A Monetary System of Equations with Inflationary Expectations for the U.S.A.

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Abstract: The paper is an empirical investigation that places Livingston's expectations of the Consumer Price Index (CPI) with the rate of inflation centre stage in a monetary system of equations with real money balances, output, employment, Federal Government debt and interest rates. The modelling approach is a Vector Auto-Regressions (VARs) scheme employing quarterly, observational data sets from U.S.A, spanning the period of 1959 to 2007. One of the important tasks is to find stationary processes for the CPI and the price expectations, which entails explaining the second-differences within the error-corrections, and using first-differences in the formation of co-integrating vectors, because the agents view them as levels in the long-run.

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Introduction[1]

Individual agents in an economy interact with each other in conditions of uncertainty, leading to the desirability of forming explicit expectations about inflation based on the Consumer Price Index (CPI). In this empirical investigation, expectations are derived from the Livingston Survey, started in 1947 (see Appendix A). The analysis entails transforming the observations into meaningful, quarterly estimates, although the raw data are from half-yearly surveys.

In a number of studies, such as Morana and Bagliano (2007),[2] there has been a neglect of expectations, as well as modelling the chosen price index (or its logarithm) in the light of the first-difference, which is not necessarily a stationary process. The empirical evidence presented in this research study, however, suggests that agents view expectations and the rate of inflation - the first-difference of the logarithm of the price index - as level variables in the long-run equilibrium process. The stationary tests imply that the second-differences are the correct variables that require modelling in the overall system of error-corrections. These encapsulate the short-run dynamics of the variables of interest in the system, which are the deviations from equilibrium (Enders, 2004).

An important part of the empirical analysis borrows concepts from the Swedish economist, Knut Wicksell (1936). His theoretical work emphasised the balance between the "market" and the "natural" rates of interest for equilibrium in an economy. This balance involves both inflation and expectations, because the "natural rate" is essentially a real variable, whereas the "market rate" is nominal. This study makes the assumption that Wicksell's "market" and "natural" rates

[1] The writers would like to thank Dr. Tim Parke and Dr. Chris Tofallis for their helpful comments on the drafting of the paper.

[2] Another recent study by De Grauwe and Polan (2005) adopts a similar approach, but neglects many important variables.

of interest involve short-term as well as long-term rates interacting with inflation and expectations.

The system involves the demand and supply of money and Federal Government debt deflated by the CPI as well as real income, output and employment, which hopefully forms important information from the product side of the economy. The employment variable is included on account of the relationship to the rate of unemployment, and therefore reflects the level of uncertainty in the formation of expectations in the economy.

The plan of the discussion, then, is to first outline the theoretical model of expectations. Second, to present the general econometric framework of the VARs system. Third, to define the variables embodied in the vectors of interest and discuss the data sets involved in the empirical analysis. Fourth, to present the statistical results relating to the long-run co-integrating mechanisms: the indicated number of vectors, the results of their estimation and possible solutions calculated from them. Fifth, various general error correction equations are subjected to 'Hendryfication' in order to derive the specific models. Those insignificant quantities that have no statistical contribution to make in the overall empirical picture are removed, leaving the significant elements of the short-run dynamics and the equilibrating co-integrating vectors. The empirical analysis is undertaken using *Microfit* Econometric Software.

The Theoretical Model

The theoretical starting point is with Keynes (1936), who depicts the significance of expectations and their unstable nature within the economy. His theoretical analyses of the level of employment, the demand for money, the level of capital expenditure and the trade cycle all depend crucially on expectations. For instance, in analysing the determination of the level of employment, Keynes wrote:

Thus the behaviour of each individual firm in deciding its daily output will be determined by its short-run expectations – expectations as to the cost of output on the various possible scales and expectations as to the sale-proceeds of this output…..It is upon these various expectations that the amount of employment which the firms offer will depend. The actually realised results of the production and scale of output will only be relevant to employment in so far as they cause a modification of subsequent expectations (Keynes, 1936, p.47).

The difficulty is that he treats expectations as exogenous, which means they are outside of the model and then cannot be determined by the mechanism of the theory. This is, however, an erroneous assumption, because expectations in the first instance are perceived as a qualitative entity, evolving from the inter-play of short-run dynamics, and arising from the imperfect information, knowledge and complexity that flows from disequilibrium of variables within the macro economy. Thus the formation of expectations is qualitative in nature, but this fact does not mean they cannot be endogenized within the edifice of the model, or transformed into quantitative constructs for analysis (Thomas, 1995).

The formation and measurement of expectations could well be based on bounded rationality and inter-dependence of economic actors. The majority of agents may in fact seek to minimise costs of expectations formulation by relying on forecasts formed by others. Such is the case of the 'naïve' agents with little or no resources available for gathering and processing data (Bywaters and Thomas, 2008). Those agents with the skills and resources to form expectations from the available, imperfect information are likely to be the 'alphas' of the 'pack'. They are a small number of professional experts who possess the economic acumen (or knowledge) to perceive expectations and understand their transformation into a quantitative format. They form expectations of inflation (π^{A}) in period t for $t+1$, that is $\pi^{A}_{t,t+1}$, based on the available, incomplete information set, I, at period t, namely

$$\pi^{A}_{t,t+1} = (\pi_{t-1} / I_{t}), \qquad [1]$$

where π_{t-1} is the actual rate of inflation from the previous period.

This 'alpha' group is small relative to the majority of agents, which means that the dissemination of expectations will initially be slow, followed by a sudden acceleration as the majority of 'followers' of the 'pack' convert to the change in predictions in an adaptive manner. This asserts that the 'followers' will formulate expectations from the 'alphas' by the extent that their prior predictions have now changed, that is

$$\pi^{F}_{t,t+1} = \pi^{A}_{t-1,t} + \lambda(\pi^{A}_{t,t+1} - \pi^{A}_{t-1,t}), \qquad (0 \le \lambda \le 1), \qquad [2]$$

where λ equals the expectations coefficient of the 'followers', which translates the fraction of the extent that the current expectations have now changed, adding to the previous period's rate to form the expected value in period t for $t+1$. This is followed by a slow-down as the 'stragglers' reluctantly fall into line, adapting to the majority's expectations of the future inflation, specifically in the form of

$$\pi^{N}_{t,t+1} = \alpha\, \pi^{F}_{t,t+1} \qquad [3]$$

This sequence of dispersion of the initial expectations formed by the 'alphas' and the degree of interdependence implies a non-linear process of diffusion[3], which can be captured by the logistic function denoted as follows[4]:

$$\pi^{A}_{t,t+1} = (1+\exp^{-(\beta_0+\beta_1\pi)})^{-1}. \qquad [4]$$

The expectations of the 'alphas' should, at some stage, reflect into the actual data, π. In fact, the expectations will be related to the true rate by a proportional coefficient, β_1 and a constant, β_0.

The key component, therefore, in generating this process of diffusion is the formation of the expectations by the 'alphas' of the 'pack'. These are the agents endowed with the capacity to perceive the direction of inflation thanks to their economic knowledge, understanding of how the economy functions, and the expertise to convert this awareness into a quantitative format. The predicament, however, is that even though these agents are experts, they will be dominated by their own beliefs in which economic theory is the correct one, leading to various subsets of macro variables that determine the formation of expectations. This means that the information set of these 'alpha' agents as a whole will be wide-ranging in terms of the focus variables that lead them to form predictions of the change in the CPI.

Nevertheless, it is not implausible to assume the 'Hendryfication' process within a system of equations as part of the empirical analysis of pinpointing the various macro entities contributing most weight in the formation of the overall average expectation that is formed by the diverse 'alphas' when put into the aggregate form. As a result, it is necessary to adopt a number of macro variables that come from a number of economic theories to expose the information set of the 'alphas' within a system of equations[5].

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[3] This factor is absent from the traditional rational expectations explanation.
[4] For further details on how to form a log-linear approach see Bywaters and Thomas (2008).
[5] The 'Hendryfication' process is the general-to-specific approach to econometric modelling, which is outlined by Hendry himself in his articles of 1983 and 1987.

Econometric Framework

It follows from above that the list of macro variables given below by various diverse theories is likely to form part of the information set that determines expectations, which can be represented in the general format as

$$\Delta Z_t = V + B_1 \Delta Z_{t-1} + \alpha\beta' Z_{t-1} + B_2 \Delta Z_{t-2} + ... + B_K \Delta Z_{t-k} + \phi d_t + \varepsilon_t , \quad [1]$$

where d_t represents a vector of non-stochastic variables such as structural break dummies, but not the intercept, which can be included separately as V if required in the general empirical analysis, or restricted to lie within Z_{t-1} of [1]. ε_t is a column vector of random values, which may be contemporaneously correlated with one another but are assumed to be non-autocorrelated over time. Z_t is also a column vector of observations at time t with K denoting the maximum lag period on the explanatory variables within the model, namely

$$Z_t = (RM_t , RB_t , RGD_t , E_t , r_{2\,t} , r_t , r_{30\,t} , r_{m\,t} , \Delta P_t , \Delta P^E_{t+1} , Y_t , C\,)', \quad [2]$$

where RM_t = Real money balances, derived by taking the log of the money as measured by M2 minus the log of the Consumer Price Index (CPI),

RB_t = Real monetary base, estimated by adopting the log of the St. Louis monetary base deflated by the logarithm of the CPI $(\ln P_t)$,

RGD_t = Real debt is based on the log format of the total public debt owed by the Federal government minus the $(\ln P_t)$,

E_t = The rate of employment, derived from the following log format: $\ln(100 - \text{U})$, where U is the rate of unemployment,

$r_{2\,t}$ = The own rate of M2,

r_t = The three-month, Treasury bill rate, representing the Federal, short-run opportunity cost of holding money,

$r_{30\,t}$ = The thirty-year, Treasury bill rate, denoting the Federal, long-run opportunity cost of possessing money[6],

$r_{m\,t}$ = Moody's corporate bond yield, indicating the commercial, long-run opportunity cost of retaining money,

ΔP_t = The rate of inflation derived by taking the logarithmic expression of $\ln p_t - \ln p_{t-1} \approx \frac{p_t - p_{t-1}}{p_{t-1}} \left(\text{or } \frac{p_t}{p_{t-1}} - 1 \right)$.

$\pi^A_{t,t+1}$ = The Livingston expectations of the rate of inflation over the next six months, based on the CPI[7], divided by two because of the use of converted, quarterly changes,

Y_t = Log of real income, based on real gross domestic product (GDP)[8],

ΔP^R_t = The logarithmic growth rate of the real price of oil, based the West Texas intermediate spot price and the CPI,

C = Intercept terms, in this case, restricted to lie within the co-integrating vectors.

All raw observations were obtained from *Fred Databank* at the Federal Bank of St. Louis, except for the Livingston expectations, which were obtained from the Philadelphia Reserve Bank[9]. All variables were seasonally adjusted except the rates of interest. They are in the form of $In(1 + r/100) \approx r$. The stationarity of the data sets was checked: all the various series achieved this condition on first-difference, except the CPI, expectations and the real price of oil, which required the second-difference to reflect the dominance of permanent shocks. Thus, the rate of inflation and the growth rate in the real price of oil along with the expectations were treated as level variables in the determination of the co-integrating vectors.

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[6] The origin of this data set employed differences in the three-month, the ten- and twenty-year rates (including the estimates) to derive the thirty-year over the period, 1959 Q1 to 1976 Q4 in the first instance, and then subsequently between 2002 Q1 and 2005 Q4 to complete the series.

[7] For more details, see Appendix [A] for the derivation of the observations used.

[8] For a summary of the "recent modifications in the U.S.A. of official measures of GDP" see Maddison (2003).

[9] For the Federal Reserve Bank of St. Louis see http://www.research.stlouisfed.org/fred2/. In the case of the Philadelphia one, see http://www.philadelphiafed.org/research-and-data/.

The next stage in the empirical analysis was to determine the number of co-integrating vectors existing between the non-stationary variables of interest in $\alpha \beta' Z_{t-1}$, representing the long-run equilibria. The number of different co-integrating vectors can be found by examining the significance of the characteristic roots, which is the rank of a matrix (Johansen, 1988; Stock and Watson, 1988). The tests for the total number of roots that are significantly different from one use the maximum and trace statistics. The results of the maximum and trace statistics give conflicting measures, although the former rather than the latter have the sharper alternative hypothesis, and are therefore, the preferred statistic when trying to discover the number of co-integrating vectors (Enders, 2004). These are reported in Table [1] below:

Table [1]: Co-integration with restricted intercepts and no trends in the VAR
Co-integration LR Test Based on Maximal Eigen-value of the Stochastic Matrix
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187 observations from 1961Q2 to 2007Q4. Order of VAR=8

List of variables included in the co-integrating vector:

$RM_t, RB_t, RGD_t, E_t, r_{2\,t}, r_t, r_{30\,t}, r_{m\,t}, \Delta P_t, \pi^A_{t,t+1}, Y_t, \Delta P^R_t, C$

List of eigen-values in descending order:

.50953 .47798 .4258 .34327 .25125 .20486 .18013 .15233 .11745 .10348
.077487 .035425

Null	Alternative	Statistic	95% Critical Value	90% Critical Value
$r = 0$	$r = 1$	133.2190	75.5100	72.0800
$r \langle= 1$	$r = 2$	121.5593	69.7600	66.4700
$r \langle= 2$	$r = 3$	103.3468	64.1100	60.6600
$r \langle= 3$	$r = 4$	78.6314	57.9700	54.9100
$r \langle= 4$	$r = 5$	54.1084	52.0600	49.0400
$r \langle= 5$	$r = 6$	42.8668	46.4700	43.4400

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The statistics in Table [1] highlight the possibility of five (or fewer) co-integrating vectors, for a lag length of eight quarters. In fact, ‘all’ five vectors were found by building up from one vector to five using the log likelihood ratio test of restrictions. Some restrictions took the form of unitary values with economic theory as the guide and because they were not significantly different from one empirically. In fact, the imposition of one and zero restrictions allowed the identification of the various vectors. Only with the construction of the fifth vector are all twentyfive restrictions imposed initially, although by the end, the procedure constrains thirty-two of the variables of interest, exhibited in Table [2] below:

Table [2]: Maximum likelihood estimates subject to over identifying restrictions
Estimates of Restricted co-integrating Relations (SE's in Brackets)
Co-integration with restricted intercepts and no trends in the VAR
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187 observations from 1961Q2 to 2007Q4. Order of VAR = 8. List of variables included in the co-integrating vectors:
$RM_t,\ RB_t,\ RGD_t,\ E_t,\ r_{2\ t},\ r_t,\ r_{30\ t},\ r_{m\ t},\ \Delta P_t,\ \pi^A_{t,t+1},\ Y_t,\ \Delta P^R_t,\ C$

	Vector e1	Vector e2	Vector e3	Vector e4	Vector e5
RM_t	1.0000 (None)	.0000 (None)	1.0000 (None)	.0000 (None)	−.0000 (None)
RB_t	.0000 (None)	.0000 (None)	−1.0000 (None)	−1.0000 (None)	−1.0000 (None)
RGD_t	.41043 (.016881)	.0000 (None)	.23273 (.024833)	.0000 (None)	1.0000 (None)
E_t	.0000 (None)	.0000 (None)	−.0000 (None)	20.5661 (7.8179)	−.0000 (None)
$r_{2\ t}$	−7.4307 (.84685)	.10935 (.0311620)	−12.4640 (1.2293)	−.0000 (None)	−23.1728 (4.5398)
r_t	−.0000 (None)	−.083985 (.032541)	.0000 (None)	−30.6151 (10.2094)	−17.5372 (3.7746)
$r_{30\ t}$	23.0559 (2.7466)	.32668 (.11113)	31.7882 (3.9610)	104.1008 (32.5294)	−45.9421 (19.2882)
$r_{m\ t}$	−11.2318 (2.0751)	−.22382 (.086950)	−18.4470 (3.0463)	−60.0431 (17.3971)	28.8418 (14.4994)
ΔP_t	.0000 (None)	1.0000 (None)	.0000 (None)	.0000 (None)	−.0000 (None)
$\pi^A_{t,t+1}$	−4.1617 (1.5310)	−1.0000 (None)	−14.0584 (2.1846)	−.0000 (None)	57.9662 (8.9436)
Y_t	−1.0000 (None)	.0034582 (.000740)	−.0000 (None)	1.0000 (None)	−.57985 (.12293)
ΔP^R_t	.0000 (None)	−.031949 (.0062488)	.50229 (.097002)	5.2637 (2.2468)	−.0000 (None)
C	1.2109 (.19768)	−.029262 (.0073501)	−4.5967 (.28942)	−7.4355 (.23956)	−4.5695 (1.1585)

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This is a relatively 'rich' system, where the vectors are a mix of different orders, as the logarithms of prices and expectations are differenced, and represent an example of multi-co-integrating relationships (Lee and Granger, 1990). All the explanatory variables in Table [2] above are statistically significant at the five per cent probability level. Using a vector to explain the normalized variables requires a corresponding change of sign of the other components.

Vector One represents the demand for real money balances, determined by positive and negative coefficients on various rates of interest, with a negative effect coming from the real Federal debt along with the positive signs on the output/income and the expectations variables. The positive coefficient on expectations means that agents are preserving the purchasing-power of their money holdings at the expense of other financial assets within their portfolio choices. The textbook answer would suggest that if the ability to spend money balances is being eroded away in the future, then households would reduce holdings by purchasing more goods and services. In the case of the debt, there is an inverse relationship because borrowing by the Authorities, which represents a rise in the Federal holdings of money to finance Government expenditure, and therefore, reduces real money balances held by the Public on account of the purchase.

Vector Two is the long-run information set of expectations and the rate of inflation of equation [1], which seems to be in the world of Wicksell (1936)[10], where the economy's expectations and the rate of inflation are stabilized by the 'balance' of rates of interest, output and the real oil price. The three-month and the corporate rates of interest indicate positive relationships between the rate of inflation and expectations. When inflation is rising with expectations, there is tightening of monetary policy, which results in the short-run rate of interest going up as part of the "damping-process". This process with the term-structure forces the thirty-year rate of interest upwards, which proxies and reflects the "natural rate" with a negative relationship in the 'Wicksellian' equilibrium on account of the imposition of indirect effects on the rate of investment expenditure, and therefore, the economy's output[11]. In other words, as the long-run (or "natural") rate of interest rises, reducing capital expenditure, and consequently, aggregate demand, which in turn pulls down the rate of growth in prices with expectations. Clearly, a definite link can be seen between the "natural" and "market" rates of interest, the rate of inflation and expectations. The 'own' rate of interest seems to be playing a similar part in the process of development, through this links with the real demand for money.

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[10] Also see Woodford (2003) for a discussion of the links to monetary policy.
[11] In Keynes's analysis (1936), it is the marginal efficiency of capital determining the rate of capital accumulation and the change in the level of aggregate demand and supply.

On the question of inflation and expectations, twin monetary forces are brought into play to explain the indirect effects on the level of economic activity. In the language of Wicksell, this is a "cumulative process" of falling activity[12]. This process also affects the aggregate supply side of the economy, where the cumulative process in terms of less output forces the rate of inflation and expectations down, as a result of the increase in the "natural rate" of interest reducing growth in the long run.

A further noticeable feature in Vector Two is the approximate one-to-one rate of inflation with the expectations variable, the so-called "unitary elasticity of expectations" in Hick's terminology (Blaug, 1997). This means that an alteration in existing prices is predicted to change future ones in the equivalent direction, and to tend towards the identical proportion from any disturbance from equilibrium, such that the 'cumulative process generates its own breeze'. Thus, individuals expect the change in prices to rise as fast as present ones (Hicks, 1968). This is Popper's prophecy of self-fulfilling expectations, dubbed by him the "Oedipus effect" (Popper, 1970).

The argument above suggests a weak form of rational expectations may exist in the long run based on bounded rationality. This might be a reasonable assumption given that the agents in the Livingston Survey, the so-called 'alphas' of the 'pack' (Bywaters and Thomas, 2008), are aware, to some degree, of the underlying economic factors that generate future inflation, and use, where possible, all the available information in the determination of forecasts of key variables. Furthermore, if the actual and expected inflation rates are roughly equal, this implies equality between the real and the potential output, given the theoretical debate. The difficulty for

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[12] In the analysis of Keynes, it is the multiplier effect of output rather than prices, which is treated as fixed because the analysis is set in a picture of idle resources (Blaug, 1997).

the empirical analysis is that the latter are trended variables whereas the former are not, and therefore only actual output is a necessary condition in the estimation of the system. Briefly, when sewn together, all the threads indicate an 'augmented Wicksell curve', that is the rate of inflation is determined by expectations and shifted by twin forces of short- and long-run (or "natural") rates of interest, directing the cumulative process of investment and output that lies backstage.

Vector Three is the supply of real money, driven by the plus one coefficient on the real monetary base, the reverse value on real Government debt, various interest rates with differing signs, but a positive rôle for expectations and a negative part for the rate growth in the real price of oil. It represents a key vector in the overall stabilization policy of the system because it encapsulates the rôle of expectations, the real monetary base and the Federal debt. Whereas the latter variables are largely under the control of the monetary authorities, the former is determined by the 'alphas'.

Vector Four determines real output as the real monetary base. Assuming the latter rather than the former, employment and real output will have similar effects. There is a mix of effects coming through from the rates of interest with an expected, negative value on the rate of growth of the real price of oil. Clearly, given the various rates of interest, the effect on output does not always show the likely crowding-out effect.

The Final Vector in the system is the supply of bonds in real terms emanating from the real debt issued by the Federal Authorities to the Public on behalf of the Government. The coefficient of plus one on real high-powered money represents the debt sold to itself in order to monetize the effect on interest rates, but leading to the creation of new money. The selling of debt to the Public, however, leads to a positive effect on the thirty-year Treasury bill rate, suggesting the possibility of crowding-out in the long-run, but the opposite value coming from the Corporate Yield. The overall end-result of the two forms of debt financing is the positive effect with income. The short-run rate of interest exhibits a negative coefficient, whereas the own rate on M2 is positive. Finally, the negative coefficient on the expectations variable suggests that the real burden of Federal debt created by the state will be reduced in the long-run.

The next part of the procedure is to derive a possible solution to the simultaneous equation system by taking linear combinations with algebra of the five estimated vectors in Table [2]. This is done by vector space column operations, either multiplying a vector by a factor κ or by adding (or subtracting) another one. This practice is analogous to changing co-ordinates in geometry, leaving the basic structure unchanged. The solution for the following subset of 'focus' variables: RM_t, r_{30t}, ΔP_t, $\pi^A_{t,t+1}$ and Y_t chosen on theoretical grounds as relatively endogenous, is shown in Table [3] overleaf.

These variables above are set equal to zero in the five vectors except in the case of the normalization, chosen to be minus one so that the other coefficients have the correct sign. For instance, in the first case, the RM_t in Vector one is the normalised variable with the other 'focus' variables set equal to zero; the second case, r_{30t} in Vector two is normalised, with RM_t, ΔP_t, $\pi^A_{t,t+1}$, and Y_t put to zero. The procedure continues until the end-variable of Y_t with RM_t, r_{30t}, ΔP_t, and $\pi^A_{t,t+1}$ equalling zero. The calculated coefficients in Table [3] can then be directly compared with theory because the normalizations are in terms of -1.

Table [3]: Possible Solutions to the Simultaneous System

	Vector e1	Vector e2	Vector e3	Vector e4	Vector e5
RM_t	–1.0000	0.0000	0.0000	0.0000	0.0000
RB_t	1.4856	0.0244	0.0280	1.3022	–0.0029
RGD_t	–0.4726	–0.0172	–0.0172	0.0078	0..0000
E_t	4.1455	–0.0912	–0.1561	0.1977	–0.1995
$r_{2\,t}$	21.4983	0.3113	0.4238	10.0728	–0.0968
r_t	–11.5615	–0.0891	–0.0782	–3.6477	0.3229
$r_{30\,t}$	0.0000	0.0000	0.0000	0.0000	–1.0000
$r_{m\,t}$	0.2194	–0.0001	–0.0357	1.9928	0.5576
ΔP_t	0.0000	–1.0000	0.0000	0.0000	0.0000
$\pi^A_{t,t+1}$	0.0000	0.0000	–1.0000	0.0000	0.0000
Y_t	0.0000	0.0000	0.0000	–1.0000	0.0000
ΔP^R_t	0.3679	0.0073	–0.0413	–0.5125	–0.0456
C	6.7082	0.1565	0.1529	7.3104	0.0012

The interesting feature to emerge from the analysis is that a number of coefficients are approaching the value of one in various vectors, namely the coefficients on RB_t, E_t, r_{2t}, r_t and r_{mt}, as highlighted in Table [3]. This suggests a subset of leading variables which are partly determined by exogenous agents, that is

RB_t - The real monetary base (or high powered money), which is controlled by the monetary authorities to manipulate the instruments of monetary policy and price expectations indirectly;

E_t - The proportion of employment;

r_{2t} - The 'own' rate of interest on M2;

r_t - The short-run rate of interest, influenced by the Federal Reserve;

r_{mt} - The rate of interest on long-term corporate bonds, which 'tracks' relatively closely the rate of interest on the Federal Government's long-run debt.

The next step in the development of the system is to include the short-run forces with the long-run factors to complete the estimation process. All the long-run vectors are used in the Hendryfication models.

The Short-run Dynamics Meet the Long-run Solutions

The study now presents the principal equations of the five normalizations shown in Table [3] to examine the overall estimation picture with the short- and long-run dynamics. It should be pointed out that the short-run dynamics are not explained by economic theory because the latter concentrates on the long-run equilibria and steady-state solutions. In fact, the coefficients within the equilibrium relationships overleaf represent the long-run elasticities with respect to the normalizations, whereas the values on the change (disequilibrium) variables denote the behaviour of the short-run elasticities. The constant terms relate to the long-run growth rates. This is outlined overleaf in Table [4], starting with the consideration of the real money balance expression.

The equation in Table [4] is well-defined given the diagnostic tests. The majority of variables seem to have the correct signs, although the values on the rate of inflation have a mix of both positive and negative contributions.

The stabilisation of real money would depend on $e5$ with its negative sign on 0.088909 in Table [4] and on its sign in the co-integrating vector five of Table [2], but real money is not there. The stability of real money flows indirectly from the negative value on the real monetary base in $e5$ of Table [2], through vector three, the money supply. Changes in the money supply affect interest rates, the level of real Government debt as well as price expectations, that will in turn lead to a change in the demand for money, vector one, but these effects are all captured in e5.

Thus, the demand for, and the supply of real money are self evidently interdependent. This suggests that the endogenous nature of the money supply interacting with the demand variables has the policy implication that monetary instruments have an impact on both income and price.

Table [4]: Money balances							
The regression coefficients with the dependent Variable: ΔRM_t From 1961 Q:2 to 2007 Q:4							
$t-i$	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$	$t-6$	$t-7$
ΔRM_{t-i}	.49205 (.063539)		.15137 (.060290)		.11611 (.058016)		
ΔRB_{t-i}						−.15153 (.036905)	.17985 (.041409)
ΔRGD_{t-i}	−.11895 (.046110)						−.10447 (.041252)
ΔE_{t-i}			.43440 (.17364)				
$\Delta r_{2\ t-i}$	1.3042 (.21761)		.82949 (.23761)				
Δr_{t-i}	−.51513 (.11115)		−.37125 (.11294)			−.23411 (.052137)	
$\Delta r_{30\ t-i}$	−.68177 (.11892)	−1.4320 (.28708)					
$\Delta r_{m\ t-i}$		1.6288 (.30376)					
$\Delta\Delta P_{t-i}$	.35968 (.07991)		−.30359 (.10280)	−.28157 (.10900)		−.16865 (.085274)	−.20398 (.084846)
ΔY_{t-i}	−.12719 (.061208)		−.25093 (.067562)		−.16891 (.054729)		
ΔP^R_{t-i}	−.0092225 (.0029694)	−.011288 (.0034599)	−.0072984 (.0030158)				−.0073410 (.0024524)
$e5(-1)$	−.0088909 (.0013217)						
$D1$	.042548 (.0043584)	Dummy variable for 1983Q1=1, 1983 Q2= -1, otherwise zero.					
$D2$	.010756 (.0039300)	Dummy variable for 2005 Q2=1, 2005 Q3= -1, otherwise zero.					
$D3$	−.04978 (.0056474)	Dummy variable for 2007 Q4=1, otherwise zero.					
$D4$	.017847 (.0040078)	Dummy variable for 1974 Q1=-1, 1975 Q2= 1, otherwise zero.					
$R^2 = .82521$, $\overline{R}^2 = .78889$, $\sigma = 0.0052465$, DW = 2.1033, T = 187. A : $\chi_4 = 6.2853$, $B : \chi_1 = .59268$, $C : \chi_2 = 1.1601$, $D : \chi_1 = .89587$.							
A: Lagrange multiplier statistic of residual auto correlation, B: Ramsey's Reset test employing the square of the fitted values, C: Based on a statistic of Skewness and kurtosis of the residuals, D: Based on the regression of squared residuals on squared fitted values.							

The outcome of the process of Hendryfication on the inflationary dynamics, with the co-integrating vectors, led to the following econometric representation in Table [5]:

Table [5]: The Change in the rate of Inflation							
The regression coefficients with the dependent Variable: $\Delta\Delta P_t$ From 1961 Q:2 to 2007 Q:4							
$t-i$	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$	$t-6$	$t-7$
ΔRB_{t-i}							−.0796949 (.035483)
ΔRGD_{t-i}	−.083570 (.034381)					.086117 (.024354)	.085968 (.032957)
$\Delta r_{2\ t-i}$		−.25800 (.094633)		−.48276 (.15515)			
Δr_{t-i}				.18860 (.073082)			.10072 (.041128)
$\Delta r_{30\ t-i}$	.95107 (.21044)	.78255 (.20625)	.62250 (.20934)	.58627 (.20962)	.57835 (.20827)	.90900 (.20728)	.69969 (.19874)
$\Delta r_{m\ t-i}$	−.56585 (.21610)	−.56185 (.21616)	−.58886 (.21991)	−.57747 (.22619)	−.49216 (.22706)	−.71794 (.22371)	−.73693 (.22400)
$\Delta\Delta P_{t-i}$	−.29888 (.091909)	−.21442 (.071165)					
$\Delta\pi^{A}_{t,t+1-i}$	.81392 (.34680)						
ΔY_{t-i}			−.13210 (.037576)				.083731 (.035901)
ΔP^{R}_{t-i}		−.011288 (.0034599)					−.0039862 (.0015763)
$e1(-1)$	−.036160 (.010385)						
$e2(-1)$	−.74651 (.10405)						
$e3(-1)$	.024150 (.0087930)						
$e4\ (-1)$	−.0055786 (.0012605)						
$D5$	.016617 (.0030234)	Dummy variable for 2005 Q3 =1, 2005 Q4 = -0.5, 2006 Q4 = -0.5, otherwise zero.					
$R^2 = .75377$, $\overline{R}^2 = .70260$, $\sigma = 0.0035165$, DW $= 1.9820$, T $= 187$. A : $\chi_4 = .63627$, $B : \chi_1 = 1.3480$, $C : \chi_2 = .99990$, $D : \chi_1 = 1.0613$.							

The empirical model in [5] above is statistically 'sound' given the values on the diagnostic tests with an adjusted-$\overline{R}^2$ of 0.70260. Once again, the short-run dynamics seem to be a mix of positive and negative impacts, partly directing the change in the rate of inflation. The interest rate variables have the expected positive coefficients apart from the M2, own and corporate rates, which display the opposite effects. The change in the expectations variable only appears once, but has a significant coefficient of 0.8139.

The major, outstanding feature, however, is the significant number of equilibrium mechanisms being retained by the empirical model, except $e5$, which represent a combination of positive and negative processes forcing the system back on the 'road of stability', depending on the quarterly values embodied in the Vectors over the sample period. It is, however, clear that $e2$ has the largest coefficient with a negative sign of 0.74651, giving rise to the most influential of the equilibrating vectors, which in Table [2] has inflation with a coefficient of plus one.

The next equation to view, resulting from the process of Hendryfication, is the particularly important one of expectations, modelled in Table [6] overleaf, exposing principal parts of the information set. This is partly determined by the rate of inflation and expectations itself along with interest rates as well as the following real variables: money, employment, debt, the monetary base and the price of oil. All the Vectors except $e3$, the real money supply, seem to play significant rôles in the equilibrating process with positive and negative contributions to the explanation of the dependent variable and driving it to the equilibrium pathway. For instance, in Vector Two of Table [2], the expectations variable has a negative coefficient. In Table [6] $e2$ has a positive coefficient, which is appropriate for determining the stability of expectations. In short, the diagnostic statistics in [6] seem to indicate a healthy, well-defined econometric model with an adjusted-$\overline{R}^2$ of 0.7306, indicating that a significant explanation of the modification process of the change in expectations of the 'alphas', the leaders of the 'pack'.

Table [6]: The change in Expectations							
The regression coefficients with the dependent Variable: $\Delta\pi^{A}_{t,t+1}$ From 1961 Q:2 to 2007 Q:4							
$t-i$	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$	$t-6$	$t-7$
ΔRM_{t-i}	.023116 (.0074717)	.020656 (.066242)			.014279 (.0069590)	−.012439 (.0060850)	
ΔRB_{t-i}		.0086820 (.0042484)			.010236 (.0042467)		
ΔRGD_{t-i}	.014234 (.0053293)						
ΔE_{t-i}		−.073917 (.016111)		.043385 (.014294)			
$\Delta r_{2\ t-i}$	.028242 (.010476)	−.074716 (.023234)				−.064466 (.023184)	
Δr_{t-i}			.056553 (.010970)		.019490 (.0065394)	.030801 (.010560)	
$\Delta r_{30\ t-i}$							.063160 (.027140)
$\Delta r_{m\ t-i}$					−.045653 (.012735)		−.061656 (.029761)
$\Delta\Delta P_{t-i}$	−.086618 (.025094)		−.044067 (.018836)	−.040917 (.014453)			
$\Delta\pi^{A}_{t,t+1-i}$	.76673 (.059551)		.14415 (.061492)				−.20702 (.049571)
ΔP^{R}_{t-i}	.0007693 (.0003492)					.0008303 (.0002710)	.0006633 (.0006633)
$e1(-1)$	−.0023304 (.0006886)						
$e2(-1)$	.15041 (.029534)						
$e4(-1)$	.00028463 (.0001789)						
$e5(-1)$	−.0015661 (.0002878)						
$D6$	.0030221 (.0003968)	Dummy variable for 1975 Q1 =-1, otherwise zero.					
$R^2 = .77878$, $\bar{R}^2 = .73016$, $\sigma = 0.000520$, DW $= 2.0635$, T $= 187$.							
A : $\chi_4 = 5.5118$, $B : \chi_1 = 0.076033$, $C : \chi_2 = 1.3523$, $D : \chi_1 = 1.1051$.							

The output equation shown overleaf in Table [7] exhibits, once again, the rôle of price expectations in the short-run dynamics with a coefficient of 1.83202. The following list of variables is also a part of the process of development: real money, the real monetary base, the real Federal debt and employment with interest rates, inflation and the real price of oil. In case of the co-integrating vectors, only $e1$ and $e3$ make significant contributions. In the case of $e1$, the coefficient in Table [7] is negative, so for stability, the coefficient on output in Table [2] needs to be positive, whereas it is actually negative. So $e1$ is not stabilising real output at least in any straightforward sense. Stability here must depend on real money, the real monetary base and employment. Real output is affected by employment and resource markets, which are known to adjust much more slowly than financial ones.

The examination of the diagnostic tests reveals that the statistical model is well-defined at the five per cent level of probability and is well within the critical limits, although the adjusted - R^2 is rather low at 0.43530.

Table [7]: The change in output							
The regression coefficients with the dependent Variable: ΔY_t From 1961 Q:2 to 2007 Q:4							
$t-i$	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$	$t-6$	$t-7$
ΔRM_{t-i}	.23508 (.052401)	.27163 (.058589)					
ΔRB_{t-i}				.10949 (.040349)			
ΔRGD_{t-i}		−.10982 (.052381)					
ΔE_{t-i}				−.41376 (.13328)			
$\Delta r_{2\ t-i}$	.60428 (.11082)						
$\Delta r_{30\ t-i}$					−.26231 (.098512)		
$\Delta\Delta P_{t-i}$				−.28157 (.10900)			
$\Delta\pi^A_{t,t+1-i}$					1.83202 (.50281)		
ΔP^R_{t-i}	−.019754 (.0035839)	−.011081 (.0031770)					
$e1(-1)$	−.0135122 (.0097669)						
$e3(-1)$	.044191 (.0056645)						
$D8$	.028503 (.0045034)	Dummy variable for 1978Q2=1, 1980 Q2= -1, otherwise zero.					
$R^2 = .47153$, $\overline{R}^2 = .43530$, $\sigma = 0.0061995$, DW $= 1.9534$, T $= 187$. A : $\chi_4 = 4.3538$, $B:\chi_1 = 1.0712$, $C:\chi_2 = .65327$, $D:\chi_1 = .31390$.							

The final expression in the Hendryfication process is the Treasury's thirty-year, the long-run rate of interest, revealing a limited source of short-run dynamics in association with the long-run, co-integrating vectors, excluding $e5$. The estimated equation is Table [8] overleaf. The dominant co-integrating vector is $e2$, with a negative coefficient. The thirty-year interest rate has a positive coefficient in Vector two of Table [2], as required for stability. Wicksell emphasised this equilibrating mechanism in his writings.

Table [8]: The long-run of Interest							
The regression coefficients with the dependent Variable: $\Delta r_{30\ t}$ From 1961 Q:2 to 2007 Q:4							
$t-i$	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$	$t-6$	$t-7$
ΔRB_{t-i}			.053938 (.026628)				
$\Delta r_{2\ t-i}$						.24420 (.071348)	
Δr_{t-i}				.13991 (.042971)			
$\Delta r_{30\ t-i}$	.74520 (.19307)						
$\Delta r_{m\ t-i}$	−.69883 (.20706)			−.18988 (.086053)			
$\Delta\Delta P_{t-i}$			.13122 (.048964)				
$\Delta \pi^{A}_{t,t+1-i}$			1.5747 (.40332)	−1.2725 (.48506)	1.4102 (.41121)		
ΔY_{t-i}						−.098123 (.036901)	
ΔP^{R}_{t-i}	−.0050647 (.0023129)						
$e1(-1)$	−.028682 (.0074067)						
$e2(-1)$	−.16886 (.073261)						
$e3(-1)$	.021592 (.0064468)						
$e4(-1)$	−.0023538 (.0008409)						
$D7$	.022481 (.0028102)	Dummy variable for 1980 Q1 = 1, 1980 Q2 = -1, otherwise zero.					
$R^2 = .46671$, $\bar{R}^2 = .41681$, $\sigma = 0.0038428$, DW $= 1.8977$, T $= 187$. A : $\chi_4 = 4.1867$, $B : \chi_1 = 2.9893$, $C : \chi_2 = 1.3686$, $D : \chi_1 = .047570$.							

At the heart of the empirical model is a combination of interest rates with positive values on the own rate, the short-run and the thirty-year rates of interest, but negative coefficients on the Corporate rates. Positive values on the real monetary base and the rate of inflation with negative responses on the real price of oil and output. In the case of expectations, coefficients are approaching the factor of one with statistical significance.

The diagnostic statistics suggest a parsimonious and sound empirical model with the R^2 equal to 0.46671, representing only approximately forty-seven per cent of the ‘focus’ variable being explained by both short- and long-run forces.

Conclusions/ Summary

One of the most important veins running through the monetary system is price expectations along with the rate of inflation, dictating the explanation and feedback mechanisms of stability. The expectations come from the Livingston Survey and are transformed into meaningful observations, firstly on the course of the rate of inflation over the next six months, and then quarterly. Furthermore, the modelling of the expectations and the rate of inflation are based on the second-difference within a VAR framework and not the first difference to ensure the necessary stationary properties.

Explaining the short- and the long-run properties, the process of Hendryfication of the five chosen error-correction equations produced well-defined, restricted empirical models using only the statistically significant variables. Arguably, the major contribution of this statistical analysis is the incorporation of expectations into the monetary side of the economy, bringing the future back into the model.

Appendix A: Livingston Survey

In June and December of each year, from 1946 to 1990, when the Federal Bank of Philadelphia took over, Livingston conducted a Survey of professional economists in academic, business, Government and finance sectors to forecast a number of key variables of the economy, such as the Consumer Price Index (CPI). They provide, for example, forecasts for the end of the current month as well as six- and twelve-months-ahead, receiving, on average, fifty replies each time (Cronshore, 1997).

Pesando (1975) suggested that the six-month-ahead forecasts were unbiased, whereas the twelve month forecasts were biased. In fact, Carlson (1977) compared statistical forecasts with the Survey predictions and found that the latter performed better than the former despite a number of problems with the Survey[13]. Given the discussion within the literature, and the results of a statistical experimentation between the two Surveys, the empirical analysis adopted the six-month-ahead statistics.

The major difficulty, however, as in the case of the CPI, is that the forecasts are for the level, P_{t+1}^{E}, rather than the rate of inflation, $\pi_{t,t+1}^{A}$. The participants predict from the latest release of the CPI, the so called base period figure, namely P_t, which is sent out to forecasters in May and November, after the release by the Government of the CPI. The results are then announced to the newspapers towards the end of June and of December.

The problem at this stage is that the series is seasonally adjusted from December 2004, but not beforehand. Therefore, it was desirable to seasonally adjust the observations up to September

[13] Carlson's article (1977) studies the advantages and dis-advantages of the Livingston Survey in particular, from the point of view of the CPI.

2004 and pin the two series together, although the actual data points are not particularly seasonal.

The mean responses of the price expectations series are converted into analytical observations, by calculating the “implied expected rates of inflation” (Carlson, 1977) using the following formula:

$$\pi^{A}_{t,t+1} = \frac{\text{Forecast of the Six level at Survey date}}{\text{Monthly Base Period}} = \left(\frac{P^{E}_{t+1}}{P_t}\right). \qquad \text{[A.1]}$$

According to the Philadelphia Federal Reserve Bank (2005), the base periods are accurate reflections of the CPI, although because of the uncertainty over the issue date of some the base figures over the historical period, no “annualisation“ of the growth rates was adopted to avoid any distortion of the information content.

The half-yearly growth rates are changed into monthly statistics by linear interpolation. Then, to transform them into quarterly observations, the following four months are extracted: March, June, September and December. This method was adopted to be consistent with the other data sets in the econometric study. Logarithms of the extracted data were taken and divided by two, to give estimated quarterly figures.

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