converted into a set of K parallel independent frequency flat MIMO channels [39]- [41]. In particular, the input-output relationship for the k-th subcarrier can be rewritten as

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{n}_k \tag{55}$$

where $\mathbf{x}_k$ and $\mathbf{n}_k$ are the $n_T \times 1$ transmitted vector and the $n_R \times 1$ AWGN vector of the k-th subcarrier, respectively. And $\mathbf{H}_k$ is modeled as

$$\mathbf{H}_k = \sum_{\ell=0}^{L-1} \rho_\ell \mathbf{H}[\ell] \exp(-j2\pi\frac{k}{K}\ell) \tag{56}$$

where $\mathbf{H}[\ell]$ denotes the channel matrix at time delay ℓ, L represents the channel delay spread, and $\{\rho_\ell^2\}$ is the power delay profile satisfying $\sum_{\ell=0}^{L-1} \rho_\ell^2 = 1$. In addition, $\mathbf{H}[\ell],\ \ell = 0, 1, \cdots, L-1$ are mutually uncorrelated and i.i.d. Rayleigh distributed. So the distribution of $\mathbf{H}_k$ can be expressed as [30]

$$\begin{aligned}\mathbf{H} &\sim \mathbb{CN}(0, \sum_{\ell=0}^{L-1} \rho_\ell^2 |\exp(-j2\pi\frac{k}{K}\ell)|^2 \mathbf{I}_{n_R} \otimes \mathbf{I}_{n_T}) \\ &= \mathbb{CN}(0, \mathbf{I}_{n_R} \otimes \mathbf{I}_{n_T}). \end{aligned} \tag{57}$$

From (55) and (57) it can conclude that the procedure of transmit power minimization for a frequency selective fading MIMO channel can be converted to that for a set of the equivalent frequency flat fading MIMO channels.

VI. Numerical Results and Comparison

In all of our simulation, we always make use of BPSK modulation for each data stream transmission, which corresponds to the modulation parameters $\xi_i = 1, \beta_i = 2, i = 1, 2, \ldots, m$. For simplicity, we let each data stream have the same constraint parameters, i.e.,

$$P_{\text{out}}^{(1)} = P_{\text{out}}^{(2)} = \ldots = P_{\text{out}}^{(i)} = P_{\text{out}}, \tag{58}$$

and

$$\overline{P}_b(1) = \overline{P}_b(2) = \ldots = \overline{P}_b(m) = P_b. \tag{59}$$

We first observe the behavior of MIMO individual outage probability using Lemma 1. In order to provide convenience for making OP comparison between SISO and MIMO systems, we first evaluate outage probability $P_{\text{out}} = 10^{-\upsilon}$ for SISO systems by setting exponent υ. And we call υ as a SISO outage exponent (OE). For example, we set the OE $\upsilon = 1$, then $P_{\text{out}} = 10^{-1}$ and $\lambda_{\text{out}} = 1.1 \cdot 10^{-1}$ for the SISO system, and under given $\lambda_{\text{out}} = 1.1 \cdot 10^{-1}$ we can compute further $P_{\text{out}} = 6.6 \cdot 10^{-5}$ for the MIMO system with $n = 6$, $m = 3$, and $i = 3$. Table II provides computed results for the MIMO system with $n = 6$, $m = 3$, and $i = 3$ when υ is set from 0.4 to 1.8. Table II shows that the MIMO system has lower individual OP as υ increases, and almost has no outage when $\upsilon \geq 1.6$. In the following, if needed, we will always set $\lambda_{\text{out}} = 1.1 \cdot 10^{-1}$, whose corresponding OE is $\upsilon = 1$.

We now consider the constraint condition of BER in optimization design, which is given in (40). Fig.2 plots the constrained BER for the MIMO system with $m = 3$ and $n = 6$ when the OE is set appropriately from 1 to 2. From this figure, the constraint condition is easily met for any of the three data streams $i = 1, 2, 3$.

We still fix the minimum antenna number $m = 3$ and the maximum antenna number $n = 6$. Fig.3 plots the individual ATP for the two adaptive transmit schemes produced using the new and old upper bounds of Q function for $i = 1, 2, 3$. It can be observed that the ATP increases gradually as i increases, which implies that the channel condition becomes worse. On the other hand, the power allocation with the new upper bound (UB) has more power saving than the one with the old UB for all $i = 1, 2, 3$.

Finally, we fix $m = 3$ and $i = 2$. Fig.4 plots the individual ATP computed by Proposition 2 for different n. It can be observed that as the maximum number of antennas n increases, the needed ATP decreases, but the amount of ATP improvement becomes gradually smaller. For comparison, this figure also includes the BER curve with the traditional UB of Q function BER $= \frac{1}{2} e^{-\widehat{\text{SNR}}}$. It can be found from this figure that all of the three different MIMO configurations have the reliable performance of exponential average BER.

TABLE I

MINIMIZATION PARAMETER c FOR THE NEW UPPER BOUND OF Q FUNCTION

BER	10^{-3}	$10^{-3.5}$	10^{-4}	$10^{-4.5}$	10^{-5}	$10^{-5.5}$	10^{-6}	$10^{-6.5}$	10^{-7}	$10^{-7.5}$	10^{-8}
SNR	6.8	7.7	8.4	9.0	9.6	10.1	10.5	10.9	11.3	11.7	12.0
c(SNR)	8.4	9.2	9.8	10.4	11.2	11.8	12.4	12.8	13.4	13.8	14.4

TABLE II

OUTAGE PROBABILITY COMPARISON BETWEEN SISO AND MIMO SYSTEMS

SISO OE	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
$\overline{\lambda}_{\text{out}}$	5.1×10^{-1}	2.9×10^{-1}	1.7×10^{-1}	1.1×10^{-1}	6.5×10^{-2}	4.1×10^{-2}	2.5×10^{-2}	1.6×10^{-2}
SISO OP	4.0×10^{-1}	2.5×10^{-1}	1.6×10^{-1}	1.0×10^{-1}	6.3×10^{-2}	4.0×10^{-2}	2.5×10^{-2}	1.6×10^{-2}
MIMO OP	2.0×10^{-2}	2.9×10^{-3}	4.3×10^{-4}	6.6×10^{-5}	1.0×10^{-5}	1.6×10^{-6}	2.5×10^{-7}	4.0×10^{-8}

VII. CONCLUSIONS

It is well known that multiple antennas can provide high multiplexing and diversity gains for wireless communications. Adaptive transmission techniques in wireless communications can utilize the system resources efficiently and provide satisfactory QoS. In this paper, we have investigated adaptive transmission mainly based on channel eigenvalues for MIMO multi-beams systems. Under the BER and OP constraints, we have presented the closed-form expressions for the minimum average transmit power and individual outage probability. Our theoretical analysis shows that in fading environments wireless communications employing multiple antennas can also achieve the exponential BER performance, as operating in non-fading AWGN channels.

APPENDIX

The proof of Proposition 1: Since $\overline{P}_b(i) \leq \frac{\xi_i}{2} \frac{\overline{\lambda}_{\text{out}}(i)}{\overline{\lambda}_{\text{mea}}(i)}$, then it follows from Lemma 2 that $\lambda_0(i) = \overline{\lambda}_{\text{out}}(i)$. Furthermore, we can have

$$\hat{\rho}^{(i)}(\overline{P}_b(i), P_{\text{out}}^{(i)}) = \int_{\overline{\lambda}_{\text{out}}(i)}^{\infty} p_i f_i(\lambda_i) d\lambda_i. \tag{60}$$

Substituting (29) into (60), we obtain

$$\begin{aligned} \hat{\rho}^{(i)}(\overline{P}_b(i), P_{\text{out}}^{(i)}) &= \int_{\overline{\lambda}_{\text{out}}(i)}^{\infty} \frac{\widehat{\text{SNR}}(i)}{\lambda_i} f_i(\lambda_i) d\lambda_i \\ &+ \int_{\overline{\lambda}_{\text{out}}(i)}^{\infty} \frac{2}{\beta_i \lambda_i} \ln(\frac{\lambda_i \Delta(i)}{\overline{\lambda}_{\text{out}}(i)}) f_i(\lambda_i) d\lambda_i. \end{aligned} \tag{61}$$

With respect to (61), we define

$$\rho_s(\widehat{\text{SNR}}(i), \overline{\lambda}_{\text{out}}(i)) = \int_{\overline{\lambda}_{\text{out}}(i)}^{\infty} \frac{\widehat{\text{SNR}}(i)}{\lambda_i} f_i(\lambda_i) d\lambda_i \tag{62}$$

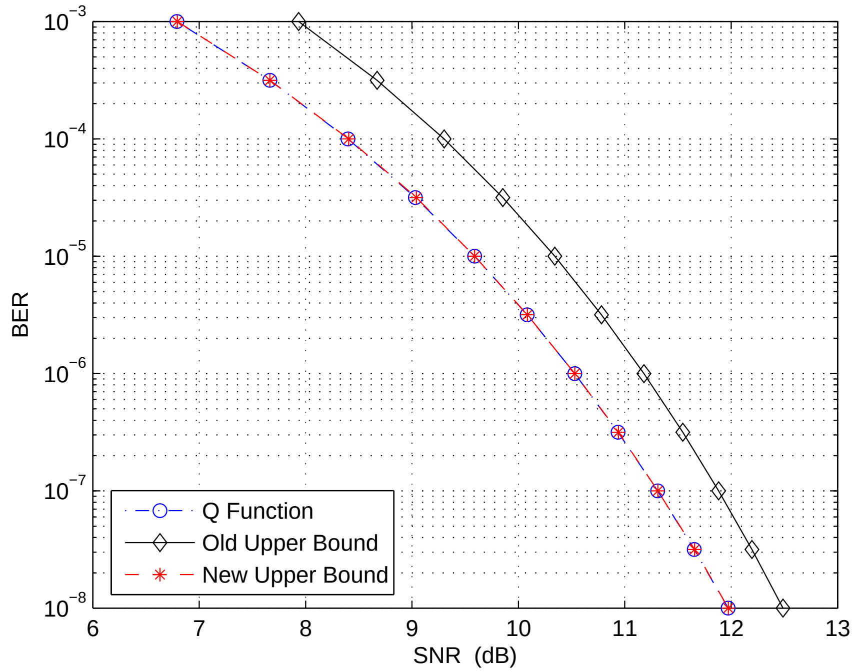

Fig. 1. BER comparison among Q function and its two upper bounds for different SNRs

and

$$\rho_{\Delta}(\overline{\lambda}_{\text{out}}(i)) = \int_{\overline{\lambda}_{\text{out}}(i)}^{\infty} \frac{2}{\beta_i \lambda_i} \ln(\frac{\lambda_i \Delta(i)}{\overline{\lambda}_{\text{out}}(i)}) f_i(\lambda_i) d\lambda_i. \tag{63}$$

In what follows, we consider to derive (62) and (63), respectively.

From Lemma 1 in [36], the marginal p.d.f. of the i-th largest eigenvalue λ_i can be written as

$$f_i(\lambda_i) = \sum_{k=i}^{m} (-1)^{k-i} \binom{k-1}{i-1} \binom{m}{k} f_{\text{min}:k}(\lambda_i) \tag{64}$$

where $f_{\text{min}:k}(x)$ denotes the p.d.f. of the smallest random variable considered in a subset of k random variables over the set of all eigenvalues, and is given by

$$f_{\text{min}:k}(x) = \frac{kC}{m!} \sum_{\alpha} \sum_{\mu} \text{sgn}(\alpha)\text{sgn}(\mu) A_k(\alpha, \mu) \times$$

$$e^{-kx} \sum\nolimits_{\tau} x^{\theta+\alpha_k+\mu_k-2+\sum_{\iota=1}^{k-1}\tau_{\iota}} \prod_{\iota=1}^{k-1} \frac{(\theta+\alpha_{\iota}+\mu_{\iota}-2)!}{\tau_{\iota}!}. \tag{65}$$

With the help of the complementary incomplete gamma function $\Gamma(q, x)$, thus we can obtain the desired result (43) after a simple derivation.

The derivation of (63) is similar, but involves a process employing the following special function

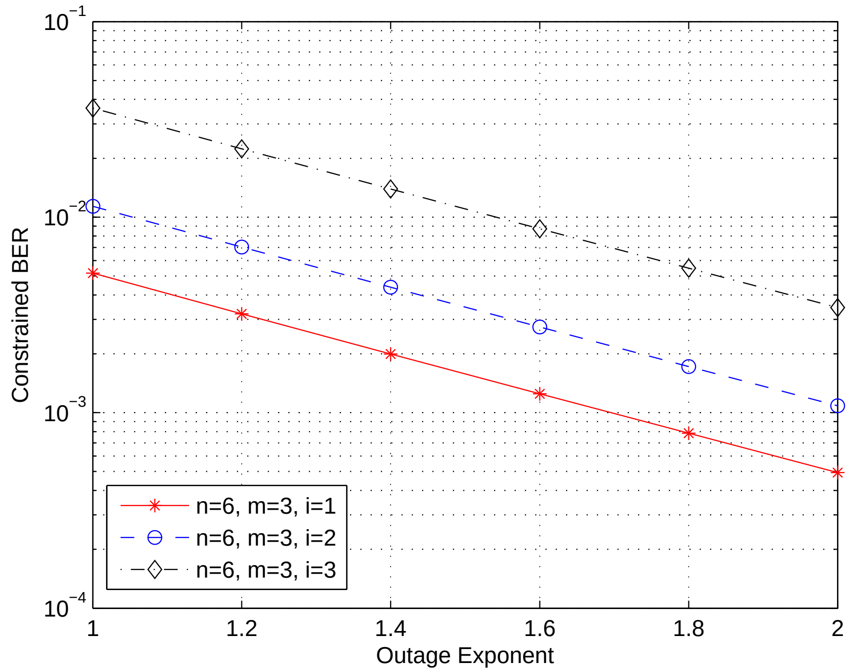

Fig. 2. Constrained BER under various SISO outage exponents

$\jmath_q(x)$ defined as [38]:

$$\begin{aligned} \jmath_q(x) &= \int_1^\infty t^{q-1} \ln t e^{-xt} dt \\ &= \frac{(q-1)!}{x^q} \sum_{k=0}^{q-1} \Gamma(k,x)/k!. \end{aligned} \tag{66}$$

Finally, again making use of (64), we can easily obtain the desired expression of $\rho_\Delta(\overline{\lambda}_{\text{out}}(i))$ (44).

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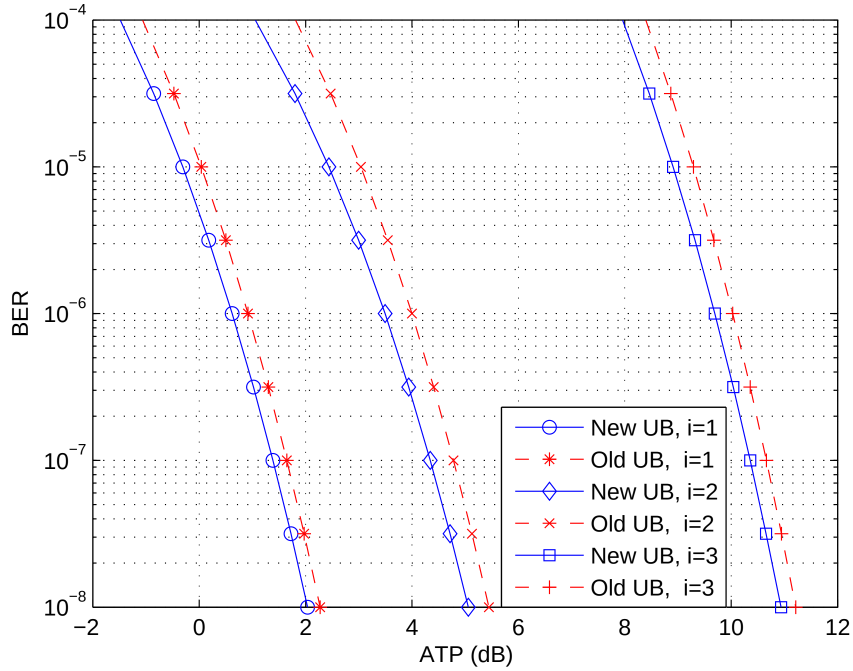

Fig. 3. ATP comparison between two power allocation schemes under different average BER constraints

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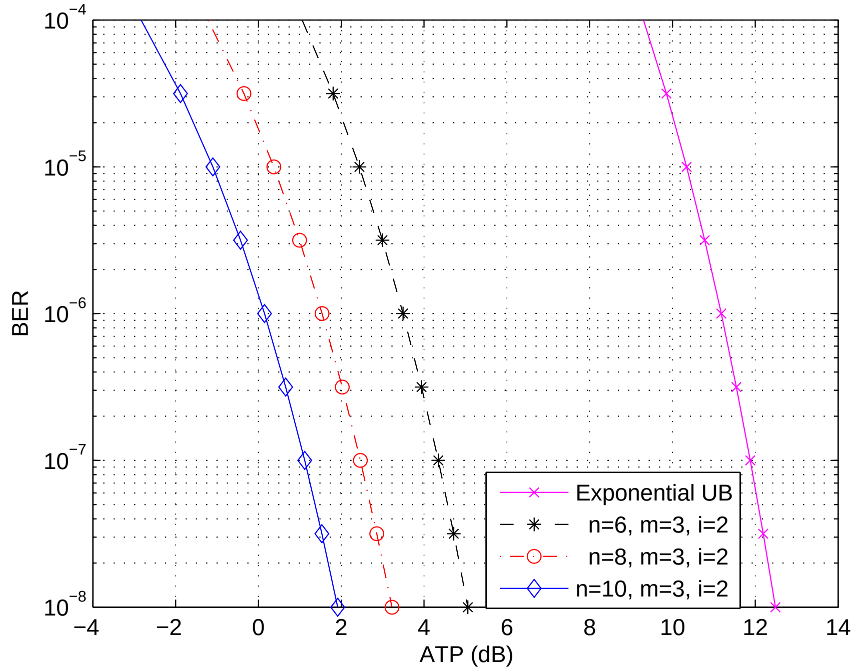

Fig. 4. ATP versus the average BER for different n

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