ELectronics EXpress

IEICE Electronics Express, Vol.VV, No.NN, 1–6

LETTER

Design of Gm-C wavelet filter for on-line epileptic EEG detection

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Abstract Analog filter implementation of continuous wavelet transform is considered as a promising technique for on-line spike detection applied in wearable electroencephalogram system. This Letter proposes a novel method to construct analog wavelet base for analog wavelet filter design, in which the mathematical approximation model in frequency domain is built as an optimization problem and the genetic algorithm is used to find the global optimum resolution. Also, the Gm-C filter structure based on LC ladder simulation is employed to synthesize the obtained analog wavelet base. The Marr wavelet filter is designed as an example using SMIC 1V 0.35μm CMOS technology. Simulation results show that the proposed method can give a stable analog wavelet filter with higher approximation accuracy and excellent circuit performance, which is well suited for the design of low-frequency low-power spike detector.

key words: Wavelet transform, ambulatory electroencephalogram, spike detection, Gm-C filter, genetic algorithm

Classification: Integrated circuits

1. Introduction

Due to the characteristic of time-frequency localization, wavelet transform (WT) has been extensively used in epileptic electroencephalogram (EEG) signal processing [1–6]. Recently, WT has been applied to on-line spike detection in wearable ambulatory EEG (WAEEG) system for epilepsy diagnosis [7–9]. To realize on-line operation, analog implementation of WT has been investigated owing to the better performance in low power design required by battery powered device such as WAEEG [10–22].

So far, the popular method for implementing WT in analog domain mainly involves the rational approximation of wavelet base (i.e. analog wavelet base) and the design of bandpass filter whose impulse response is the analog wavelet base (i.e. wavelet filter). As a first step, the construction of analog wavelet base plays an important role in wavelet filter design, since low-order analog wavelet base with high approximation precision will generate accurate wavelet coefficients while achieving low power consumption. Maclaurin series approximation (MSA) presented in [7] can achieve ultra-low power dissipation and has been employed to design the wavelet filter for spike detection in WAEEG [8,19]. Compared with other approximation methods, MSA can give a simple analog wavelet base with only one term in the numerator, which can facilitate circuit design and decrease power consumption. However, MSA cannot achieve high approximation accuracy and has problem in guaranteeing approximation system stable for arbitrary filter order and time delay.

This Letter aims to propose a novel approximation method to overcome the problems with MSA, which converts the construction of analog wavelet base to an optimization problem. Then, optimization techniques can be used to find the stable analog wavelet base with high approximation accuracy at arbitrary filter order and time delay. The Marr wavelet base is used as an example, and the genetic algorithm (GA) is employed to find the optimum approximation. Finally, the Gm-C filter structure based on LC ladder simulation is used to synthesize the obtained analog wavelet base using SMIC 1V 0.35μm CMOS technology. Simulation results show that the designed Marr wavelet filter achieves high approximation accuracy with the ultra-low power consumption of 58.5pW at scale a=0.1.

2. Rational approximation of wavelet base

Assuming $\psi(t)$ is wavelet base, the continuous WT (CWT) of signal $f(t)$ at scale a and time-shift b can be defined by the convolution of $f(t)$ with a dilated wavelet [23]

$$WT_f(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t)\psi^*(\frac{t-b}{a})dt \tag{1}$$

Then, the analog implementation of CWT at scale a can be realized by analog filter whose impulse response is $\frac{1}{\sqrt{a}}\psi(\frac{-t}{a})$ [7, 12].

For biomedical signal processing, Gaussian-family wavelet bases are normally considered as the best choices. Particularly, the Marr wavelet transform has been employed in on-line spike detection for WAEEG [7, 8].

Generally, the Marr wavelet base at scale a in frequency-domain can be given as

$$\Psi_a(s) = -\pi^{\frac{1}{4}}\sqrt{\frac{8}{3}a^5}s^2 exp(a^2s^2/2) \tag{2}$$

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DOI: 10.1587/elex.16.20190560
Received September 6, 2019
Accepted November 8, 2019
Publicized November 20, 2019

Apparently, the Marr wavelet is non-causal, and cannot be implemented by analog filter directly. To make wavelet causal, a time delay t_0 should be introduced, which delays the filter output and has no change in amplitude. Then, the delayed transfer function in frequency-domain at scale a can be written as [7]

$$H(s) = \frac{-\pi^{\frac{1}{4}}\sqrt{\frac{8}{3}a^5}s^2}{e^{st_0 - a^2s^2/2}} \tag{3}$$

To synthesize by analog filter, the denominator of Eq.(3) should be approximated by a polynomial in terms of s. In [7], the MSA method is proposed to approximate the exponential term by Maclaurin series expansion, which has the rational form as

$$H_a(s) = \frac{-\pi^{\frac{1}{4}}\sqrt{\frac{8}{3}a^5}s^2}{B_n s^n + B_{n-1}s^{n-1} + \cdots + B_1 s + 1} \tag{4}$$

Experiment results show that the MSA method can give a reasonable approximation accuracy. Also, the simple rational function with only one term in the numerator can yield a compact circuit, the feature that is desired in low power implementation of spike detection [19].

Although successful in many aspects, MSA method has the shortcoming in obtaining stable analog wavelet base. The coefficients $B_i (i = 1, 2.., n)$ in Eq.(4) derived from Maclaurin series expansion are determined values at each time delay and filter order. Thus, the stability of approximated transfer function depends strongly upon the time delay and filter order. In other words, stable analog wavelet base may not exist at several time-delay values and filter orders selected according to application requirement. Meanwhile, MSA method cannot realize high approximation accuracy that may result in false spike detection.

To alleviate the difficulty encountered by MSA, this Letter proposes a novel method to find the stable rational approximation of Marr wavelet base while keeping high accuracy and simple structure as Eq.(4).

3. Construction of analog wavelet base

3.1 Mathematical approximation model

In frequency domain, the optimal approximation can be found when analog wavelet base and ideal wavelet base have the same magnitude-frequency characteristic, which means the denominators of Eq.(3) and Eq.(4) have the same magnitude along with frequency variation. Hence, the core task for constructing analog wavelet base can be considered as the polynomial fitting of the denominator in Eq.(3).

L_2-norm is used to quantify the fitting error between polynomial and exponential term in frequency domain. Then, the construction of analog wavelet base can be realized by minimizing the L_2-norm of the error function, i.e.

$$E(x) = \|abs(D(\omega)) - abs(D_a(\omega))\|_2 \tag{5}$$

in which the operator abs means magnitude, x represents the parameters B_i $(i = 1, 2.., n)$ in Eq. (4) need to be determined and

$$D(\omega) = e^{st_0 - a^2s^2/2} \tag{6}$$

$$D_a(\omega) = B_n s^n + B_{n-1}s^{n-1} + \ldots + B_1 s + 1 \tag{7}$$

Note that the numerical approach is usually used to calculate the L_2-norm error of Eq. (5). Herein, the discretized resolution is setting as frequency interval $\Delta\omega = 0.01$rad/s and frequency points N=1001 to cover the dominant frequency range.

Also, the poles of analog wavelet base should be located at the left half of s-plane to ensure the stability of wavelet filter. Thus, the rational approximation of the Marr wavelet base can be regarded as an optimization problem with respect to the parameters B_i $(i = 1, 2.., n)$ in Eq. (4), and can be written as

$$\begin{cases} min \quad E(x) = \sqrt{\sum\limits_{k=0}^{1000}[abs(D(k\Delta\omega)) - abs(D_a(k\Delta\omega))]^2} \\ s.t. \quad real(z_i) < 0, i = 1, 2, \ldots n \end{cases} \tag{8}$$

where n is the order of wavelet filter, z_i represents the zero of $D_a(\omega)$.

To find the optimal solution to above optimization problem, genetic algorithm is utilized in this Letter.

3.2 Genetic algorithm

Genetic algorithm is a heuristic optimization method which mimics the process of natural evolution. As a global search method, GA can rapidly locate the region where the global optimum exists by means of bio-inspired operators, mainly including [24, 25]:

1) Fitness function. Fitness function returns the fitness score of each individual. The individuals with high fitness score have the chance to produce new individuals as the offspring. Herein, the fitness function is defined by the approximation error as shown in Eq. (5).

2) Selection. Genetic algorithm selects individual genomes according to the fitness score calculated by fitness function. The highly fit individuals have more chances for later breeding.

3) Crossover. A pair of parents are selected from the candidate individuals, whose chromosomes are recombined to produce new off-springs. Normally, only part of the selected individuals are performed crossover according to the preset crossover probability.

4) Mutation. To maintain genetic diversity, mutation operator is usually employed to introduce changes in the chromosome, which can avoid local minima during the process of optimization.

The requirement of stability introduces a constraint to the process of optimization. To guarantee the poles stable, a penalty term is added to the fitness function of the individuals who break the constraint condition. Thus, the candidate

solutions with unstable poles have lower probability of being selected to the next generation.
To perform with random initial solution including infeasible population as required by GA, the exterior penalty function is used in this Letter. Then, the fitness function with penalty term can be expressed as

$$\begin{cases} E(x) & x \in X \\ E(x) + (gen/2)^2 \cdot max(real(z_i), 0) & x \notin X \end{cases} \tag{9}$$

where X defines the feasible range of optimal solution, i.e. $real(z_i) < 0$, gen is an increasing positive real number and equals to the number of generation in each iteration.

3.3 Design of analog Marr wavelet base using GA
The proposed method can be employed to approximate Marr wavelet base at arbitrary filter order and time delay. In this Letter, the seventh-order Marr wavelet filter at t_0=4 and a=1 is taken as the example. Then, the mathematical approximation model can be derived as

$$\begin{cases} min \quad E(x) = \sqrt{\begin{array}{l}\sum\limits_{k=0}^{1000}[abs(D(k\Delta\omega)) - abs(D_a(k\Delta\omega)]^2 \\ +(gen/2)^2 \cdot max(real(z_i), 0)\end{array}} \\ D(\omega) = e^{4s - s^2/2} \\ D_a(\omega) = B_7 s^7 + B_6 s^6 + \ldots + B_1 s + 1 \end{cases} \tag{10}$$

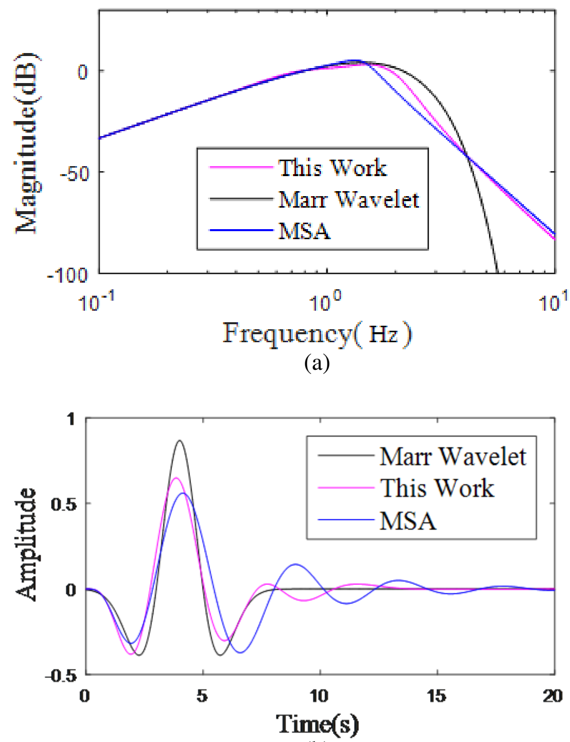
Genetic algorithm is used to find the globally-optimal solution of Eq. (10), in which selection operator is tournament, crossover and mutation operators are arithmetic with probability of 0.8 and adaptive method, respectively. The initial population size of 80 is selected experimentally based on the empirical guideline that initial population size is approximately 10 times the number of dimensions [26, 27].
After 100 iterations, the optimal solution obtained by GA is

$$\begin{array}{l} B_1 = 3.9, B_2 = 6.9, B_3 = 8.6, B_4 = 5.7 \\ B_5 = 3.6, B_6 = 1, B_7 = 0.34 \end{array} \tag{11}$$

Then, the seventh-order analog Marr wavelet base can be written as

$$H(s) = \frac{-2.1741s^2}{0.34s^7 + s^6 + 3.6s^5 + 5.7s^4 + 8.6s^3 + 6.9s^2 + 3.9s + 1} \tag{12}$$

Fig. 1 shows the approximation result of proposed method compared with MSA. As for EEG analysis, the approximation of frequency response over the range 40dB below center frequency gain is most important [28]. It can be seen from Fig. 1(a) that the proposed method has higher approximation accuracy in frequency domain than MSA, especially in the higher frequencies within the range between 0 and -50dB.
Although originated from frequency domain, the proposed approximation method also has good performance in time domain, as illustrated in Fig. 1(b). Compared with MSA, the proposed method yields a better fit in the domain of support, while vanishing faster with lower oscillation outside of the support interval, the feature that well suited for the characteristic of wavelet.
The obtained transfer function Eq. (12) has stable poles (i.e.



(a)

(b)

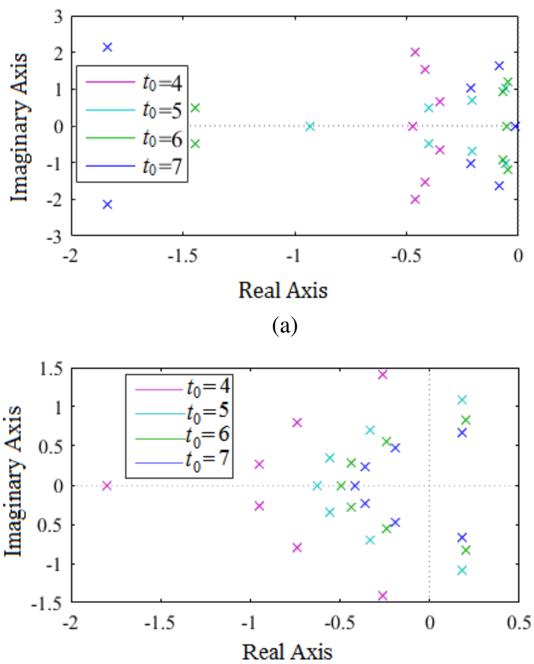
Fig. 1: Approximation of Marr wavelet in (a) frequency domain (b) time domain.

real part is negative) due to the penalty function used in constraint condition. Fig. 2 shows poles' positions of the seventh-order Marr wavelet base at different time-delay values obtained by proposed method and MSA method. Obviously, all of the poles generated by proposed method are located at the left half of s-plane, while some of the poles by MSA are not.

4. Wavelet filter design

As for low-frequency application such as EEG analysis, Gm-C filter is the most popular technique since its center frequency can reach a few hertz by using Gm cell with low transconductance. Herein, to compare with MSA, the Gm-C filter structure based on LC ladder simulation proposed in [19] is used to synthesize analog wavelet base.

4.1 Filter structure and synthesis
Doubly resistively terminated LC ladder filter structure has the very low sensitivity to inexact component value, which can facilitate future chip fabrication [29]. Fig. 3 gives the Gm-C filter structure derived from doubly terminated LC ladder [19]. One can realize wavelet filter at any scale by denormalizing Eq. (12) to related centre frequency. Hereby,



(b)

Fig. 2: Poles map for Marr wavelet base at different time delays obtained by (a) proposed method (b) MSA method.

centre frequency f_0=2.1Hz (i.e. a=0.1) used in spike detection is selected. Then, using coefficient matching [19], the design formulas for the parameters in Fig. 3 can be determined as,

$$
\begin{cases}
3.4\times10^{-8} = C_1C_2C_3C_4C_{L1}C_{L2}C_{L3}/g_m^7 \\
1\times10^{-6} = (C_1C_2C_3C_{L1}C_{L2}C_{L3} + C_2C_3C_4C_{L1}C_{L2}C_{L3})/g_m^6 \\
3.6\times10^{-5} = (C_2C_3C_{L1}C_{L2}C_{L3} + C_2C_3C_4C_{L2}C_{L3} \\
\quad +C_1C_2C_4C_{L1}C_{L3} + C_1C_2C_4C_{L1}C_{L2} \\
\quad +C_1C_3C_4C_{L1}C_{L3} + C_1C_2C_3C_{L1}C_{L2} \\
\quad +C_2C_3C_4C_{L1}C_{L3})/g_m^5 \\
5.7\times10^{-4} = (C_1C_2C_{L1}C_{L3} + C_2C_4C_{L1}C_{L3} \\
\quad +C_1C_3C_{L1}C_{L3} + C_2C_3C_{L1}C_{L3} \\
\quad +C_3C_4C_{L1}C_{L3} + C_2C_3C_{L2}C_{L3} \\
\quad +C_2C_3C_{L1}C_{L2} + C_1C_2C_{L1}C_{L2} \\
\quad +C_2C_4C_{L1}C_{L2})/g_m^4 \\
0.0086 = (C_2C_4C_{L2} + C_2C_3C_{L2} + C_2C_{L1}C_{L2} \\
\quad +C_2C_3C_{L1} + C_1C_2C_{L1} + C_3C_{L1}C_{L3} \\
\quad +C_2C_{L1}C_{L3} + C_1C_3C_{L1} + C_2C_4C_{L1} \\
\quad +C_3C_4C_{L3} + C_1C_4C_{L1} + C_2C_{L3}C_4)/g_m^3 \\
0.069 = (2C_2C_{L1} + C_2C_{L2} + C_4C_{L1} + C_1C_{L1} + C_3C_{L3} \\
\quad +C_3C_{L1} + C_2C_{L3})/g_m^2 \\
0.39 = (C_2 + C_3 + C_4 + C_{L1})/g_m
\end{cases}
\tag{13}
$$

It is worth noting that coefficient matching in the numerator of Eq. (12) is not considered in Eq. (13), since mismatch of numerator coefficient does not affect spike detection result. Numerator coefficient can be viewed as the filter gain which only changes the absolute amplitude, not the relative amplitude of wavelet transform coefficient that usually used in threshold-dependent spike detection algorithm.

To improve transconductors matching and facilitate design automation, choose 100pS for the transconductance of Gm cells in Fig. 3. Obviously, solving simultaneous equations of Eq. (13) is a nontrivial, often difficult task. Herein, the solving process is converted to an optimization problem, which minimize the sum of the residuals' square between both sides of the seven simultaneous equations in Eq. (13). Genetic algorithm clarified in Section 3 is used to find the optimum solution to capacitance values, which is given as

$$
\begin{aligned}
&C_1 = 8.07\text{pF}, C_2 = 5.60\text{pF}, C_3 = 14.64\text{pF}, C_4 = 5.88\text{pF} \\
&C_{L1} = 13.18\text{pF}, C_{L2} = 14.35\text{pF}, C_{L3} = 4.88\text{pF}
\end{aligned}
\tag{14}
$$

4.2 Gm cell with low transconductance

To implement 100ps transconductance, the Gm cell realized by simple differential pair circuit is employed, as illustrated in Fig. 4.

The Gm cell is operated in deep weak inversion, thus the drain current of M_1 and M_2 are expressed as

$$I_{ds1} = I_{S0}exp(\frac{V_{GS1}-V_{TH}}{nU_T}) \tag{15}$$

$$I_{ds2} = I_{S0}exp(\frac{V_{GS2}-V_{TH}}{nU_T}) \tag{16}$$

where I_{S0} is the specific current [30].

The relationship between I_{ds1} and I_{ds2} can be given as

$$I_{ds1} + I_{ds2} = I_{bias} \tag{17}$$

$$I_{ds1} - I_{ds2} = I_{out} \tag{18}$$

Based on Eq. (15)~Eq. (18), one can deduce that

$$V_{in} = V_{GS1} - V_{GS2} = nU_T ln(\frac{I_{out}+I_{bias}}{I_{out}-I_{bias}}) \tag{19}$$

Then,

$$I_{out} = I_{bias} tanh(\frac{V_{in}}{2nU_T}) \tag{20}$$

By using Maclaurin expansion, Eq. (20) can be written as

$$I_{out} \approx \frac{I_{bias}V_{in}}{2nU_T} \tag{21}$$

Then, the transconductance can be deduced as

$$g_m = \frac{I_{bias}}{2nU_T} \tag{22}$$

For the used CMOS process, n and U_t are 1.25 and 26mV, respectively. Thus, to realize the transconductance of 100pS, $I_{bias}$is calculated as 6.5pA.

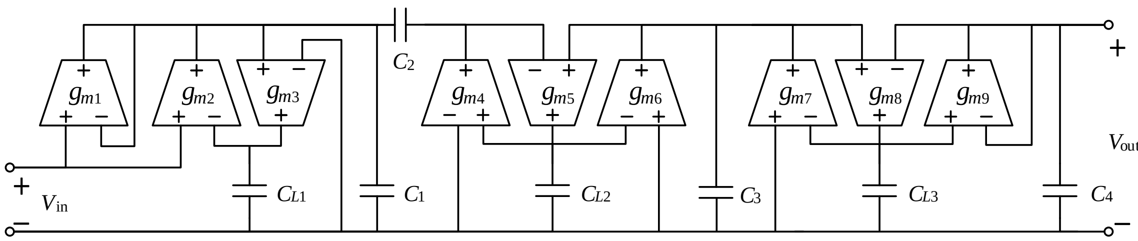


Fig. 3: Gm-C filter structure based on LC ladder simulation for synthesizing the 7th order analog wavelet base.

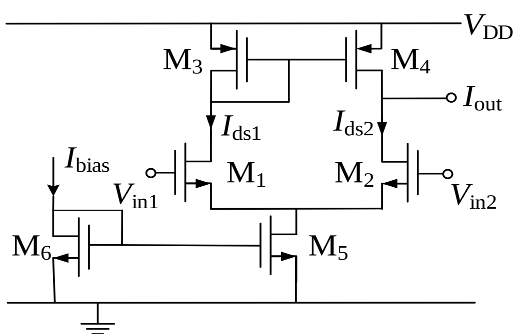


Fig. 4: Circuit structure of Gm cell.

5. Simulation result

The wavelet filter is designed and simulated using standard SMIC 1V 0.35μm CMOS process model. Fig. 5 gives the frequency and impulse response of designed Marr wavelet filter, respectively. The centre frequency of 2.1Hz (i.e. a=0.1) has been achieved at I_{bias}=6.5pA, where the power consumption is only 58.5pW, a figure that can be negligible compared with other blocks in WAEEG system. The bias circuit is generated off-chip, and thus the power consumption of bias circuit is not included. Simulation results show that the dynamic range is about 42.5dB, which meets the requirement of EEG analysis. Table I summarizes the detailed performance.
By adjusting the transconductance value of Gm cells, the wavelet filter at different scales can be implemented. Fig. 6 illustrates the frequency response of designed wavelet filter at dyadic scales covering EEG bandwidth, i.e. 1-64 Hz. Obviously, the presented approach can implement wavelet transform coefficients at different scales conveniently with higher accuracy and excellent circuit performance required by WAEEG system.

6. Conclusion

This Letter has presented a novel method for designing analog wavelet filter used in on-line spike detection. Compared with existing methods, the proposed approach has some advantages as below:
1) Different from other approximations with complex structure, the proposed analog wavelet base has simple structure with only one term in the numerator. Thus, the proposed analog wavelet base is suitable for the synthesis by LC ladder filter structure, which has very low sensitivity to component variation, and well suits for future fabrication of spike detector in the IC process.
2) Different from MSA, the construction of analog wavelet base in this Letter is modeled as an optimization problem. Then, optimization technique can be used to find the optimal approximation. Hence, the proposed method gives a more flexible way than MSA in searching the candidate so-

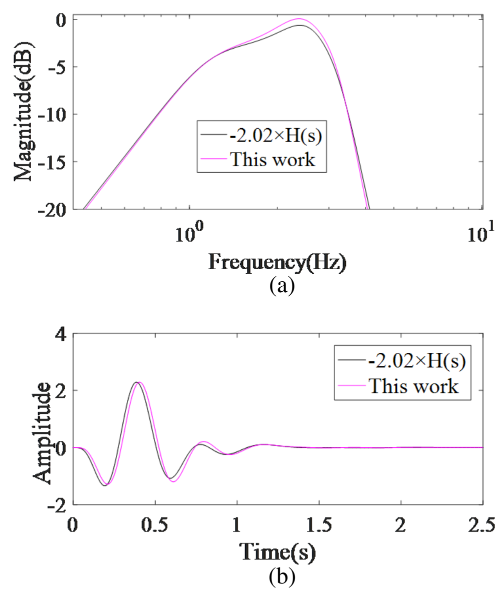


Fig. 5: Simulated response of Marr wavelet filter (a) frequency response (b) impulse response.

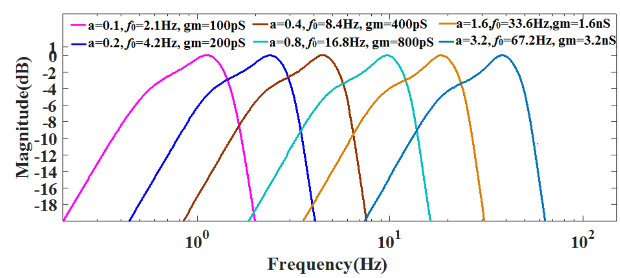


Fig. 6: Simulated frequency response of Marr wavelet filter at six scales covering 1-64 Hz EEG bandwidth.

Table I: Summary of simulated response for designed wavelet filter

Parameter	Simulated performance
Power supply	1V
Signal input range	Up to 20m Vpp
SNR	Up to 42.4dB
Input referred noise (integrated over 1.5-3 Hz passband)	53.3μ Vrms
THD(2Hz,20mVpp input)	0.29%
IMD3(2, 2.1Hz inputs , 20mVpp total)	-29.8dBc
Dynamic range	42.5dB
Power Consumption	58.5pW
FOM	0.94×10^{-13}

lutions to achieve minimum approximation error. Also, the introduction of penalty function can eliminate the unstable solutions from the candidates, and thus guarantee the analog wavelet base stable. Experiment results show that the proposed method can yield a stable analog wavelet base with higher approximation accuracy, which will enhance the performance of on-line epileptic EEG detection.

Acknowledgments

This work was supported by the Fundamental Research Funds for the Central Universities (Grant No. 2018JBM005).

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