A Kosambi-Karhunen–Loève Learning Approach to Cooperative Spectrum Sensing in Cognitive Radio Networks

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***Abstract*—This paper focuses on the issues of cooperative spectrum sensing (CSS) in a large cognitive radio network (CRN) where cognitive radio (CR) nodes can cooperative with neighboring nodes using spatial cooperation. A novel optimal global primary user (PU) detection framework with geographical cooperation using a deflection coefficient metric measure to characterize detection performance is proposed. It is assumed that only a small fraction of CR nodes communicate with the fusion center (FC). Optimal cooperative techniques which are global for class deterministic PU signals are proposed. By establishing the relationship between the CSS technique design issues and Kosambi-Karhunen–Loève transform (KLT) the problem is solved efficiently and the impact on detection performance is evaluated using simulation.**

Keywords—Cognitive radio networks; cooperative spectrum sensing; global cooperation; learning; Kosambi-Karhunen–Loève transform.

I. INTRODUCTION

Wireless communication systems ought to collect information about the radio spectrum in order to adapt their operations and behaviour to provide an improved match to the prevailing conditions. Consequently, cognitive radio (CR) is becoming increasingly important to current and future wireless communication systems for identifying underutilised spectrums, characterising interference and consequently achieving reliable and efficient operations [1-3]. One of the main conditions for the successful operation of CR's is that their transmission should not cause harmful interference to primary users (PU's). Secondary users (SU's) or CRs can utilize the licensed spectrum as long as the PU is absent. However, when the PU comes back into operation, the SU should vacate the spectrum to avoid interference with the PU. As such, spectrum sensing is a key element in CR networks as it must initially be carried out before giving the unlicensed SU access to the vacant licensed channel [4].

In wireless channels, the hidden terminal problem which leads to very low SNR is one of the greatest challenges of implementing spectrum sensing. In a case whereby a single CR sensing is shadowed, in severe multipath fading, Doppler or shadowing effects it may not reliably detect the PU signal and access the channel when there is a primary signal present causing interference to the licensed PU hence reducing the utilisation of the radio frequency spectrum [5]. To overcome the hidden terminal problem and increase the spectrum sensing reliability, cooperative spectrum sensing (CSS) schemes have been studied in [4-8].

In traditional spectrum sensing, the goal was to design a system for detecting a PU signal of interest. The performance of such systems degrades if the PU signal changes over time or for other PU signals. Due to the predicted heterogeneous nature of future Fifth Generation (5G) wireless networks, modern systems are expected to perform spectrum sensing for different PU signal models [9]. Therefore, it is desirable to build a global system which is flexible enough to generalise several PU signal models e.g. satellite, WIFI, WiMAX, WLAN, Bluetooth, mobile, cellular etc. A type of global decision known as hard decision (HD) in CSS smoothes the progress of design and lowers the communication overhead, but at the cost of lost data and reduction in performance [5]. The soft data (SD) fusion CSS schemes in previous research works [6-8] provide a considerable enhancement in the probability of detection (P_D) at the expense of increased bandwidth required for transmitting the sensing measurements to a fusion centre (FC).

This paper considers a cognitive radio network (CRN) consisting of a number of CRs who forward SD to a central FC, the FC then processes the observed information. CRN often operate with certain constraints, such as minimal sensing time to reduce interference and increase throughput [6]. Consequently, reducing system complexity in terms of communication is vital. For example, bandwidth resources can be conserved if CRs do not transmit irrelevant or redundant information. Such transmission can be avoided through reducing the number of random variables under consideration. The problem of variable reduction at the local CR sensors was considered in the context of cooperative estimation in [10-11] and cooperative detection in [12]. In CRN networks architectures CRs can cooperate with neighboring CR nodes and form a network wide low-variable projection of the observed PU signal [13]. The resulting low-variable projection measurement is transmitted by small fractions of CRs to the FC. For large CRN, it is not always feasible to modify the CSS technique for each individual CR for different PU signals. Therefore, CRs can be designed to acquire data applicable to a hypothesis test without being aware of the PU signal. In such scenarios, a practical approach is to design a global CSS technique which is effective for a wide range of PU signals. The main contribution of this paper can be summarized as follows:

- A global PU detection framework with geographical CSS using a deflection coefficient metric is proposed.
- The relationship between CSS deflection coefficient and Kosambi-Karhunen–Loève transform (KLT) is evaluated.
- A novel optimal CSS matrix $\mathbf{W}$ is proposed such that the P_D performance of a CRN is increased.

The rest of this paper is organized as follows: After the presentation of the system model of CSS techniques for the PU signal detection in Section II, the global cooperative techniques are presented Section III. In Section IV, the proposed global CSS techniques for PU signal detection are presented. In section V, Monte-Carlo simulation results and analyses concerning the performance the CRN system are presented. Conclusions are discussed in section VI.

II. COOPERATIVE TECHNIQUES FOR PU SIGNAL DETECTION

In this section, a system model for hypothesis testing and CSS techniques are presented. A global PU detection framework with geographical CSS using a deflection coefficient metric is also proposed.

A. Hypothesis Testing

The considered scenario for a CRN is that individual CRs perform local spectrum sensing in a distributed manner for detecting a high-dimensional PU $\mathbf{s}$ signals. The detection problem for local sensing is in effect a binary hypotheses testing predicament that can be represented as follows:

$$\begin{cases} H_0: & \mathbf{y}=\mathbf{n} \\ H_1: & \mathbf{y}=\mathbf{s}+\mathbf{n} \end{cases} \tag{1}$$

where $\mathbf{y} \in \mathbb{R}^N$ is the observed signal, $\mathbf{n} \sim \mathcal{N}(0,\sigma^2\mathbf{I}_N)$ denotes the additive white Gaussian noise (AWGN) with covariance $\sigma^2\mathbf{I}_N$, and $\mathbf{s} \in \mathrm{R}^N$ is the PU signal of significance. H_0 or H_1 are the hypothesis of indicating a vacant channel and occupied channel of the PU's signal significance.

B. Spectrum Sensing

Considering a CRN with *N* CRs sensing nodes, *N* CRs each sensing a scalar variable combine to sense an *N*-dimensional signal, it is assumed that each CR node can forward its observation of the PU signal of significance in noise to the FC through an ideal communication link. The FC then processes the observed information and decides in favour of either H_0 or H_1. Conversely, in large CRNs, due to a variety of reasons including power, multipath fading and network architecture, it may not be possible for all CRs to communicate with the FC. This is addressed by CSS [8].

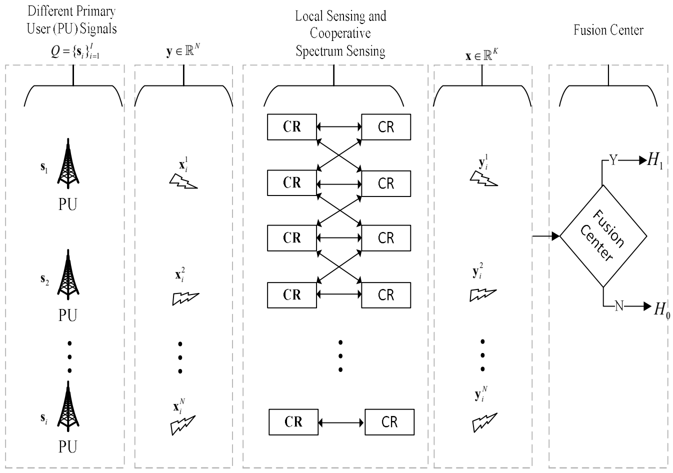

Figure 1. System model for proposed CSS detection.

C. Cooperative Spectrum Sensing

It is assumed that only a fraction *K* of the *N* CRs are allowed to transmit to the FC, where $K < N$, and can cooperate with each other, which refers to the process of cooperation of their observation with those from their neighboring CR nodes. It is assumed that without loss of generality, only the initial CR nodes can communicate with the FC. Where $\mathrm{W} \in \mathbb{R}^{K\times N}$ is the CSS matrix whose elements correspond to the weights to linearly combine the CR observations. $\mathbf{W}$ projects the high-variable signal $\mathbf{y} \in \mathbb{R}^N$ onto $\mathbf{x} \in \mathbb{R}^K$ as $\mathbf{x}=\mathbf{W}\mathbf{x}$, as shown in the system model in Figure 1. At the FC a global hypotheses test is carried out and a decision about the PU signal based on *K* low-variable measurements of $\mathbf{x}$ is carried out.

The aim is to design an optimal cooperative matrix $\mathbf{W}$ such that the P_D performance is increased. Initially, the problem for the case of a single CR is evaluated using deflection coefficient as the performance measure [14]. The increase in the deletion coefficient at the FC is equivalent to the decrease of the probability of error (P_E) for a Gaussian representation. The problem of detecting a known signal $\mathbf{s}$ is given by:

$$\max_{W} \frac{\mathbf{s}^T\mathbf{W}^T}{\mathbf{W}\mathbf{W}^T}\mathbf{W}\mathbf{s} \tag{2}$$

The system model is defined to obtain a global CSS technique for a wide class Q of signals $\{\mathbf{s}_i\}_{i=1}^I$. It is assumed that the signal $\mathbf{s}$ belongs to a class $Q=\{\mathbf{s}_i\}_{i=1}^I$ of PU signal where $\mathbf{s}_i$ are probable and the FC has knowledge of elements of set.

III. GLOBAL COOPERATIVE TECHNIQUES

In this section, a performance measure and global performance of the CSS is presented.

A. Performance measure

Deflection coefficient of a detector is a measure to characterize the detection performance [14]. The larger the deflection coefficient indicates the easier differentiation between two hypotheses, and thus the better the detection performance. It is assumed that the signals under H_I come from a set of signals $\{\mathbf{s}_i\}_{i=1}^{I}$. Let the deflection coefficient D_{ss} for a signal class $Q=\{\mathbf{s}_i\}_{i=1}^{I}$ be defined by:

$$D_{ss}=\frac{\mathbf{s}^T\mathbf{W}^T}{\mathbf{W}\mathbf{W}^T}\mathbf{W}\,\mathrm{s} \tag{3}$$

Hence, the CSS deflection coefficient D_{css} is a summation of each individual CRs coefficient for $\mathbf{s}_i$

$$D_{css}=\sum_{i=1}^{I}D_{ss} \tag{4}$$

B. Global performance

The D_{css} is maximized which takes into account the performance of the systems for all I signals. A global CSS technique will incur a certain level of loss compared to a signal detection performance. Hence, a measure for the global performance that quantifies the performance decrease of the system as I increase is calculated. The global performance $(G_q(I))$ is the performance loss when a single spectrum sensing technique is used for a set of I signals defined by:

$$G_q(I)=\frac{D_{cc}}{\sum_{i-=1}^{I}\mathbf{s}_i^T\mathbf{s}_i} \tag{5}$$

where $\sum_{i-=1}^{I}\mathbf{s}_i^T\mathbf{s}_i$ represents the summation of deflection coefficients when the CSS technique is optimized for each PU signal and D_{cc} is the deflection coefficients when a global spectrum sensing technique is considered. Using Cauchy-Schwartz inequality [15],

$$\|\mathbf{P}_{\mathrm{w}}\mathbf{s}_i\|_2^2\le\|\mathbf{P}_{\mathrm{w}}\mathbf{s}_i\|_2\,\|\mathbf{s}_i\|_2 \tag{6}$$

where the projection matrix $\mathbf{P}_{\mathrm{w}}=\dfrac{\mathbf{W}^T}{\mathbf{W}\mathbf{W}^T}\mathbf{W}$.

Hence, $D_{cc}\le\sum_{i=1}^{I}\|\mathbf{s}_i\|_2^2$. The ith CR shares it data as indicated by the CSS matrix **W.** The cooperative performance (G_{css}) in the CRN system can be defined as:

$$(G_q(I))\sum_{i=1}^{K}|\gamma_i| \tag{7}$$

where γ_i is specified by the row of the cooperation matrix **W.** As the number of CR capable of transmitting information to the FC (K) increases, the P_D will increase.

IV. GLOBAL COOPERATIVE SPECTRUM SENSING TECHNIQUES FOR PU SIGNAL DETECTION

In this section, the relationship between CSS deflection coefficient and KLT is evaluated and a novel optimal CSS matrix such that P_D performance is of a CRN is increased is proposed. A traditional technique to design the cooperative matrix **W** is to use a random construction of elements of W which are generated from a certain probability density function (PDF) [16]. A matrix $\mathbf{E}\in\mathbb{R}^{K\times N}$ satisfies $\mathbf{Q}\in\mathbb{R}^{N}$ if,

$$(1-\zeta)\|\mathbf{s}_i\|_2^2\le\|\mathbf{E}\mathbf{s}_i\|_2^2\le(1+\zeta)\|\mathbf{s}_i\|_2^2 \tag{8}$$

where $\xi\in(0,1)$ and $\mathbf{s}_i\in\mathbf{Q}$. For a random CSS technique where $\sqrt{\dfrac{K}{N}}\mathbf{P}_{\mathrm{w}}$ satisfies the deflection coefficient, D_{css} in equation (4). D_{css} can be approximated as:

$$\approx\frac{K}{N}\sum_{i=1}^{I}\|\mathbf{s}_i\|_2^2.\ D_{css}=\sum_{i=1}^{I}\frac{\mathbf{s}^T\mathbf{W}^T}{\mathbf{W}\mathbf{W}^T}\mathbf{W}\,\mathrm{s}_i\approx\frac{K}{N}\sum_{i=1}^{I}\|\mathbf{s}_i\|_2^2. \tag{9}$$

The aim of increasing the deflection coefficient can be formulated by:

$$\max_{W}\sum_{i=1}^{I}\frac{\mathbf{s}^T\mathbf{W}^T}{\mathbf{W}\mathbf{W}^T}\mathbf{W}\,\mathrm{s}_i \tag{10}$$

The optimisation problem formulated in equation (10) is equivalent to Kosambi-Karhunen–Loève transform (KLT) as shown by [17]:

$$\max_{\mathbf{W}}\sum_{i=1}^{I}\frac{\mathbf{s}^T\mathbf{W}^T}{\mathbf{W}\mathbf{W}^T}\mathbf{W}\,\mathrm{s}_i=\max_{\mathbf{W}^T\in\mathbf{s}_K^N}Tr(\mathbf{W}\phi\mathbf{W}^T) \tag{11}$$

where $\phi=\sum_{i=1}^{I}\mathbf{s}_i\mathbf{s}_i^T$ and $\mathbf{s}_K^N=\{\mathbf{W}^T\in\mathbb{R}^{N\times K}\,|\,\mathbf{W}\mathbf{W}^T=I_K\}$.

To show that optimality is not lost, the search geographical space is constrained so that $\mathbf{W}^T\in\mathbf{S}_K^N$. Using properties of projection matrices $(\mathbf{P}_{\mathrm{w}})^2=\mathbf{P}_{\mathrm{w}}$ and $\mathbf{P}_{\mathrm{w}}=\mathbf{P}_{\mathrm{w}}{}^T$ the aim function can be rewritten as

$$\max_{W}\sum_{i=1}^{I}\|\mathbf{P}_{\mathrm{W}}\mathbf{s}_i\|_2^2. \tag{12}$$

$\mathbf{W}^T$ can be represented by $\mathbf{W}_{ort}^T\Re^T$ using Gram-Schmidt orthogonalization [18], where $\mathbf{W}_{ort}^T\Re^T=\mathbf{I}_K$ and $\Re^T$ is an upper triangular matrix. Hence,

$$\mathbf{P}_{\mathrm{w}}=\frac{\mathbf{W}_{ort}^T\Re^T}{(\Re^T\mathbf{W}_{ort}\mathbf{W}_{ort}^T\Re^T)}\Re\mathbf{W}_{ort} \tag{13}$$

Therefore, the optimization problem can be represented by:

$$\max_{W}\sum_{i=1}^{I}\|\mathbf{P}_{\mathrm{W}}\mathbf{s}_i\|_2^2=\max_{\mathbf{W}^T\in\mathbf{s}_K^N}\sum_{i=1}^{I}\mathbf{s}_i^T\mathbf{W}^T\mathbf{W}\mathbf{s}_i \tag{14}$$

$$=\max_{\mathbf{W}^T\in\mathbf{s}_K^N}Tr(\mathbf{W}\phi\mathbf{W}^T) \tag{15}$$

which is equivalent to KLT.

V. SIMULATION RESULTS

In this section, numerical simulations are used to demonstrate the performance of the proposed scheme. Various PU signals with varying characteristics are modelled with in the CRN. Monte-Carlo simulations are used to analyze the proposed CSS techniques. The total number of zero entries in the optimal CSS matrix is obtained. Each element of the I signal $Q=\{\mathbf{s}_i\}_{i=1}^{I}$ are obtained from a standard normal distribution and each is assumed to be a known signal in the set.

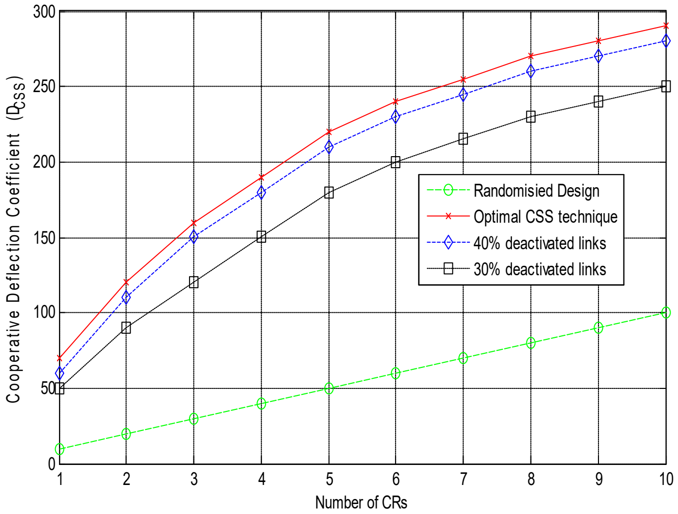

Figure 2. Cooperative deflection coefficient vs Number of CRs capable of transmitting to the FC

Firstly, the performance of the CSS in CRN technique is evaluated. The graph in figure 2 represents the CSS deflection coefficient D_{css} vs number of CRs K capable of transmitting to the FC for $I = 10$ and $N = 40$. The performances are evaluated when there is a constraint of 30% and 40%, which in other words represents when the links are deactivated by $\frac{1}{K \times N}\sum_{i=1}^{K}\|\mathbf{w}_i\|_0 = 0.4$ and $\frac{1}{K \times N}\sum_{i=1}^{K}\|\mathbf{w}_i\|_0 = 0.3$.
The randomized cooperative matrix design without constraints i.e. 100% of the link activated is also presented. It was observed that the proposed CSS technique performs significantly better that a traditional random design even with 40 and 30 % link deactivation. Secondly, taking into account the variable reduction it has been observed from figure 2 that as the number of CRs decrease, the CSS deflection coefficient D_{css} also degrades. The larger the deflection coefficient indicates the easier differentiation between two hypotheses, and thus the better the detection performance. The 40% link deactivation is very close to the optimal D_{css}, where D_{css} is the deflection coefficient achieved with 100% of the link activated. The reduction in link deactivation as shown when comparing the case of 30% link deactivation and the 40% link activation the increase in the D_{css}.

In figure 3, a graph representing the global performance G_q vs number of different PU signals I has been presented for K CRs capable of communicating with the FC, with 40% and 30% of links deactivate, the number of PU signal I for $N = 40$ and $K = 10$.

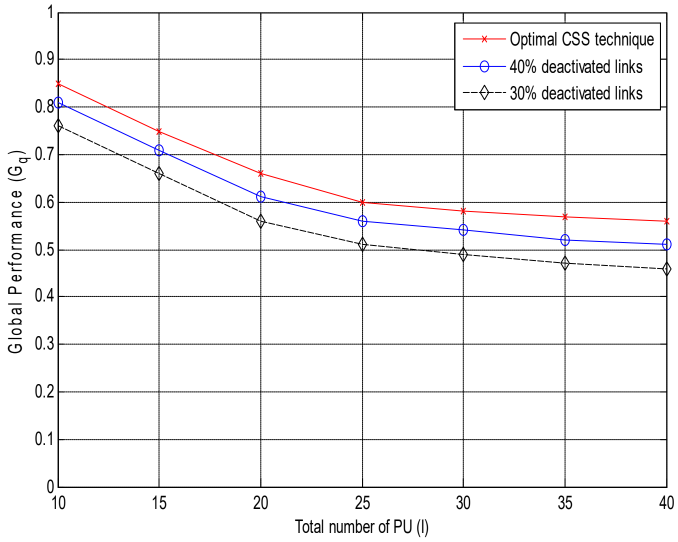

Figure 3. Global performance vs Number of different PU signals.

It can be observed from figure 2 that the proposed CSS technique performs significantly better than both 30% and 40% link deactivation even as the total number of different PU signal increase. It can also be observed that as the total number of PU increase G_q decreases which is expected.

VI. Conclusion

The radio frequency spectrum is a very valuable resource in wireless communication systems. CR, which is one of the methods employed in the utilization of the available spectrum more efficiently through opportunistic spectrum usage, has become an exciting and promising concept in the realization of heterogeneous 5G networks. One of the important elements of the CR is sensing the available spectrum opportunities under constraints. In this paper, the issue of designing a global cooperative spectrum sensing technique for high-variable signal detection has been considered. By establishing the equivalence between cooperation matrix design and KLT formulations, techniques have been adapted from the sparse learning literature to efficiently solve and optimize the problem. A novel optimal CSS for CRN matrix such that P_D performance is of a CRN is increased has been proposed. New performance measurement has been defined and though simulation it was observed that the proposed collaboration techniques provided significant gains in detection performance in comparison to traditional random designs.

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