

Citation for published version:

Z. Jiang, et al., "A two-dimensional analytical model for prediction of the radiation heat transfer in open-cell metal foams", *Applied Thermal Engineering*, Vol. 93: 1273-1281, October 2015.

DOI:

<https://doi.org/10.1016/j.applthermaleng.2015.09.043>

Document Version:

This is the Accepted Manuscript version.

The version in the University of Hertfordshire Research Archive may differ from the final published version. **Users should always cite the published version of record.**

Copyright and Reuse:

Copyright © 2015 Elsevier Ltd. All rights reserved.

This Manuscript version is distributed under the terms of the Creative Commons Attribution licence

(<https://creativecommons.org/licenses/by-nc-nd/3.0/>).

Enquiries

If you believe this document infringes copyright, please contact the Research & Scholarly Communications Team at rsc@herts.ac.uk

1 **A Two-dimensional Analytical Model for Prediction of the**
2 **Radiation Heat Transfer in Open-cell Metal Foams**

3 Z. Jiang^a, K. Wang^{a,*}, H. Wu^{b,*}, Y. Wang^a, J. Du^a

4 ^aInstitute of Engineering Thermophysics, Chinese Academy of Sciences
5 Beijing, 100190, China.

6
7 ^bInstitute of Engineering and Energy Technologies, School of Engineering and Computing,
8 University of the West of Scotland, Paisley, PA1 2BE, United Kingdom,

9

10 **Abstract**

11 In this article, a new simple two-dimensional (2D) explicit analytical model for the
12 evaluation of the radiation heat transfer in highly porous open-cell metal foams is
13 formulated and validated. A correction factor, C , is introduced to correct the deviation
14 of the specific area for the purpose of simplification. The numerical results are
15 compared with published experimental data and three-dimensional (3D) model
16 proposed in previous works, and the present two-dimensional model is proved to be
17 relatively accurate in estimating the radiative conductivity for all the investigated
18 structures. In the current work, the effects of the control parameters, such as the
19 number of order in the iterative procedure, solid emissivity, the temperature
20 difference, shape of solid particle and correction factor on the predictions of radiation
21 characteristics are well discussed.

22

23 **Keywords:** Modelling; Thermal radiation; Porous medium; Open-cell metal foam;
24 Radiation heat transfer.

Nomenclature

a	side length [m]	X, Y	Cartesian coordinates [-]
A_{sf}	specific area [m^{-1}]	Greek symbols	
b	bottom face of the unit cell [-]	α_i	dimensional coefficient [-]
C	correction factor [-]	β_i	dimensional coefficient [-]
d	side length of the unit cell [m]	ε	solid emissivity[-]
d_f	diameter of strut [m]	ρ	solid reflectance [-]
d_p	characteristic cell size [m]	σ	Stefan-Boltzmann constant[W/ m^2K^4]
F	configuration factor [-]	ϕ	porosity [-]
H	foam sample thickness [m]	Subscripts	
i	sequence of the unit cell [-]	bt	void face b to void face t
J	irradiation from void face[W/m^2]	bj	void face b to solid particle j
k_r	radiative conductivity [$W/m K$]	c	cold side
l_b	length of bottom void face [m]	h	hot side
l_j	length of solid particle [m]	jk	solid particle j to solid particle k
l_s	length of side void face [m]	jt	solid particle j to void face t
l_t	length of top void face [m]	kt	solid particle k to void face t
N_c	total number of cells [-]	sj	void face s to solid particle j
q_r	radiation heat flux [W/m^2]	st	void face s to void face t
Q_r	irradiation [$m s^{-2}$]	Superscripts	
s	side face of the unit cell [-]	-	negative direction
t	top face of the unit cell [-]		
T	temperature [K]		

27 **1. Introduction**

28 Metal foams are extensively used for many industrial applications involving
29 numerous technological fields over more than 50 years due to their attractive physical
30 properties such as, high porosity, large specific surface, flow mixing enhancement,
31 attractive stiffness properties and low cost [1]. Their averaged thermo-physical
32 properties are also important for many applications, e.g., compact heat exchangers [2],
33 solar receivers [3], and catalytic reactors [4]. The main characteristic of heat transfer
34 in metal foams is dictated by the enhanced effective thermal conductivity (ETC). The
35 ETC used to quantify the magnitude of heat conduction in metal foams is studied
36 through model prediction [5-13], numerical simulation [14-16] and experimental
37 research [16-18].

38 Previous publications reported on the thermal properties of metal foams at high
39 temperature where conduction and radiation heat transfer may occur are relatively
40 weak [19]. To overcome the experimental difficulties, Coquard et al. [20] proposed an
41 innovative method to evaluate the conduction and radiation contribution in metal
42 foams. They developed an identification method using thermograms obtained from
43 laser-FLASH measurements to minimize the discrepancy between experimental and
44 theoretical thermograms. Coquard et al. [19], afterwards, presented a detailed review
45 on the radiation and conduction heat transfer from ambient to high temperature. They
46 also proposed an analytical model for the real foams to predict the conduction and

47 radiation heat transfer at high temperature. Their predicted results agreed well with
48 the experimental results [20].

49 Several studies have been devoted to the radiation heat transfer in metal foam
50 [21-24]. Coquard et al. [21] modelled the radiation heat transfer in open cell metal
51 foams and closed cell polymer foams utilizing two approaches, i.e., Homogeneous
52 Phase Approach (HPA) and Multi-Phase Approach (MPA). The radiation heat
53 transfer of these two types of foams was investigated using three-dimensional (3D)
54 tomographic images. The calculated results were compared with the results of direct
55 Monte Carlo (MC) simulations and the suitability of the two approaches was then
56 evaluated. Tancrez et al. [22] developed a general method with direct identification of
57 the radiation properties, i.e., absorption, scattering coefficients and phase function of
58 porous medium using Monte Carlo (MC). This method was applied to both sets of
59 Dispersed radius Overlapping Opaque Spheres (DOOS) in a transparent fluid phase
60 and Dispersed radius Overlapping Transparent Spheres (DOTS) in an opaque solid
61 phase. Zhao et al. [23] measured the ETC of metal foams with a range of pore sizes
62 and porosities between 300 and 800 K. The radiative conductivity was decoupled
63 from the equivalent conductivity due to conduction. As for the equivalent
64 conductivity due to conduction contribution alone, the model proposed in [6] was
65 used. At the same time, Zhao et al. [24] used the Rosseland equation to calculate the
66 equivalent radiative conductivity based on the experimentally obtained spectral

67 transmittance and reflectance. The calculated results were found to be in satisfactory
68 agreement with the experimental data [23].

69 Although many significant results in the modelling radiation heat transfer of
70 open-cell metal foam have already been obtained, the aforementioned approaches are
71 not quite suitable for engineering applications. Thus, Zhao et al. [25] proposed an
72 explicit analytical model based on the simplified cubic structure. In this model, the
73 fundamental foam parameters and the emission and reflectance in metal foam
74 structure were considered to establish functional relationships between the structure
75 and the radiation characteristics of open-cell metal foams. The calculated equivalent
76 radiative conductivity showed that in general there was a good agreement between the
77 predicted and experimental data. Most recently, as an extension of the simplified
78 analytical approach of [25], Contento et al. [26] made further improvements by
79 recalculating the configuration factors that involved in the dimensionless coefficients
80 and a close agreement between predicted result and measured data was achieved. As
81 the same time, Contento et al. [27] developed a new radiative heat transfer model
82 based on a more realistic Lord Kelvin representation of open cell metal foams instead
83 of the simplified cubic structure using the same analytical approach. This explicit
84 simple approach that initially proposed by Zhao et al. [25] can be relatively suitable
85 for engineering applications.

86 Based on the brief literature review, it can be seen that much effort has been made
87 to develop models for estimation of radiation heat transfer in open-cell metal foam.

88 From an engineering perspective, however, due to the complex nature of the
89 configuration factors for implementation in three-dimensional modelling, research on
90 modelling radiation heat transfer has been far from complete. More effort needs to be
91 made in this area. In this study, a newly simplified two-dimensional model is
92 proposed and could serve as an efficient alternative to evaluate the radiative
93 characteristics in porous open-cell metal foams for engineering applications. For the
94 assessment of the new model, the comparisons between numerical predictions with
95 experimental data [23] and previously proposed model [25] are carried out.

96 **2. Model description**

97 *2.1. Structure simplification*

98 The microstructure of typical open cell metal foam is shown in Fig. 1. Porous
99 medium such as metal foams has a complex microstructure made up of solid
100 ligaments and pores generally filled with fluid. In order to simplify the analysis of
101 radiation heat transfer in metal foam, the microstructure can be assumed to be
102 consisted of randomly oriented cells with characteristic size d_p which are mostly
103 homogeneous in size and shape, whilst the solid of the metal foam can be treated as
104 particles with simple geometry (circle, square and rectangle etc.) distributed in fluid
105 zone regularly or randomly. In the current work, the connection of the solid phase of
106 the metal foam can be neglected since the thermal radiation in metal foam mainly
107 passes through the void due to the large porosity ($\phi \geq 90\%$) of metal foam.

108 Based on the above simplification, a new 2D structure with regularly distributed
 109 square particles with side length of a are selected to develop the analytical model for
 110 analysing the radiation heat transfer, as presented in Fig. 2(a). Since the structure is
 111 periodic, Fig. 2(b) shows the details of two neighbouring square unit cells. Within
 112 each cell, there are four quarters of solid particle at four corners which are labelled
 113 with 1-4 respectively. As for the four faces, two side faces are referred as s , whereas
 114 the top and bottom faces are represented by t and b . The relationship between d and
 115 measured d_p based on the same area is shown as:

$$116 \quad d^2 = \frac{\pi}{4} d_p^2 \quad (1)$$

$$117 \quad d = \frac{\sqrt{\pi}}{2} d_p \quad (2)$$

118 Then, a is obtained based on the porosity for the two-dimensional structure as:

$$119 \quad \frac{4a^2}{d^2} = 1 - \phi \quad (3)$$

$$120 \quad a = \sqrt{\frac{1 - \phi}{4}} d \quad (4)$$

121 where ϕ is the porosity of the metal foam.

122 2.2. Assumptions

123 In order to simplify the heat transfer mechanism in open-cell metal foam, the
 124 following major assumptions were made in the derivations of the governing
 125 equations:

- 126 (i) The diffraction is neglected. The characteristic size of porous medium is
 127 considered as large compared to the heat radiation wavelengths.

128 (ii) The solid particles are assumed as grey and opaque since they are metallic. The
129 void zone is considered as vacuum.

130 (iii) Surface of solid particles reflecting diffusely the incident radiation is assumed
131 since surface roughness at 10 μ m scale is being taken into account [26].

132 (iv) Steady-state heat flow is assumed in a specific zone of the metal foam
133 sandwiched between two plates with cold boundary temperature (T_c) for the top
134 plate and hot boundary temperature (T_h) for the bottom plate. Sample is thermally
135 insulated at side walls, which means that there exists a radiation heat flux in the
136 positive Y direction.

137 (v) It is assumed that the radiation is decoupled from the conduction and the
138 temperature varies linearly with Y direction [25].

139 (vi) Temperature difference within unit cell can be neglected since the porous foam
140 sample is sufficiently thick. This means that each unit cell has a unique value of
141 temperature in the same layer [26].

142 Other simplifications are described in the due course in the rest of the paper.

143 2.3. *Mathematical formulations*

144 2.3.1 *Basic formulations*

145 Based on the assumptions, the temperature difference between the two cells in
146 adjacent planes in Y direction is represented by equation:

$$147 \Delta T = \frac{T_h - T_c}{N_c} \quad (5)$$

148 where ΔT is the temperature difference between two cells in adjacent planes, N_c

149 denotes the total number of cells in Y direction which is given by:

$$150 \quad N_c = \frac{H}{d} \quad (6)$$

151 where H is the thickness of the porous medium sample. The temperature of the i th
152 ($i=1,2,3 \dots N_c$) cell is:

$$153 \quad T[i] = T_h - (i-1)\Delta T \quad (7)$$

154 Thus, the radiative conductivity k_r can be obtained by:

$$155 \quad k_r = \frac{q_{r,net}}{(T_h - T_c)/H} \quad (8)$$

156 where $q_{r,net}$ is the net radiation heat flux.

157 The net radiation heat flux $q_{r,net}$ will be calculated based on the top void face t of
158 the i th cell. Since the radiation heat fluxes in both directions are not identical, the net
159 radiation heat flux can be mathematically expressed by the following equation:

$$160 \quad q_{r,net} = q_r - q_r^- \quad (9)$$

161 where q_r is the radiation heat flux in the positive Y direction and q_r^- is the
162 radiation heat flux in the negative Y direction, respectively.

163 2.3.2 Derivation

164 Firstly, radiation in the positive Y direction will be analysed, as radiation in the
165 negative Y direction is familiar with that in positive Y direction. As shown in Fig. 2(b),
166 the total irradiation on the void face t of the i th cell includes both the emission and
167 reflectance from the solid particles $1-4$ to the void faces s, b . The total irradiation Q_r
168 on t is given by:

$$169 \quad Q_r = (Q_r)_{emission} + (Q_r)_{reflectance} \quad (10)$$

170 where,

$$171 \quad (Q_r)_{emission} = \sum_{j=1}^4 l_j F_{jt} \cdot \varepsilon \sigma T^4 + l_b F_{bt} J_b + 2l_s F_{st} J_s \quad (11)$$

172 where $l_j(j=1,2,3,4)$ is the length of the j th solid particle within a unit cell, ε is the solid
173 emissivity, σ is Stefan-Boltzmann constant equal to 5.669×10^{-8} W/m²K⁴, T is the
174 temperature of the unit cell, l_b and J_b are the length and irradiation of the void face b ,
175 l_s and J_s are the length and irradiation of the void faces, F is the configuration factor.

176 The three terms on the right side of Eq. (11) are the emission on the void face t
177 from four solid particles in four corners, bottom void face b and side void faces s ,
178 respectively.

$$179 \quad (Q_r)_{reflectance} = \sum_{j=1}^4 \sum_{k=1, j \neq k}^4 \rho l_j F_{jk} F_{kt} \cdot \varepsilon \sigma T^4 + \sum_{j=1}^4 \rho l_b F_{bj} F_{jt} J_b + 2 \sum_{j=1}^4 \rho l_s F_{sj} F_{jt} J_s \quad (12)$$

180 where $\rho = 1 - \varepsilon$ is the solid reflectivity. Similarly, the three terms on the right side of Eq.
181 (12) represent the reflectance of incident radiation on the solid particles from each
182 other, bottom void face and two side faces, respectively.

183 Considering the model is two-dimensional, the unit of Q is W/m.

184 In the current study, the configuration factors can be analysed geometrically. The
185 following formulations are used:

$$186 \quad F_{12} = F_{21} = F_{13} = F_{31} = F_{34} = F_{43} = F_{24} = F_{42} = F_1 \quad (13)$$

$$187 \quad F_{14} = F_{41} = F_{23} = F_{32} = F_2 \quad (14)$$

$$188 \quad F_{1t} = F_{2t} = F_3 \quad (15)$$

$$189 \quad F_{3t} = F_{4t} = F_4 \quad (16)$$

190 $l_1 = l_2 = l_3 = l_4$ (17)

191 $l_t = l_b = l_s$ (18)

192 $F_{s1} = F_{s3} = F_{b3} = F_{b4} = \frac{l_1}{l_s} F_3$ (19)

193 $F_{s2} = F_{s4} = F_{b1} = F_{b2} = \frac{l_1}{l_s} F_4$ (20)

194 where l_t is the length of the top void face in the unit cell.

195 Radiation in the positive Y direction is given by:

196 $q_r = \frac{Q_r}{l_t}$ (21)

197 Substitute Eqs. (13-20) to Eq. (21), the radiation in the positive Y direction can be

198 expressed in the following manner:

199 $q_r = \frac{Q_r}{l_t} = \frac{l_1}{l_t} (2 + 4\rho F_1 + 2\rho F_2)(F_3 + F_4)\varepsilon\sigma T^4$
 200 $+ (F_{bt} + 4\frac{l_1}{l_t}\rho F_3 F_4)J_b + \left[2F_{st} + 2\frac{l_1}{l_t}\rho(F_3 + F_4)^2 \right] J_s$ (22)

201 For the simplification of Eq. (22), dimensionless coefficients $\beta_1, \beta_2, \beta_3$ are introduced

202 and defined as:

203 $\beta_1 = l_1(2 + 4\rho F_1 + 2\rho F_2)(F_3 + F_4)/l_t$ (23)

204 $\beta_2 = F_{bt} + 4l_1\rho F_3 F_4 / l_t$ (24)

205 $\beta_3 = 2F_{st} + 2l_1\rho(F_3 + F_4)^2 / l_t$ (25)

206 Thus, Eq. (22) can be further reduced to:

207 $q_r = \beta_1\varepsilon\sigma T^4 + \beta_2 J_b + \beta_3 J_s$ (26)

208 In order to calculate the radiation in the positive Y direction q_r, J_b and J_s which are in

209 the right side of Eq. (26) need to be calculated first. Similarly, the irradiation from
 210 void face s , J_s can be analyzed

$$\begin{aligned}
 211 \quad J_s &= \frac{l_1}{l_s} (2 + 4\rho F_1 + 2\rho F_2)(F_3 + F_4)\varepsilon\sigma T^4 \\
 212 \quad &+ (F_{bt} + 4\frac{l_1}{l_s}\rho F_3 F_4)J_s + \left[F_{st} + \frac{l_1}{l_s}\rho(F_3 + F_4)^2 \right] J_b \quad (27)
 \end{aligned}$$

213 The quantity of J_s can be calculated from Eq. (27) which is written as following
 214 equation:

$$215 \quad J_s = \frac{l_1(2 + 4\rho F_1 + 2\rho F_2)(F_3 + F_4)/l_s}{1 - F_{bt} - 4l_1\rho F_3 F/l_s} \varepsilon\sigma T^4 + \frac{2F_{st} + 2l_1\rho(F_3 + F_4)^2/l_s}{1 - F_{bt} - 4l_1\rho F_3 F/l_s} J_b \quad (28)$$

216 Eq. (28) can be further written as:

$$217 \quad J_s = \alpha_1 \varepsilon\sigma T^4 + \alpha_2 J_b \quad (29)$$

218 where α_1 and α_2 are the dimensionless coefficients, defined as:

$$219 \quad \alpha_1 = \frac{l_1(2 + 4\rho F_1 + 2\rho F_2)(F_3 + F_4)/l_s}{1 - F_{bt} - 4l_1\rho F_3 F/l_s} \quad (30)$$

$$220 \quad \alpha_2 = \frac{2F_{st} + 2l_1\rho(F_3 + F_4)^2/l_s}{1 - F_{bt} - 4l_1\rho F_3 F/l_s} \quad (31)$$

221 Substitute Eq. (29) to Eq. (26) lead to:

$$222 \quad q_r = (\beta_1 + \beta_3\alpha_1)\varepsilon\sigma T^4 + (\beta_2 + \beta_3\alpha_2)J_b \quad (32)$$

223 2.3.3 Iteration process

224 For the convenience of iteration process, q_r , T , J_b of the i th unit cell can be
 225 rewritten as $q_r[i]$, $T[i]$, $J_b[i]$, thus, Eq.(32) can be rewritten as:

$$226 \quad q_r[i] = (\beta_1 + \beta_3\alpha_1)\varepsilon\sigma (T[i])^4 + (\beta_2 + \beta_3\alpha_2)J_b[i] \quad (33)$$

227 As the bottom face b of the i th unit cell is the top face of the $(i-1)$ th unit cell.

228 Therefore, the Eq. (33) can be expressed as:

$$229 \quad q_r[i] = (\beta_1 + \beta_3\alpha_1)\varepsilon\sigma(T[i])^4 + (\beta_2 + \beta_3\alpha_2)q_r[i-1] \quad (34)$$

230 Similarly,

$$231 \quad q_r[i-1] = (\beta_1 + \beta_3\alpha_1)\varepsilon\sigma(T[i-1])^4 + (\beta_2 + \beta_3\alpha_2)q_r[i-2] \quad (35)$$

$$232 \quad q_r[i-2] = (\beta_1 + \beta_3\alpha_1)\varepsilon\sigma(T[i-2])^4 + (\beta_2 + \beta_3\alpha_2)q_r[i-3] \quad (36)$$

233 ...

234 where the bottom face of the first unit cell is the bottom boundary of the porous
235 medium sample with the temperature T_h , thus:

$$236 \quad q_r[1] = (\beta_1 + \beta_3\alpha_1)\varepsilon\sigma(T_h)^4 + (\beta_2 + \beta_3\alpha_2)q_r[1] \quad (37)$$

237 Thus, the quantity of $q_r[i]$ can be calculated implementing an iterative procedure from
238 the boundary.

239 In the case of the radiation flux in the negative y direction, it can similarly be written
240 as:

$$241 \quad q_r^-[i] = (\beta_1 + \beta_3\alpha_1)\varepsilon\sigma(T[i+1])^4 + (\beta_2 + \beta_3\alpha_2)J_b^-[i] \quad (38)$$

242 where $J_b^-[i]$ is the irradiation on void face t of i th unit cell from the top void face of
243 the $(i+1)$ th unit cell, as shown in Fig. 2(b).

244 Similarly,

$$245 \quad q_r^-[i] = (\beta_1 + \beta_3\alpha_1)\varepsilon\sigma(T[i+1])^4 + (\beta_2 + \beta_3\alpha_2)q_r^-[i+1] \quad (39)$$

$$246 \quad q_r^-[i+1] = (\beta_1 + \beta_3\alpha_1)\varepsilon\sigma(T[i+2])^4 + (\beta_2 + \beta_3\alpha_2)q_r^-[i+2] \quad (40)$$

247 ...

$$248 \quad q_r^-[N_c] = \varepsilon\sigma T_c^4 \quad (41)$$

249 The determination of $q_r^-[i]$ is the same as that of $q_r[i]$. Then $q_{r,net}$ can be
250 calculated by Eq. (9). Consequently, the equivalent radiative conductivity is
251 determined by Eq. (8).

252 **3. Determination of coefficients**

253 In the analytical solution of the equivalent radiative conductivity, the dimensionless
254 coefficients, i.e., $\beta_1, \beta_2, \beta_3$ and α_1, α_2 need to be determined. As previously mentioned,
255 the coefficients are the functions of the configuration factors, geometric parameters
256 and the solid reflectance according to Eqs. (23-35) and Eqs. (30,31). In order to
257 determine these coefficients, the configuration factors, $F_1, F_2, F_3, F_4, F_{bt}$ and F_{st} ,
258 should be firstly determined. The crossed strings method is utilized to calculate the
259 configuration factors for a two-dimensional geometric structure with known
260 geometric parameters of the unit cell.

261 As for the solid reflectance, it is recognized that the solid reflectance is related to
262 the emissivity ($\rho + \varepsilon = 1$ for opaque material). However, the emissivity of a solid
263 material depends on many other factors such as temperature and orientation. The
264 influence of the emissivity on the radiation heat transfer is discussed in the next
265 section.

266 **4. Results and discussion**

267 *4.1. Model validation*

268 In the current work, the validation of the model is based on the FeCrAlY (Fe 75%,
269 Cr 20%, Al 5%, Y 2%) metallic foam produced via the sintering route which is

270 studied by Zhao et al. [23] and the test conditions employed for the current simulation
271 are listed in Table 1. Due to the fact that the real values of the geometric parameters
272 of the metal foam usually are different from that supplied by manufacturers, the
273 measured values instead of the nominal values will be considered. The currently
274 developed model will be evaluated through the comparison of the equivalent radiative
275 conductivity between the experimental data [23] and previous numerical results
276 [25,26].

277 The predicted results for all samples are shown in Figs. 3-6. It is clearly seen that
278 there is a large deviation between experimental data and predicted results for all
279 samples. It reveals that the currently developed model does not fully show the
280 geometrical characteristics of three-dimensional structure of metal foam. Thus, this
281 model needs to be corrected and modified.

282 In the simplified 2D model, the specific surface area can be defined as the ratio of
283 the total side length of solid particles and the area:

$$284 \quad A_{sf,2D} = \frac{8a}{d^2} \quad (42)$$

285 As for 3D structure of metal foam, following reference [28], the specific surface area
286 is defined as:

$$287 \quad A_{sf,3D} = \frac{3\pi d_f [1 - e^{-(1-\phi)/0.04}]}{(0.59d_p)^2} \quad (43)$$

288 where d_f is diameter of the strut. It is noted that the specific surface area in the
289 present 2D model is different from that in 3D structure, which results in the deviation
290 of the emission from solid particles in the calculation of radiation. In order to reduce

291 this deviation, a correction factor (C) is introduced to correct the emission from solid
 292 particles, which is defined as:

$$\begin{aligned}
 293 \quad C &= \frac{A_{sf,3D}}{A_{sf,2D}} \\
 294 \quad &= \frac{3\pi d_f [1 - e^{-(1-\phi)/0.04}] d^2}{8a(0.59d_p)^2} \\
 295 \quad &= 1.0773\pi^{2.5} (1-\phi)^{-0.5} [1 - e^{-(1-\phi)/0.04}] \frac{d_f}{d_p} \quad (44)
 \end{aligned}$$

296 Thus, the previous analysis needs to be reconsidered. The proposed correction factor
 297 C is added into the item of emission radiation in Eqs. (11-12), then Eqs. (11-12) are
 298 rewritten as:

$$299 \quad (Q_r)_{emission} = \sum_{j=1}^4 Cl_j F_{jt} \cdot \varepsilon \sigma T^4 + l_b F_{bt} J_b + 2l_s F_{st} J_s \quad (45)$$

$$300 \quad (Q_r)_{reflectance} = \sum_{j=1}^4 \sum_{k=1, j \neq k}^4 C \rho l_j F_{jk} F_{kt} \cdot \varepsilon \sigma T^4 + \sum_{j=1}^4 \rho l_b F_{bj} F_{jt} J_b + 2 \sum_{j=1}^4 \rho l_s F_{sj} F_{jt} J_s \quad (46)$$

301 The rest of the derivation is similar to the previous analysis. The same iteration is
 302 carried out to obtain the radiative conductivity. Firstly, the effect of the correction is
 303 observed. Fig. 7 shows the predicted radiative conductivity with and without the
 304 correction factor for S1. It can be seen that the effect of correction is significant. It
 305 reveals that, in the process of simplification, the geometrical characteristics needs to
 306 stay consistent to ensure the validity of simplified model.

307 Figs. 8-11 show the comparison of the radiative conductivity versus temperature at
 308 different pores per inch (PPI) and porosity between the present predicted results of
 309 corrected model and experimental data [23] as well as previously numerical

310 results[25,26]. The results in Figs. 8-11 clearly show that the proposed model and
311 model from reference [26] perform well in predicting the experimental data in all
312 cases, while the initial model proposed by Zhao et al. [25] did not perform well for the
313 cases of S2 and S4. Percent differences between the predicted results and the
314 experimental data are reported in Table 2. And it is noted that there may have been a
315 slight over-estimation or under-estimation of the radiative conductivity. This could be
316 mainly due to the fact that the current model assumes uniform distribution of the solid
317 particles in the porous media and uses the average particle diameter whereas in the
318 real case the particle size is within a certain range. Despite this, it can be seen that in
319 general there is a good agreement between the currently predicted and the
320 experimental data.

321 Then the effects of the control parameters such as, the number of the orders, the
322 solid emissivity, temperature gradient, and the geometry on the radiative conductivity
323 will be examined in detail.

324 *4.2. Effect of number of orders*

325 As analyzed in section 2, the radiative conductivity is determined by implementing
326 an iterative procedure which takes into account the irradiation from other unit cells up
327 to the ones in contact with the boundaries. We define that the model has first-order
328 accuracy if the $(i-1)$, i , $(i+1)$, $(i+2)$ th unit cells are reserved which implies that the i th
329 cell and $(i+1)$ th cell share the face t that only accounts for the contributions from the
330 adjacent neighbouring cells($(i-1)$ th, $(i+2)$ th) in both directions. Geometrically, the

331 face t is the central face within these four cells along y direction. Thus, the bottom
332 face of the $(i-1)$ th cell and the top face of $(i+2)$ th cell are boundaries. Similarly, for
333 second-order accuracy, one more unit cell in both directions is included in the
334 calculation. For the other numbers of the orders, they can be defined in a same
335 principle. Fig.12 shows that the radiative conductivity of sample 1 varies with the
336 number of the order at two different temperatures, i.e.550K and , 750K at a solid
337 emissivity of 0.6. It reveals that the numbers of cells above and below the central face
338 need to be considered to obtain the stable values of radiative conductivity. Thus, in
339 order to stabilize the calculated values of the radiative conductivity, the number of
340 orders of 25 is used for the current model.

341 *4.3. Effect of the solid emissivity*

342 As previously mentioned, the effect of the solid emissivity on the radiative
343 conductivity needs to be addressed. Generally, the emissivity of the steel varies
344 between 0.3 and 0.8[29]. Fig.13 shows the effect of the solid emissivity on the values
345 of radiative conductivity at two temperatures of 550 K and 750 K. It is clearly seen
346 that the value of the radiative conductivity increases with increasing solid emissivity
347 even though a large emissivity can lead to a smaller reflectance. It reveals that the
348 proportion of the emission in total radiation is relatively large. In addition, the effect
349 of the solid emissivity on the radiative conductivity is significant at temperature of
350 750 K, while it is relatively mild at temperature of 550 K. The reason could be that
351 the emitting radiation is in proportion to the biquadrate of temperature. For the

352 purpose of comparison, a solid emissivity of 0.6 is assumed in present work, which is
353 consistent with the previous study of [25] and [26].

354 *4.4. Effect of temperature gradient*

355 For a fixed thickness with the same mean temperature, the effect of the temperature
356 difference on the predicted radiative conductivity at fixed temperature of 750 K is
357 shown in Fig. 14. A specific mean temperature can be determined in different
358 temperature difference between the top and bottom boundaries of the foam samples. It
359 can also be concluded from Fig. 14 that the radiative conductivity is not sensitive to
360 the temperature difference. In the current model, therefore, a 10 K temperature
361 difference is used for the iterative procedure.

362 *4.6. Effect of geometry*

363 As mentioned in Section 2.1, the shape of the solid particles can be other simple
364 geometries. For example, two shapes, such as circle, rhombus are assumed based on
365 the same porosity and characteristic size to investigate the effect of shape of solid
366 particles as seen in Fig. 15. The calculations are shown in Fig. 16 for the case of S1. It
367 can be seen that shape of the solid particles has insignificant effect on the thermal
368 radiation in the present model. It is noted that different shapes of the solid particles
369 may lead to different geometry structure for the present simplified 2D model, which
370 implies that the configuration factors may be different. However, due to the large
371 porosity of metal foam, the influence of different structures is insignificant in general.

372 Fig. 17 demonstrates the variation of radiative conductivity with the change of the
373 PPI for the same porosity of 95%. For comparison purposes, two PPI are used i.e. 30
374 and 60. Comparison shows that the radiative conductivity increases monotonously
375 with decreasing PPI at the same temperature, such a result is due to the smaller PPI
376 results in a bigger pore size. And the bigger pore size would lead to a large
377 “penetration thickness” which implies that more heat can be directly transferred by
378 thermal radiation to a deeper thickness of the foam before it decays to a lower level
379 [25].

380 **5. Conclusions**

381 A newly developed two-dimensional model is employed for the calculation of the
382 radiation heat transfer in highly porous open-cell metal foams and comparing these
383 results with available experimental data as well as three-dimensional numerical
384 solution proposed in the previous work. A correction factor, C , is introduced for the
385 correction of the deviation of the specific area between simplified two-dimensional
386 structure and three-dimensional structure. The results demonstrated that using a
387 two-dimensional analytical model instead of a three-dimensional approach leads to a
388 relatively minor discrepancy. Besides, the calculation is simpler than the
389 three-dimensional model because of the simpler determination of configuration
390 factors and coefficients due to the nature of two-dimensional structure, which is
391 significant for engineering applications. The effect of the solid emissivity on the
392 radiative conductivity is more significant at higher temperature. The radiative

393 conductivity is not sensitive to the temperature difference during the iterative
394 procedure. The effect of the shape of the solid particle is observed and it is relatively
395 small. It is found that the samples with smaller PPI could lead to a higher value of
396 radiative conductivity. In addition, the correction factor C is found to be significant
397 for the present model. Overall, the biggest advantage of the proposed
398 two-dimensional model is its simplicity and convenience of calculation with good
399 accuracy compared to the previous three-dimensional model. However, the present
400 model is only suitable for vacuum condition. Future work needs to be done to
401 investigate the thermal radiation in metal foam in atmospheric pressure. Besides,
402 more experimental data of different metal foams (material, PPI, porosity etc.) are
403 needed in order to validate the present model.

404

405 **Acknowledgements**

406

407 The authors would like to acknowledge the financial support from National Natural
408 Science Foundation of China (No. 51206164).

409 **References**

- 410 [1] C. Y. Zhao, Review on thermal transport in high porosity cellular metal foams
411 with open cells, *Int. J. Heat Mass Transfer*, 55(13-14) (2012) 3618-3632.
- 412 [2] S. Y. Kim, J. W. Paek and B. H. Kang, Flow and heat transfer correlations for
413 porous fin in a plate-fin heat exchanger, *J Heat Trans- ASME*, 122(3) (2000)
414 572-578.

- 415 [3] C. Albanakis, D. Missirlis, N. Michailidis, K. Yakinthos, A. Goulas, H. Omar, D.
416 Tsipas and B. Granier, Experimental analysis of the pressure drop and heat
417 transfer through metal foams used as volumetric receivers under concentrated
418 solar radiation, *Exp. Therm Fluid Sci.*, 33(2) (2009) 246-252.
- 419 [4] X. H. Yu, Z. Z. Wen, Y. Lin, S. T. Tu, Z. D. Wang and J. Y. Yan, Intensification
420 of biodiesel synthesis using metal foam reactors, *Fuel*, 89(11) (2010) 3450-3456.
- 421 [5] A. Bhattacharya, V. V. Calmidi and R. L. Mahajan, Thermophysical properties of
422 high porosity metal foams, *Int. J. Heat Mass Transfer*, 45(5) (2002) 1017-1031.
- 423 [6] K. Boomsma and D. Poulikakos, On the effective thermal conductivity of a
424 three-dimensionally structured fluid-saturated metal foam, *Int. J. Heat Mass*
425 *Transfer*, 44(4) (2001) 827-836.
- 426 [7] V. V. Calmidi and R. L. Mahajan, The effective thermal conductivity of high
427 porosity fibrous metal foams, *J Heat Trans-T Asme*, 121(2) (1999) 466-471.
- 428 [8] Z. Dai, K. Nawaz, Y. G. Park, J. Bock and A. M. Jacobi, Correcting and extending
429 the Boomsma-Poulikakos effective thermal conductivity model for
430 three-dimensional, fluid-saturated metal foams, *Int. Commun. Heat Mass*
431 *Transfer*, 37(6) (2010) 575-580.
- 432 [9] M. A. A. Mendes, S. Ray and D. Trimis, A simple and efficient method for the
433 evaluation of effective thermal conductivity of open-cell foam-like structures, *Int.*
434 *J. Heat Mass Transfer*, 66 (2013) 412-422.

- 435 [10] M. A. A. Mendes, S. Ray and D. Trimis, An improved model for the effective
436 thermal conductivity of open-cell porous foams, *Int. J. Heat Mass Transfer*, 75
437 (2014) 224-230.
- 438 [11] J. W. Paek, B. H. Kang, S. Y. Kim and J. M. Hyun, Effective thermal
439 conductivity and permeability of aluminum foam materials, *Int. J.*
440 *Thermophysics*, 21(2) (2000) 453-464.
- 441 [12] R. Singh and H. S. Kasana, Computational aspects of effective thermal
442 conductivity of highly porous metal foams, *Appl. Therm. Eng.*, 24(13) (2004)
443 1841-1849.
- 444 [13] B. Dietrich, G. Schell, E. C. Bucharsky, R. Oberacker, M. J. Hoffmann, W.
445 Schabel, M. Kind and H. Martin, Determination of the thermal properties of
446 ceramic sponges, *Int. J. Heat Mass Transfer*, 53(1-3) (2010) 198-205.
- 447 [14] R. Coquard and D. Baillis, Numerical investigation of conductive heat transfer in
448 high-porosity foams, *AcMat*, 57(18) (2009) 5466-5479.
- 449 [15] P. Kumar and F. Topin, Simultaneous determination of intrinsic solid phase
450 conductivity and effective thermal conductivity of Kelvin like foams, *Appl.*
451 *Therm. Eng.*, 71(1) (2014) 536-547.
- 452 [16] M. A. A. Mendes, V. Skibina, P. Talukdar, R. Wulf, U. Gross, D. Trimis and S.
453 Ray, Experimental validation of simplified conduction–radiation models for
454 evaluation of Effective Thermal Conductivity of open-cell metal foams at high
455 temperatures, *Int. J. Heat Mass Transfer*, 78 (2014) 112-120.

- 456 [17] E. Solorzano, M. Hirschmann, M. A. Rodriguez-Perez, C. Korner and J. A. de
457 Saja, Thermal conductivity of AZ91 magnesium integral foams measured by the
458 Transient Plane Source method, *MatL*, 62(24) (2008) 3960-3962.
- 459 [18] R. Wulf, M. A. A. Mendes, V. Skibina, A. Al-Zoubi, D. Trimis, S. Ray and U.
460 Gross, Experimental and numerical determination of effective thermal
461 conductivity of open cell FeCrAl-alloy metal foams, *Int. J. Thermal Sciences*, 86
462 (2014) 95-103.
- 463 [19] R. Coquard, D. Rochais and D. Baillis, Conductive and Radiative Heat Transfer
464 in Ceramic and Metal Foams at Fire Temperatures, *Fire Technol.*, 48(3) (2010)
465 699-732.
- 466 [20] R. Coquard, D. Rochais and D. Baillis, Experimental investigations of the
467 coupled conductive and radiative heat transfer in metallic/ceramic foams, *Int. J.*
468 *Heat Mass Transfer*, 52(21-22) (2009) 4907-4918.
- 469 [21] R. Coquard, D. Baillis and J. Randrianalisoa, Homogeneous phase and
470 multi-phase approaches for modeling radiative transfer in foams, *Int. J. Thermal*
471 *Sciences*, 50(9) (2011) 1648-1663.
- 472 [22] M. Tancrez and J. Taine, Direct identification of absorption and scattering
473 coefficients and phase function of a porous medium by a Monte Carlo technique,
474 *Int. J. Heat Mass Transfer*, 47(2) (2004) 373-383.

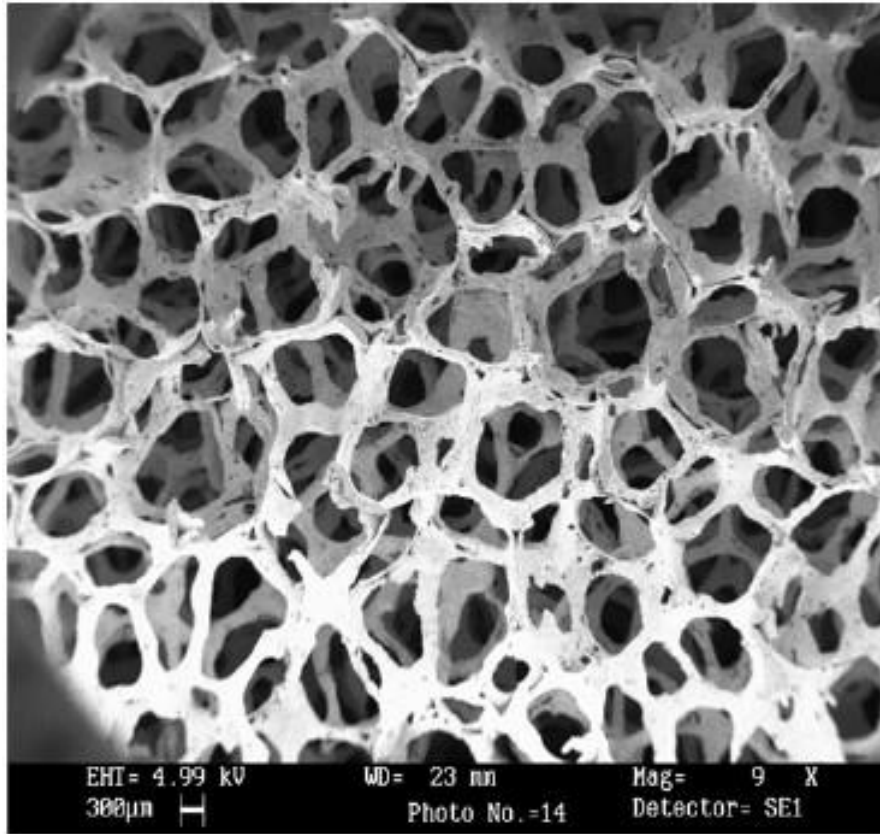
- 475 [23] C. Y. Zhao, T. J. Lu, H. P. Hodson and J. D. Jackson, The temperature
476 dependence of effective thermal conductivity of open-celled steel alloy foams,
477 *Mat Sci Eng a-Struct*, 367(1-2) (2004) 123-131.
- 478 [24] C. Y. Zhao, T. J. Lu and H. P. Hodson, Thermal radiation in ultralight metal
479 foams with open cells, *Int. J. Heat Mass Transfer*, 47(14-16) (2004) 2927-2939.
- 480 [25] C. Y. Zhao, S. A. Tassou and T. J. Lu, Analytical considerations of thermal
481 radiation in cellular metal foams with open cells, *Int. J. Heat Mass Transfer*,
482 51(3-4) (2008) 929-940.
- 483 [26] G. Contento, M. Oliviero, N. Bianco and V. Naso, Prediction of radiative heat
484 transfer in metallic foams, *Int. J. Thermal Sciences*, 76 (2014) 147-154.
- 485 [27] G. Contento, M. Oliviero, N. Bianco and V. Naso, The prediction of radiation
486 heat transfer in open cell metal foams by a model based on the Lord Kelvin
487 representation, *Int. J. Heat Mass Transfer*, 76 (2014) 499-508.
- 488 [28] V.V. Calmidi, R.L. Mahajan, Forced convection in high porosity metal foams,
489 *ASME J. Heat Transfer* 122 (2000) 557 – 565.
- 490 [29] M. L. Hunt and C. L. Tien, Effects of Thermal Dispersion on Forced-Convection
491 in Fibrous Media, *Int. J. Heat Mass Transfer*, 31(2) (1988) 301-309.

492

493

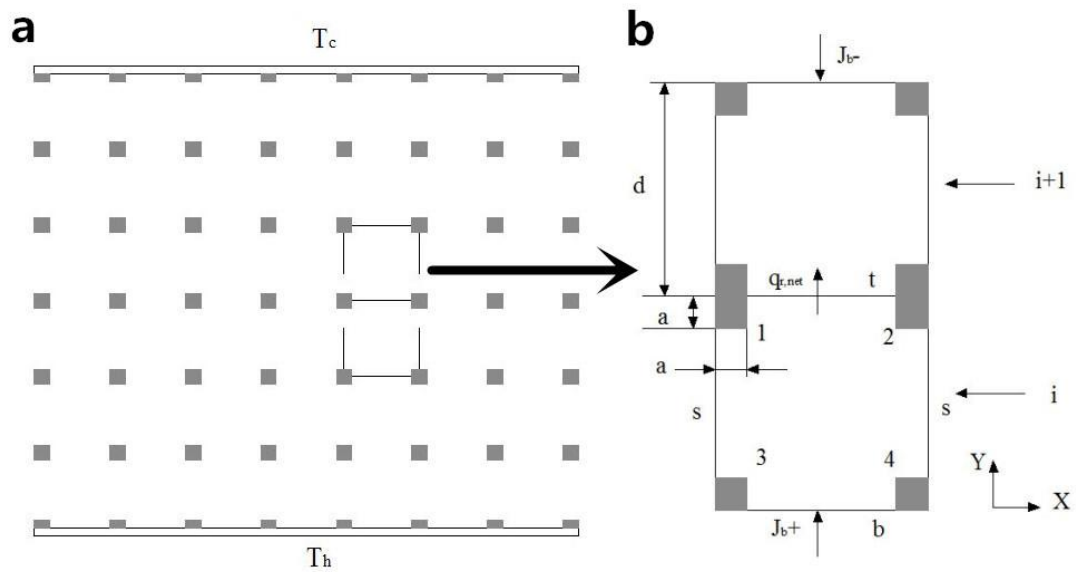
494

495



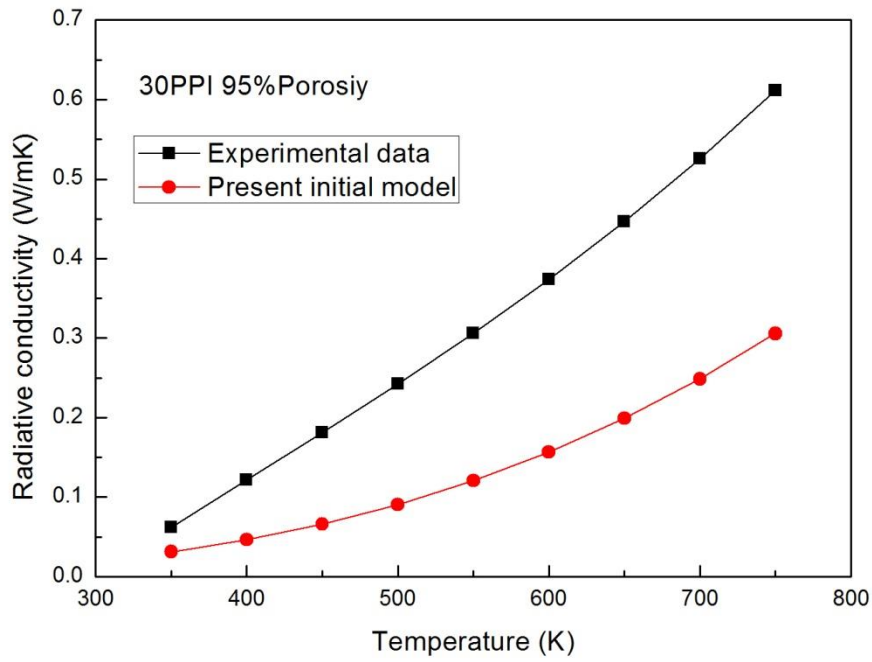
496
497

Fig. 1. Typical open-cell metallic foam morphology [25].



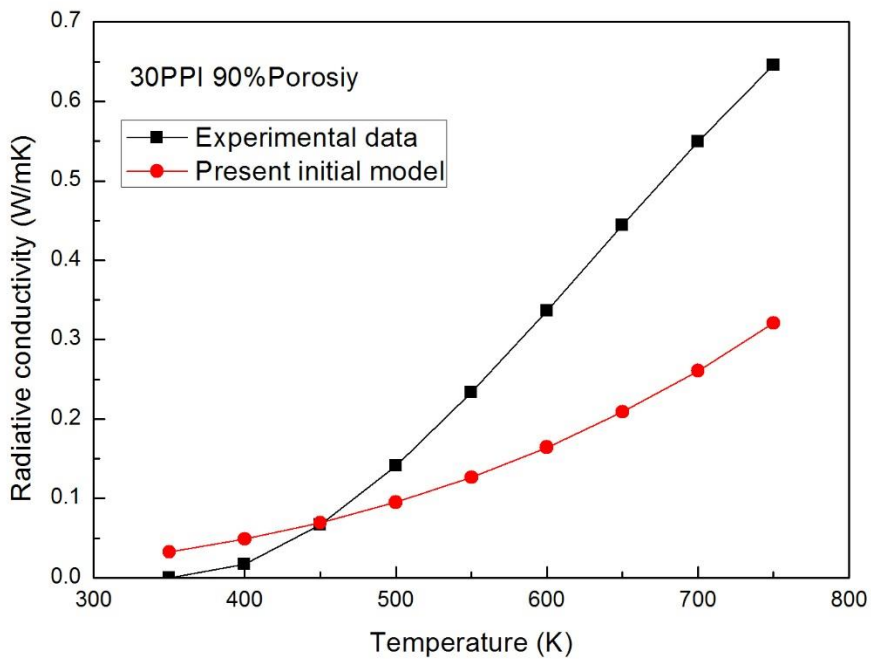
498
499
500
501

Fig. 2. (a) Two-dimensional idealized structure of porous medium; (b) Model foam structure and notations.



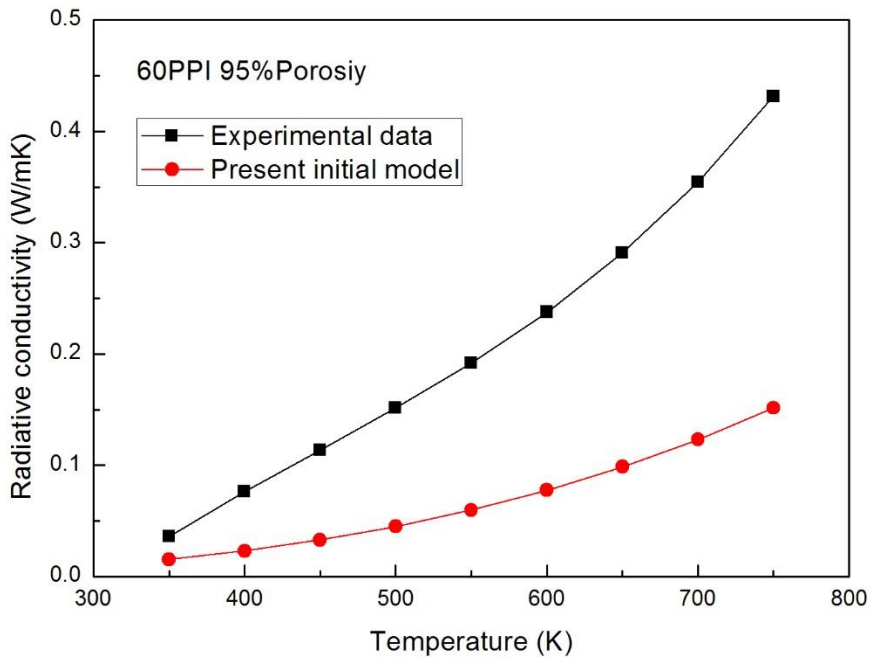
502
503
504

Fig. 3. Comparison between experimental data and predicted results of present initial model for S1.



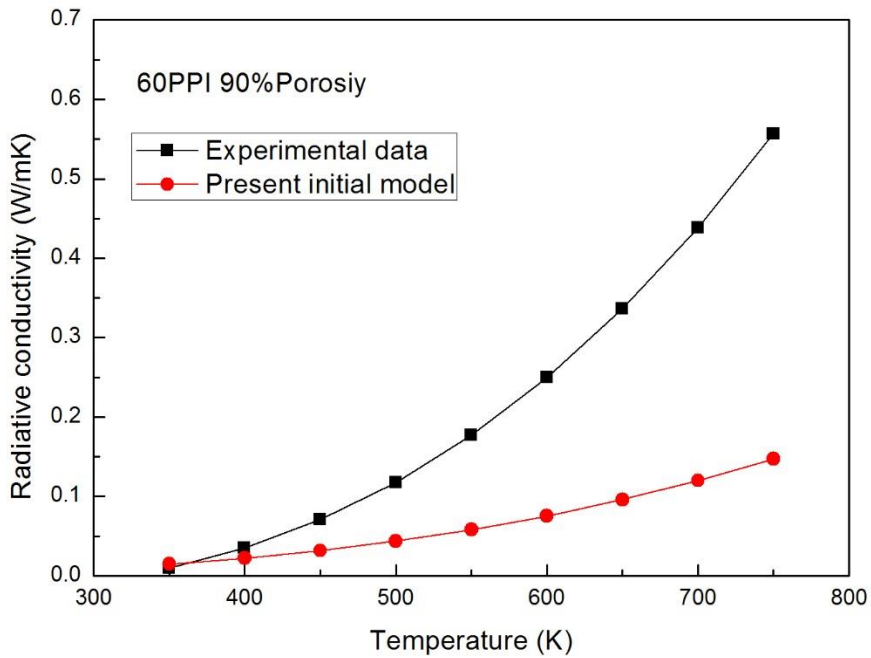
505
506
507

Fig. 4. Comparison between experimental data and predicted results of present initial model for S2.



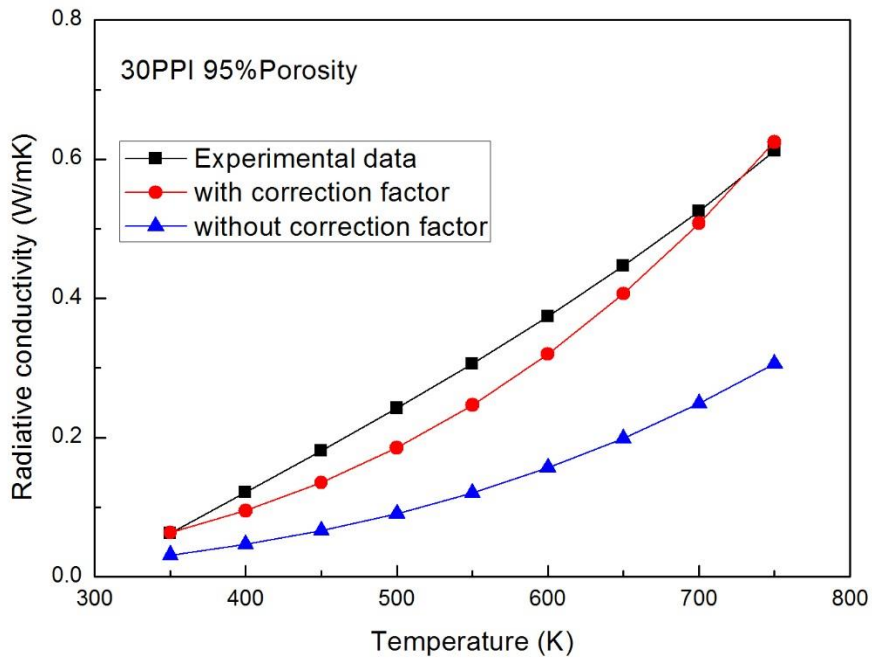
508
509
510

Fig. 5. Comparison between experimental data and predicted results of present initial model for S3.



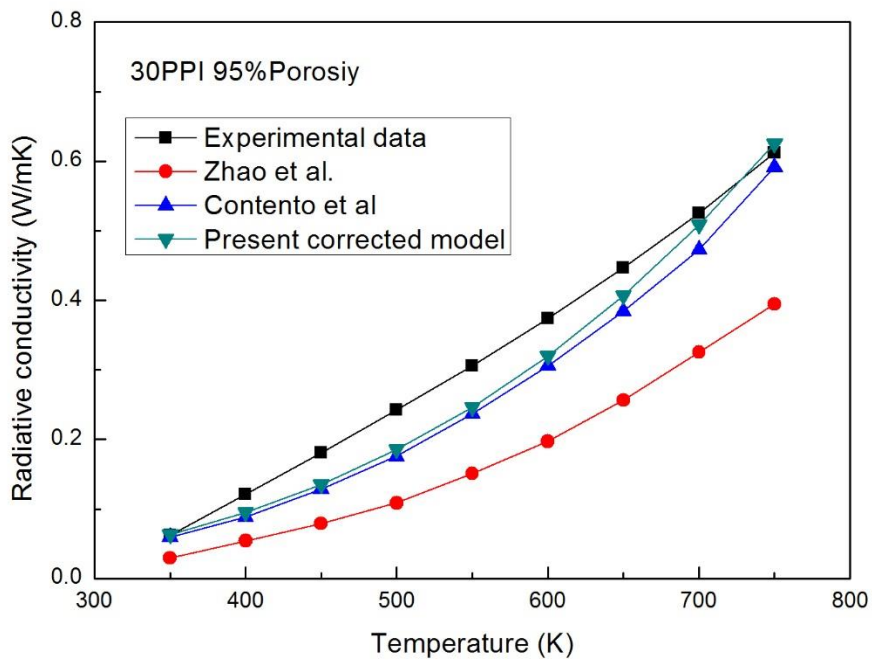
511
512
513

Fig. 6. Comparison between experimental data and predicted results of present initial model for S4.



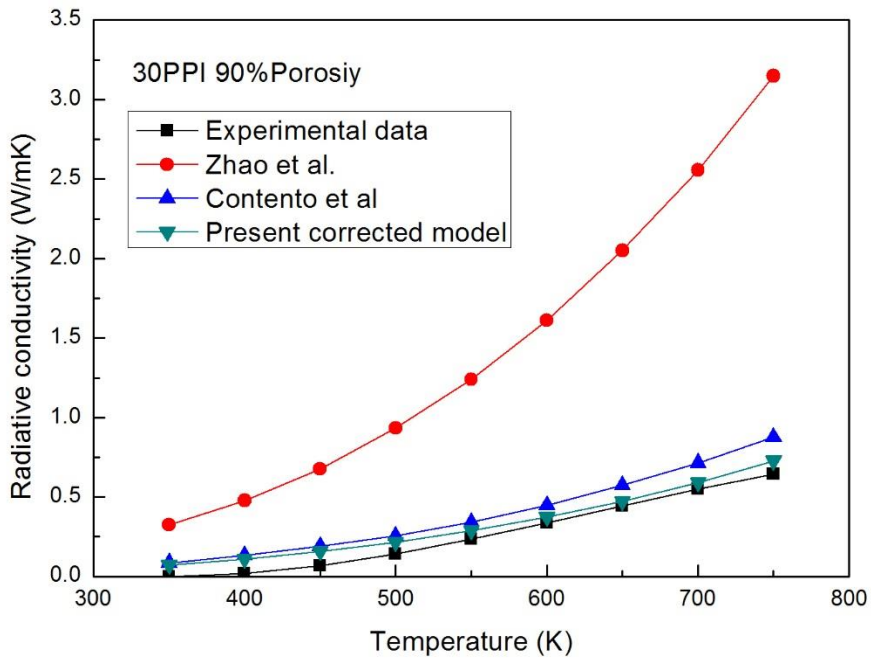
514
515

Fig. 7. Effect of correction factor on radiative conductivity for S1.



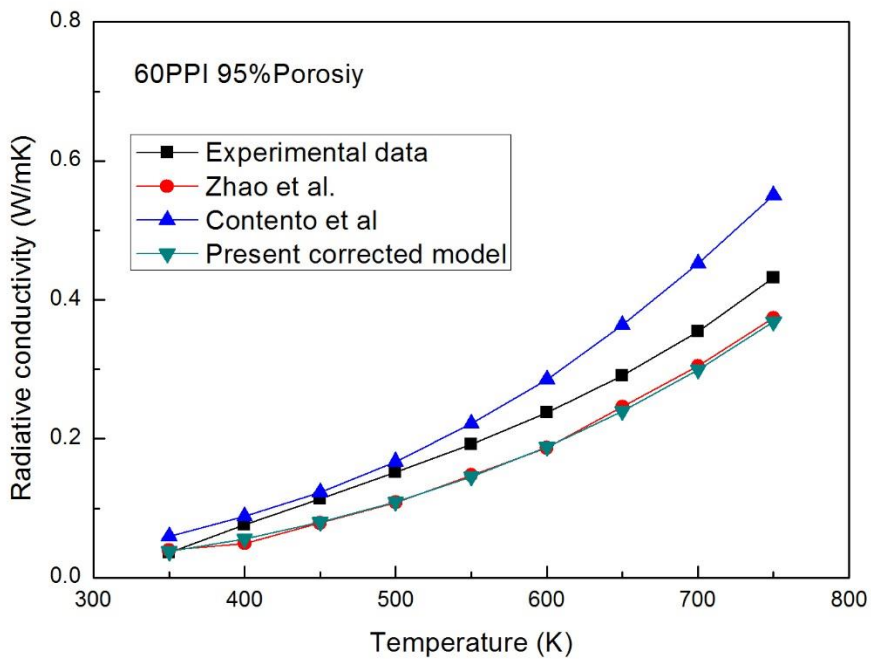
516
517
518

Fig. 8. Comparison between predicted results of present corrected model and experimental data, results of previous 3D models for S1.



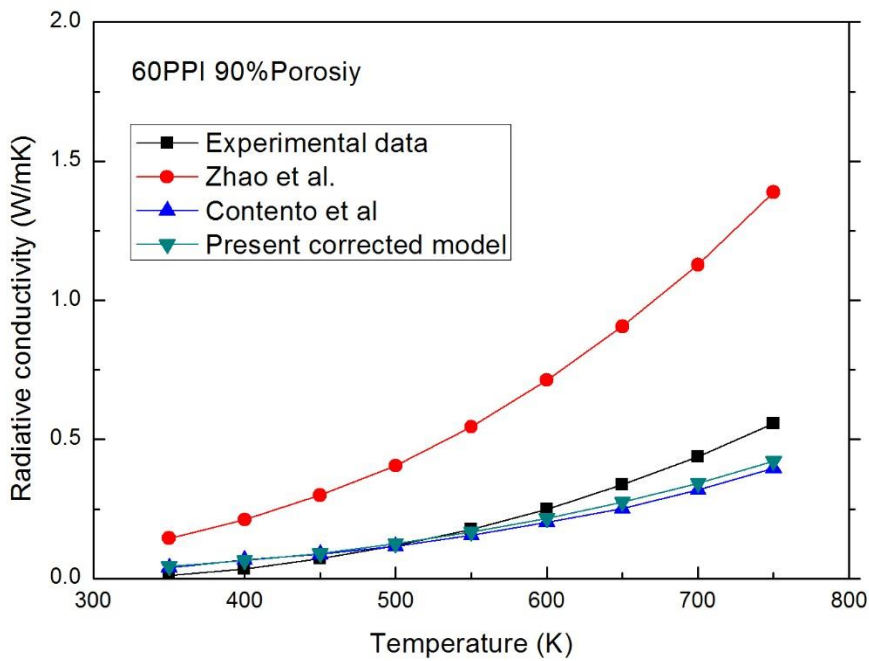
519
520
521

Fig. 9. Comparison between predicted results of present corrected model and experimental data, results of previous 3D models for S2.



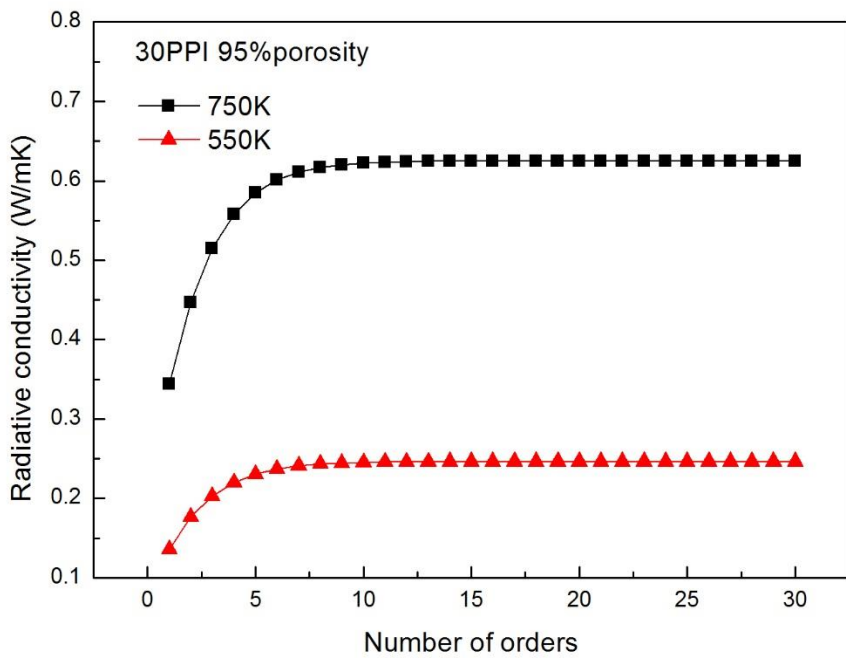
522
523
524

Fig. 10. Comparison between predicted results of present corrected model and experimental data, results of previous 3D models for S3.



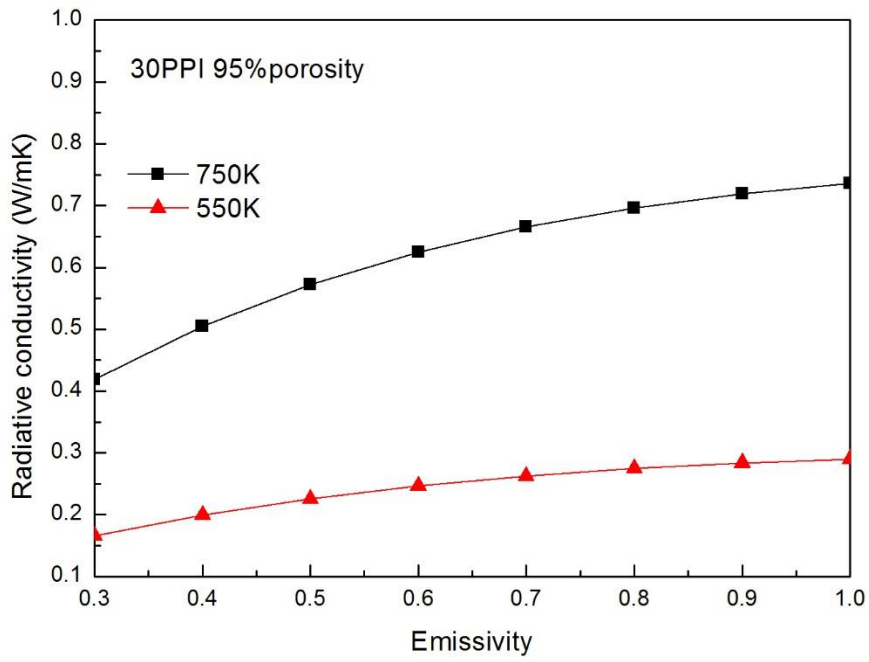
525
526
527

Fig. 11. Comparison between predicted results of present corrected model and experimental data, results of previous 3D models for S4.



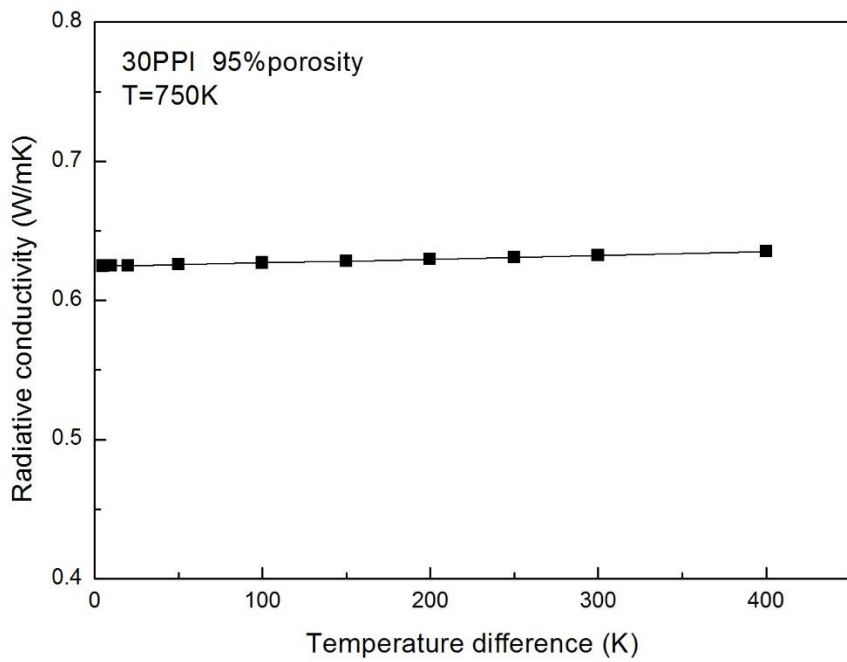
528
529
530

Fig. 12. Radiative conductivity vs. the number of orders at fixed solid emissivity of 0.6 and different temperatures for S1.



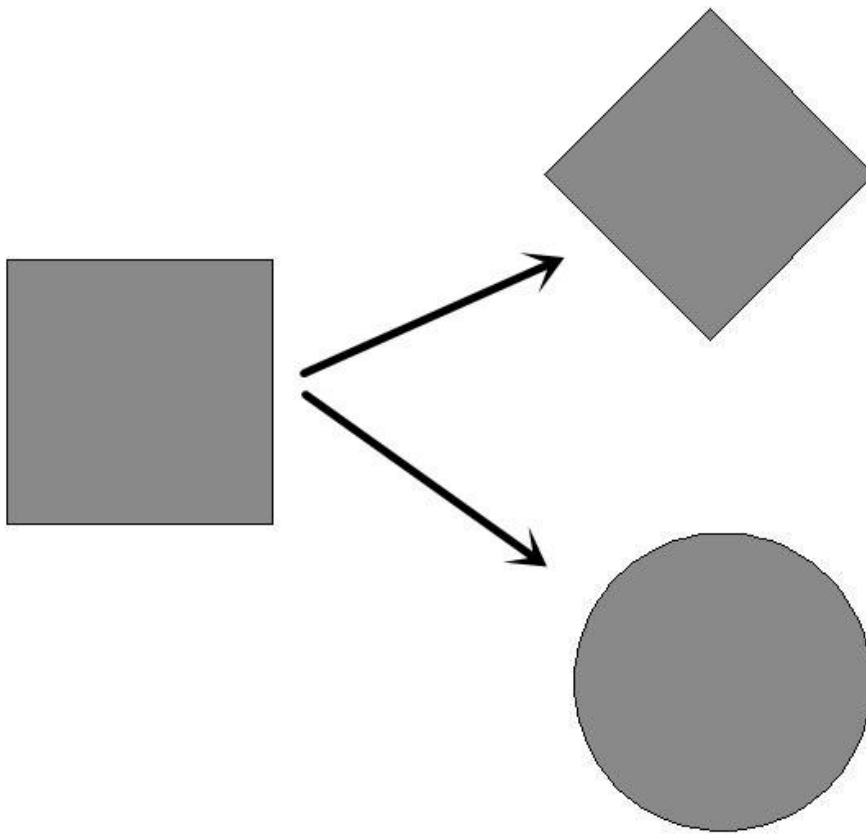
531
532

Fig. 13. Radiative conductivity vs. solid emissivity at different temperatures for S1.



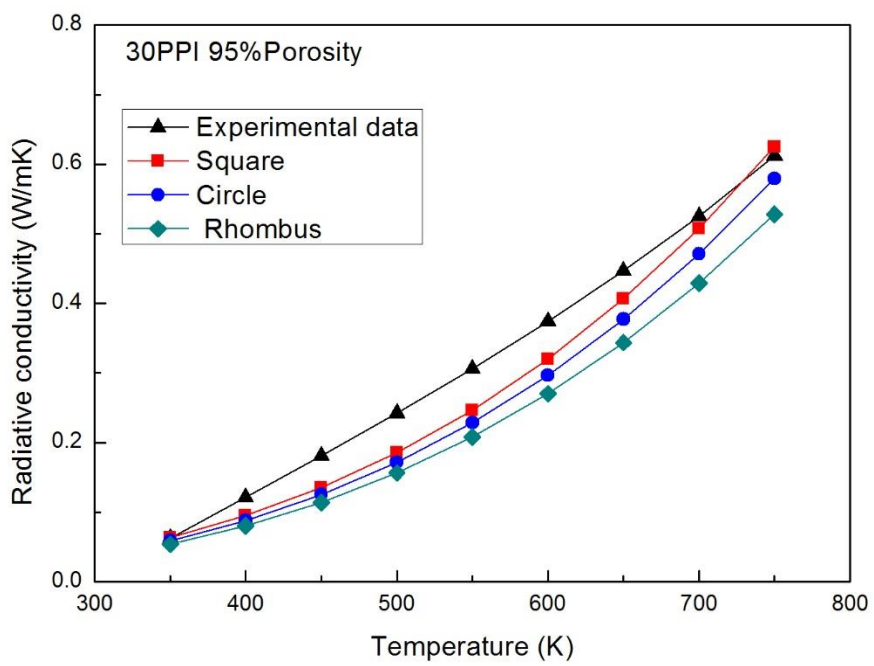
533
534
535

Fig. 14. Radiative conductivity vs. temperature difference at fixed mean temperature for S1.



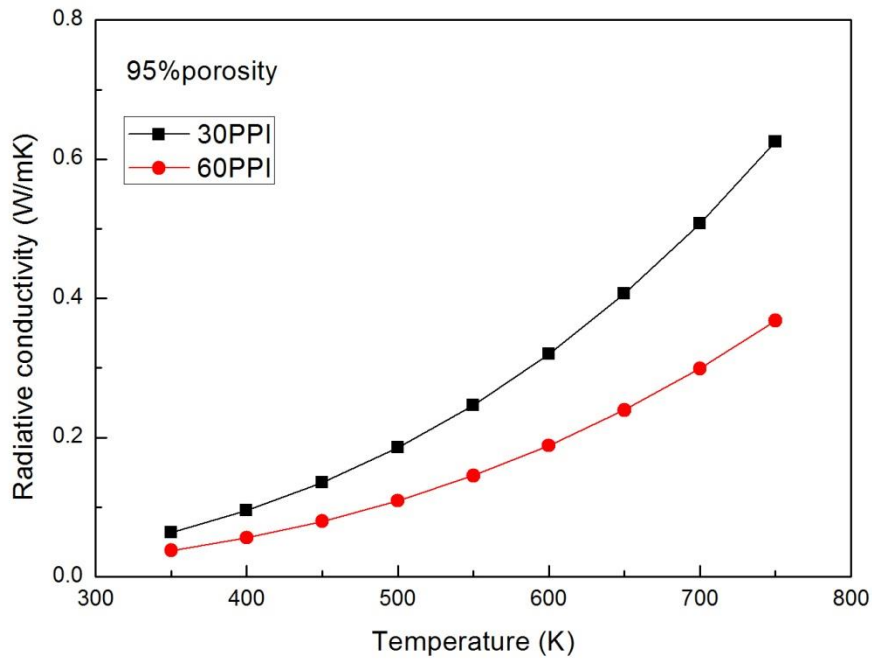
536
537

Fig. 15. Different shapes of solid particle.



538
539

Fig. 16. Effect of shape of solid particle on radiative conductivity for S1.



540

541

Fig. 17. Radiative conductivity vs. temperature at different PPI.

542

Table 1

543

Geometric properties of different foam samples [26].

544

	Sample			
	S1	S2	S3	S4
Pores per inch (PPI)	30	30	60	60
Nominal porosity (%)	95	90	95	90
Measured porosity (%)	95.9	90.7	94.5	90.8
Nominal cell size(mm)	0.847	0.847	0.423	0.423
Measured cell size(mm)	1.999	2.089	0.975	0.959
Equivalent cell size(mm)	1.772	1.851	0.864	0.850
Measured diameter of the strut(mm)	0.215	0.267	0.124	0.154

545

546

547

548

549

550

551

552

553

554

555

556

Table 2

557

Percent differences between predicted results and experimental data.

Sample	Zhao et al.'s model [25]	Contento et al.'s model [26]	Present corrected model
S1	-48.16	-17.35	-12.49
S2	485.95	63.37	35.57
S3	-19.14	23.98	-19.23
S4	205.50	-13.17	-7.07

558

559